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Micromechanical Model for Simulating Hydraulic Fractures of Rock

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Abstract. A numerical model is developed to study hydraulic fracturing in permeable and heterogeneous rocks, coupling with the flow and failure process. The effects of flow and in-situ stress ratio on fracture, material homogeneity and breakdown pressure are specifically studied.

Introduction

Hydraulic fracturing has been widely used in determining in-situ stresses in rock masses and stimulating reservoir production. Understanding hydraulic fracture mechanisms and, then, finding ways to predict the geometry of the hydraulically induced fracture and the initiation pressure are important both for stress measurements and for improving well production. There are different theories for hydraulic fracturing, the oldest one and the most frequently used one being that proposed by Hubbert and Willis [1]. Although Haimson [2] improved this theory by taking into account the effect of fluid penetration. These improvements, according to Zoback et al. [3], were not sufficient for explaining the hydraulic fracturing results published by Haimson and Fairhurst [4] and others [5]. Among many of the theories in connection with hydraulic fracturing, the one based on Linear Elastic Fracture Mechanics (LEFM) was widely used [6-9]. This theory considers a pressurized hole in an infinite space and the existence of only two symmetric radial cracks. Based on this LEFM approach, Degue and Ladanyi [5] proposed a new theory able to take into account the effect of fluid penetration and the pressurization rate.

There are at least two drawbacks in most of the hydraulic fracture theories. First, the materials studied in most of the hydraulic fracture models are assumed to be impermeable. Therefore, the theories cannot explain the effect of fluid permeability on the hydraulic fracture behavior. Generally speaking, in the case of an impermeable rock, the influence of permeability on fracture behavior may be ignored. The pressure - time curve is quite sufficient for determining the breakdown pressure [10]. Fracture initiation is characterized by a sharp peak which is then followed by a pressure drop. This peak is obviously related to unstable fracture propagation. However, as pointed out by Charlez [10], in the case of permeable rocks, the fracturing fluid percolating into the rock, equilibrium between well and formation is continuously maintained. As a result, fracture initiation is often stable. Even for rocks with low permeability, the fluid flow behavior has to be taken into account when it is loaded to failure considerably. Experimental results for rock samples with low permeability show that there is no drastic change in permeability as the loading stress increases within the elastic deformation range. However, significant permeability changes occur if there is so-called plastic deformation (more micro-fractures appear in this stage) [11]. Although there has been a great deal of interest in the last ten years in the coupling of fluid flow and geomechanical deformation processes in a single model where the dependence of flow and deformation (stress) on each other can be modeled simultaneously, most hydraulic fracturing models remain uncoupled. However, in a large number of hydraulic fracturing problems there is a strong interaction between deformation (stress) and fluid flow. Therefore, standard modeling of these various processes without considering the interaction effect can lead to significant errors in many cases [11,12].

Secondly, the influence of heterogeneity existed in rock on the fracture pattern or hydraulic fracture path cannot be taken into account in most of the existing flow-coupled models. It is well known that rock is a heterogeneous geological material containing many natural weaknesses, such as pores, grain boundaries, and pre-existing cracks. When rock is subjected to hydraulic loading, these pre-existing defects can induce crack or fracture growth, which can in turn change the structure of the rock and alter the fluid flow properties of the rock [13]. This heterogeneity related flow properties could influence the hydraulic fracturing behaviour in many ways. For example, in the rock immediately around the borehole or the initiated fracture tips, micro-cracks or micro-fractures may initiate and grow. Consequently, a highly permeable damage zone or crack propagation zone is created around the fracture tip.

Due to the difficulty in obtaining a complete analytical solution for hydraulic fracturing problems, numerical simulation methods are widely used. Sophisticated hydraulic fracturing simulators, which can model the fracturing process numerically either in two dimension or three dimension, have been developed to optimize the benefit of the hydraulic fracturing treatment. In these models, the influence of the permeability of rock on the fracture propagation is assumed to be practically negligible.

In this paper, a numerical model, which couples the flow and the failure process, is developed to study hydraulic fracturing in permeable and heterogeneous rocks. Analyses have been performed to simulate the hydraulic fracturing in rocks with various degrees of heterogeneity. The effects of the in-situ stress ratio and the non-uniform stress field are also investigated. The results indicate that both the rock heterogeneity and the permeability affect fracture initiation and propagation significantly, and that the simplistic premise that rock is homogeneous and impermeable may apply to limited, but not general cases in hydraulic fracturing.

Theory

Based on the general observations, a model of coupling between flow, stress and damage is proposed. The formulation of the model is based on the following assumptions :

- The rock mass is fully saturated.
- The flow of the fluid is governed by the Biot's consolidation theory [14].
- The rock is assumed to be brittle-elastic material with residual strength, and its loading and unloading behaviour are described by elastic damage mechanics.
- An element is considered to have failed in the tension mode when its minimum principal stress exceeds the tensile strength of the element, and to have failed in the compression-shear mode when the shear stress satisfies the strength criterion defined by the Mohr Coulomb failure envelope.
- The permeability varies as function of the stress states in elastic deformation, and increases dramatically when the element fails.
- The local heterogeneity in the properties of rock masses is defined by Weibull function, that is

$$\varphi(s,m)=\frac{m}{s_o}\left(\frac{s}{s_o}\right)^{m-1}\exp\left[-\left(\frac{s}{s_o}\right)^{m}\right] \tag{1}$$

where s is the element strength f or elastic modulus E, and s_0 is the mean strength f0 or elastic modulus of elements E0. The parameter m is defined as homogeneity index. Figure 1 shows the variations of φ with respect to m and it obvious that a higher m value represents a more homogeneous material.

Basic equations. Coupled seepage and stress processes in saturated geological media could be interpreted with Biot's theory of consolidation [14]. By extending Biot's theory of consolidation to include stress effects to permeability, the following governing equations could be obtained [14]:
Balance equation:

$$\frac{\partial \sigma_{ij}}{\partial x_{ij}} + \rho X_j = 0 \quad (i, j = 1,2,3) \tag{2}$$

Geometrical equation:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$
$$\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \tag{3}$$

Constitutive equation:

$$\sigma'_{ij} = \sigma_{ij} - \alpha p \delta_j = \lambda \delta_j \varepsilon_v + 2G\varepsilon_j \tag{4}$$

Seepage equation:

$$k\nabla^2 p = 0 \tag{5}$$

Coupling equation:

$$k(\sigma, p) = \xi k_o e^{-\beta(\sigma_{ii}/3 - \alpha p)} \tag{6}$$

where σ_1 and σ_3 are the major and minor principal stresses. ϕ is the friction angle and fc is the compressive strength. k is the permeability coefficient, k0 is the permeability coefficient when the stress is zero,ξis the damage factor of permeability, which reflects the increase in permeability induced by damage, p is the pore-fluid pressure, α is the Biot's constants, β is the coupling parameter that reflects the influence of stress on coefficient of permeability.

The above Equations (1) to (5) are from the Biot's theory of consolidation [14]. An additional equation (6) was introduced to represent the influence of stress on permeability. Furthermore, experimental results show that the permeability cannot be a constant but a function of stresses since the fracture aperture is most likely to change when the stress conditions vary. Based on this observation, various permeability-stress relationships have been established [11], and the relationship between permeability and stress is assumed to follow a negative exponential function. The model was loaded in a quasi-static fashion and a flow chart outlining the pertinent steps of the analysis is given in Figure 2. At each loading increment, the seepage and stress equation in the elements were solved and the coupling analysis was performed. The stress field for each element was then examined. According to the stress level, the elements could be classified into four phases:

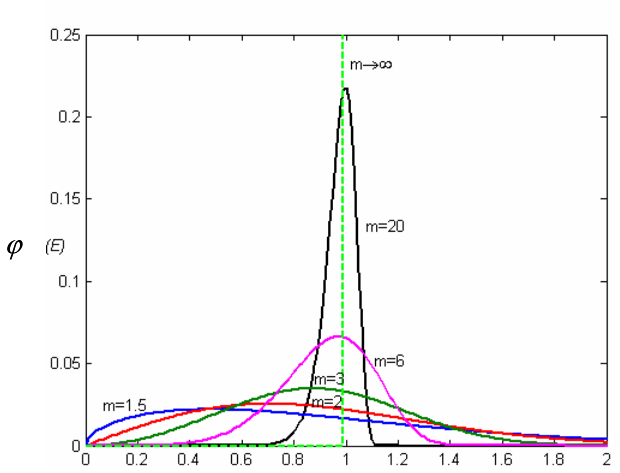


Figure 1: Weibull's distribution for rock specimens with different homogeneous

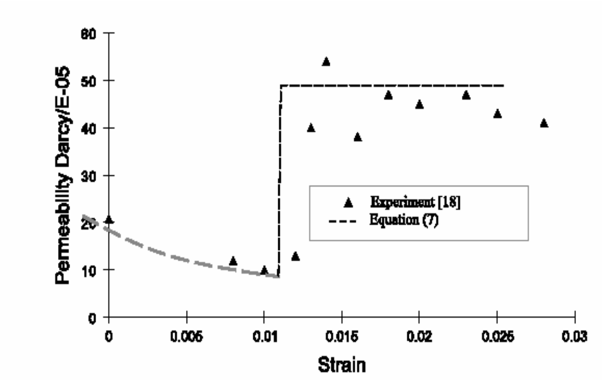


Figure 3: comparison of the permeability-strain experimental obtained by Yang [18] and equation (6)

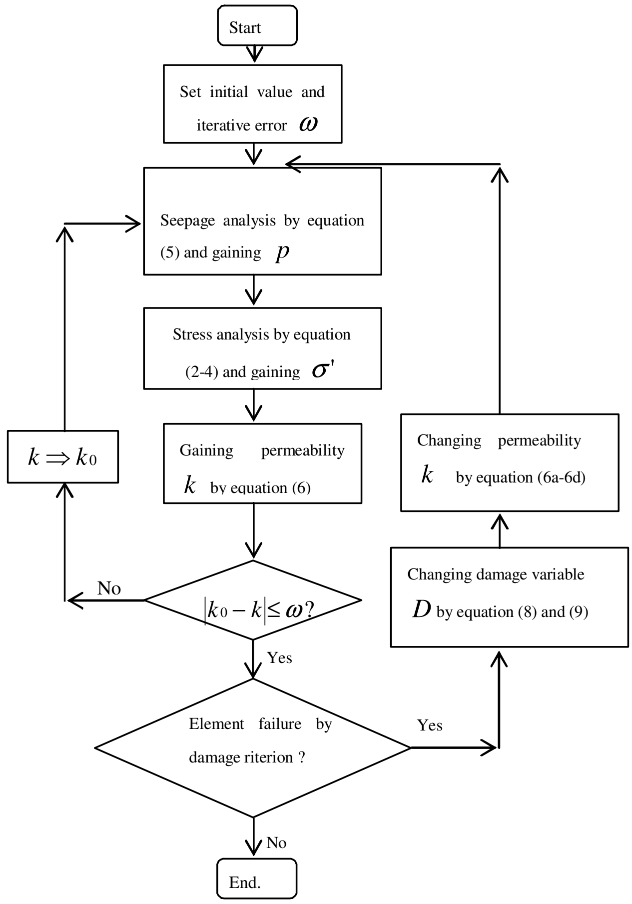


Figure 2: The flow chart of F-RFPA2D Program under uniaxial compressive stress and tensile stress

(1) Elastic phase : If $\sigma_1 - \sigma_3 \dfrac{1+\sin\phi}{1-\sin\phi} < f_c$ or $\sigma_3 > -f_t$, the element is in the elastic phase and Eo and ν are the elastic modulus as well as the Poisson ratio of the element, respectively. As the permeability will decrease with the stress, it is assumed that α = 0 and ξ = 1. Therefore, the permeability is given by

$$k(\sigma, p) = k_o e^{-\beta(\sigma_{ii}/3)} \tag{6a}$$

A comparison of the experimental results shows that Equation (6a) can approximate the change in permeability fairly well (Figure 4). Finally, the authors would like to emphasize that the variation of permeability during damage and cracked phase is most important and meaningful for hydraulic fracturing process in rocks when equation (6) was employed.

(2) Damage Phase : If $\sigma_1 - \sigma_3 \dfrac{1+\sin\phi}{1-\sin\phi} \ge f_c$, the element is in the damage phase and the failure is due to compression-shear. The elasticity modulus of the element will decrease according to the constitutive law for brittle failure. Mathematically, the modulus can be written as

$$E = (1 - D)E_o \tag{7}$$

where D represents the damage variable. E is the elastic modulus of the damaged elements. The damage variable can be defined as :

$$D = 1 - \frac{f_{cr}}{E_o \varepsilon} \tag{8}$$

where fcr is the residual compressive strength.

On the other hand, its permeability will undoubtedly increase as fractures begin to form and develop. To reflect the increase in permeability, ξ is taken to be 5 and the permeability is computed by

$$k(\sigma, p) = 5k_o e^{-\beta(\sigma_{ii}/3)} \tag{6b}$$

The variation in permeability predicted by Equation (6b) is also in good agreement with the experimental results (Figure 4).

If $\sigma_3 \le -f_t$, the element fails in tensile failure mode. The elastic modulus can still be given by Equation (7) but the damage variable has to be defined in terms of the residual tensile strength ftr:

$$D = 1 - \frac{f_{tr}}{E_o \varepsilon} \tag{9}$$

As the change in permeability should be independent on the mode of failure, it is assumed that the change in permeability after damage can also be given by Equation (6b).

(3) Cracked phase : In the cracked phase, macro fractures begin to form and the element will loss its capacity and stiffness. Therefore, the elastic modulus is assigned a very small value. Due to the existence of macro fractures [15], the permeability will increase significantly and it can be obtained by assuming ξ=100 and α=1, that is

$$k(\sigma, p) = 100k_o e^{-\beta(\sigma_{ii}/3 - p)} \tag{6c}$$

(4) Closed-crack phase : When the compressive strain of the crack element ε is greater than ε_{cu}, the crack can be considered to be closed. The element can transfer stress again and its modulus will increase as the compressive strain increase. The modulus can be computed by

$$E = \frac{f_{cr}}{\varepsilon_{cu}} \times \frac{\varepsilon}{\varepsilon_{cu}} \tag{10}$$

To compute the permeability in this phase[15], we can assume that ξ=0.01 and α=0, that is

$$k(\sigma, p) = 0.01k_o e^{-\beta(\sigma_{ii}/3)} \tag{6d}$$

As the elements may change from one phase to the other, an iteration process is required to modify the elastic modulus and permeability by taking into account such changes. If there is a change in phase for an element, the elastic modulus and the permeability coefficient of the element will be adjusted accordingly and re-analyze will be carried out (Figure 2). The iteration will continue until there are no changes in phase at an equilibrium strain field. When above equations are extended to three dimensional cases, the intensity and residual intensity of the element increases proportionally according to the Hookean model.

Numerical Model

A benchmark case of fracturing of an elastic rock stratum by applying pressure in a borehole has been analyzed by using the present model (Figure 4). The horizontal, σ_k, and vertical, σ_v, in-situ stresses are 1.0MPa and 2.0MPa, respectively. After drilling a hole, a pressure of 1 MPa is applied in the hole. The homogeneous index is chosen to be 1000, that is the rock stratum is almost homogeneous. Other parameters adopted in the study are given in Table 1. The variations of the tangential and radial stress along the horizontal diameter are plotted in Figure 5. The computed results are in good agreement with the published one and the analytical solution [10].

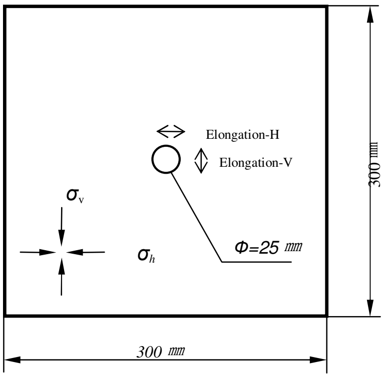


Figure 4: Sample geometry for simulating hydraulic fracturing [16]

Having the model validated, further analyses have been carried out to study the effects the flow in fracture, the material homogeneity, the in-situ stress ratio and the pore pressure field on the breakdown pressure and the development of the fractures. The findings are reported as follows:

Hydraulic fracture of homogeneous materials. The progressive growth of the fractures under the action of pressure with water flowing in them is investigated. The parameters adopted in the analysis are tabulated in Table 1. The hole had a small crack at its crown (Figure 6). In the first case, the homogeneity index is taken to be 1000, two straight fractures open at both ends of the vertical diametrical line where the tensile stress is highest. These fractures initiate at a pressure of 21.1 MPa and it grows to a length of 15 mm at 21.2 MPa. The path of propagation is rather straight. The results indicate that the growth of the crack is controlled by the stress concentration due to the in-situ stresses. A plot of the variations of diametric (vertical and horizontal) lengths versus the water pressure (Figure 7) shows that these lengths increase gradually. The plot confirms this point as the curves increase rapidly after 21.2 MPa, the pressure required for initiating the fracture.

Since we use the loading condition of constant rate of pressure, the loading process becomes unstable as soon as the breakdown pressure is reached. Therefore, no fracture closing stage as observed in the experiments of Zhao et al. [16] is confirmed in our simulation.

Hydraulic fracture of non-homogeneous rocks.
(1) Stress field
Unlike the analytical solution which is restricted to homogeneous materials, the present model can easily simulate the heterogeneity by varying the homogeneous parameters (m). Therefore, further

analyses have been conducted to study the effect of heterogeneity by taking m to be 3. Other parameters remain the same as those tabulated in Table 1. The horizontal and vertical in-situ stresses are 1.0MPa and 2.0MPa, respectively. Figure 8 shows the variation of the stresses along the horizontal diameter for a heterogeneous medium. Due to the variation in the elastic modulus of the elements, there are obvious fluctuations in the stresses though the average stresses are fairly close to the homogeneous one. The fluctuation is over ±10%.

Table 1: Input material mechanical properties – mean values

Parameter	value
Homogeneity index/ m	1.5,3,1000
Young's modulus/ E0	6 GPa
Poisson's ratio / v	0.25
internal friction angle/φ	30°
Compression strength/ fc	60MPa
Tension strength/ ft	6MPa
Coefficient of residual strength / f cr/ fc = f tr/ ft	0.1
Permeability/ k0	0.01m/s
Initiate water pressure/ p i	5MPa
water pressure /Δp	0.1MPa
Water pressure coefficient/α	0.1
Biot's constants/β	0.01

From the computer pattern, one can conclude that the stress in each element is affected not only by the in-situ stresses but also its stiffness. The lack of stress concentration has a strong bearing on the interpretation of the hydraulic fracture [16] and it is further discussed in the next section.

(2) Hydraulic fracture paths with different In-situ stress ratio

Further simulations have been carried out to study the effect of the σ_h / σ_v stress ratio on the fracture pattern. The stress ratio varies from 1.5 (uniform pressure) to 1.0. The fracture patterns for different stress ratio are shown in Figure 9. In the analysis, the homogeneity index is taken to be 3. At high stress ratios, that is stress ratio equal to 1.5, a pair of fractures extends in the maximum tensile stress direction from both ends of the hole (Figure 9a). The fractures are also rather straight.

As the stress ratio decreases, the main fractures, while oriented in the maximum horizontal stress direction, are no longer straight and show a tendency to branch out along the grain boundaries. Figure 10b shows the fracture pattern for stress ratio equal to 1.25. Though cracks open at both end of the vertical diameters, they branch out after growing for a short distance. One can also note that isolated fractures also open within the rock mass. Such fractures should represent the existence of weak elements. Figures 9c and10d show the hydraulic fracture path of rock models with the stress ratio equal to 1.13 and 1.0 respectively. The figures indicate multiple major traces without any preferred orientations are formed. There are significant branching and isolated fracturing. Comparison of the present results with the stress patterns [17] shows that the present model can predict the initiation and development of fractures fairly accurately. These results indicate that the crack pattern depends on the homogeneity when the stress ratio is close to one.

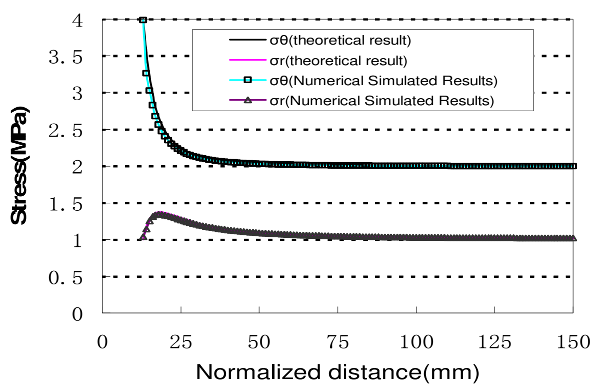
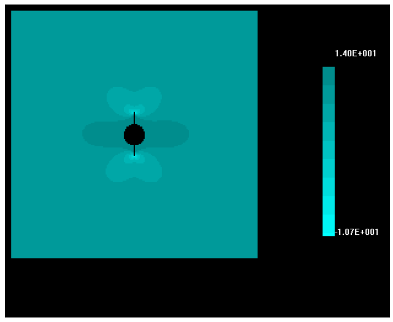
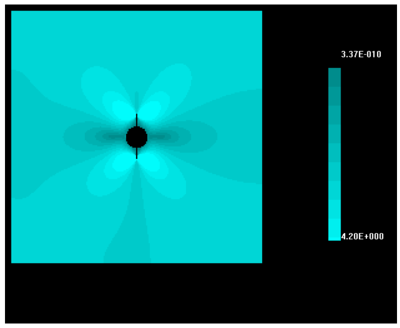


Figure 5: Stress distribution along borehole in the homogenous materials



(a)minimum principal stress



(b) shear stress

Figure 6: Numerically obtained stress fields in simple (m=1000)

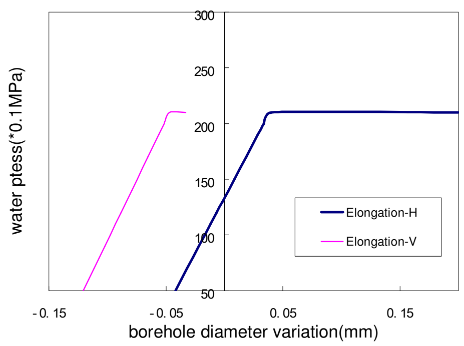
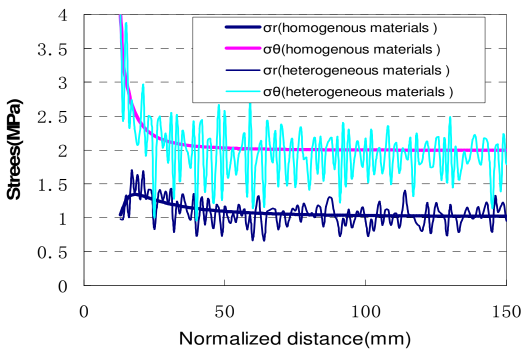


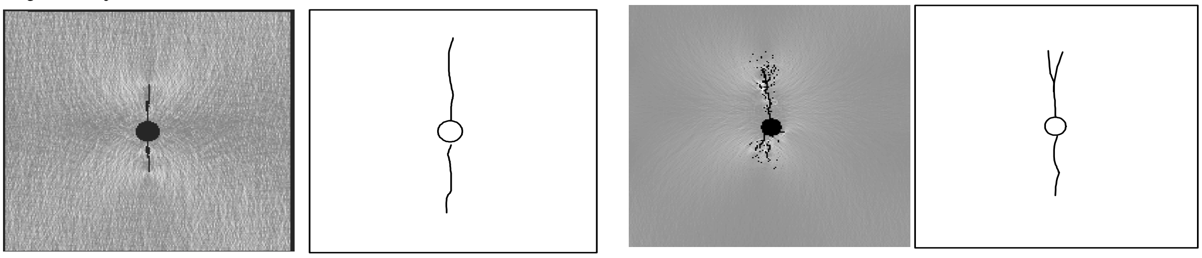
Figure 7: Relationships among water pressure and borehole diameter variation of samples (m=1000)



Figuer 8: Stress distribution along borehole in the heterogeneous materials.

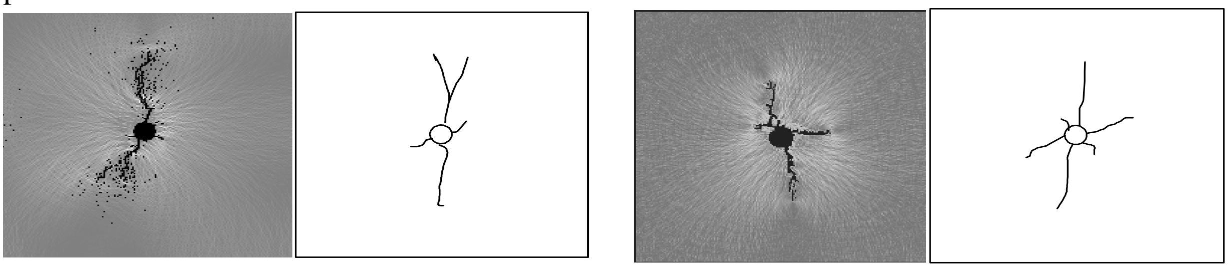
In Figure 10, the breakdown pressures, which is the pressure that fractures are initiated, predicted by the present model are compared to those obtained by the experiment and analytical approaches. When parameter m=1.5 and m=3, the numerical simulation results are accord with those obtained from experiment. The results show that the present method has a more accurate prediction than the analytical ones.

Due to the heterogeneity of rocks, the hydraulic fracture paths are irregular. The numerically obtained random nature of hydraulic fracture path during hydraulic fracturing and its dependence on mescoscopic homogeneity is clearly illustrated in Fig.10. As we can see in the figure, the hydraulic fracture propagation is controlled by the pre-existing field of defects. The hydraulic fracture deterministically selects a path of least resistance through the material with statistical features, and the random location of the individual in homogeneities results in an irregular hydraulic fracture trajectory.



Shears stress evolution during failure Experimentally obtained result Shears stress evolution during failure Experimentally obtained result

(a)σh=10.3MPa,σv=15.5MPa,σv/σh=1.5,pb=18.8MPa (b)σh=10.3MPa,σv=12.9MPa,σv/σh=1.25, pb =21.8MPa



Shears stress evolution during failure Experimentally obtained result Shears stress evolution during failure Experimentally obtained result

(c)σh=10.3MPa,σv=11.6MPa,σv/σh=1.13,pb=22.1MPa (d)σh=10.3MPa,σv=10.3MPa,σv/σh= 1.00, pb =24.4MPa

Figure 9: Experiment [17] and Numerical Simulated Results with different pressure ratio.

Conclusions

The numerical simulations using flow-coupled Rock Failure Process Analysis code (F-RFPA2D) clearly indicates the results indicate that both the rock heterogeneity and the permeability affect fracture initiation and propagation significantly. Our findings are:

(1) Heterogeneity of rock has a significant influence on initiation and breakdown pressure. For heterogeneous rock samples, hydraulic fracture initiation occurs considerably earlier before breakdown pressure is reached. While for homogeneous samples, fracture initiation pressure and breakdown pressure are indistinguishable. Both fracture initiation and breakdown pressure values are much higher for the homogeneous sample than that for the heterogeneous sample.

(2) The hydraulic pressure in the crack plays an important role to make the sample fracture. It can also be deduced previously that the pressurizing fluid flowing into the cracks is the main factor.

(3) The F-RFPA2D clearly simulates the hydraulic fracturing in heterogeneous rocks in a more realistic way than other numerical models. The capability of F-RFPA2D in identifying hydraulic fracturing mechanisms, rather than prejudicing towards certain mechanisms, is obvious. Therefore,

it is concluded that the flow-coupled F-RFPA2D is a useful tool in understanding the physics of hydraulic fracturing, especially in heterogeneous and permeable materials.

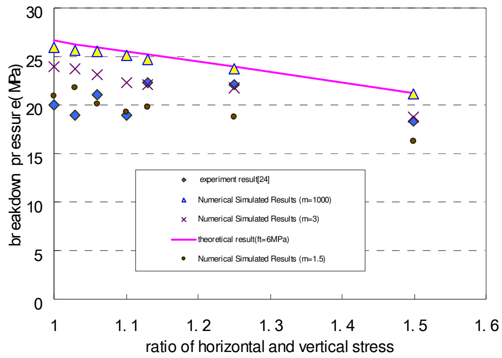


Figure 10: breakdown pressure with different pressure ratio(σv/ σh =1.0-1.5)

Finally, this work is intended to demonstrate a new model for coupling stress, flow and damage. The parameters selected in this paper are just examples for the demonstration of the model. Even if specific parameters such as m or k have been selected in this paper, it does not imply that we recommend these parameters to be used by others who use our code for their own modeling.

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