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Consolidation of vertical drain with depth-varying stress induced by multi-stage loading

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ABSTRACT

An explicit analytical solution is developed for the consolidation of vertical drain with both radial and vertical drainage by adopting a depth-varying stress induced by multi-stage loading. The well resistance and smear effect are also considered. The smear effect is described by three decay patterns of horizontal permeability towards drains within the smeared zone, including a reduced constant pattern, a linear decay pattern and a parabolic decay pattern. A parameter analysis is performed to investigate the consolidation behavior of the vertical drain. The convergence of the proposed series solution is discussed.

1. Introduction

Vertical drains, such as sand drain wells and prefabricated vertical drains (PVDs), are commonly used to accelerate the consolidation of soft soils combination with preloading. The mechanism of this improvement is usually attributed to the following two aspects: (a) providing an additional drainage path in the radial direction, along which the soil layer usually has a larger permeability than along the vertical direction [1] and (b) greatly reducing the length of the drainage path from the thickness of soil layer to half of the drain spacing. The most well-known theoretical study on the radial consolidation of vertical drains was first carried out by Barron [2]. Since that study, a large number of studies have been conducted by many researchers. By incorporating the radial and vertical drainage in a coupling fashion, Leo [3] presented a series of closed-form solutions for equal strain consolidation of vertical drains subjected to instantaneous and ramp loading and analyzed the smear effect and well resistance. And furthermore, this solution was later extended by Lei and Jiang [4] to consider a time- and depth-dependent loading. Zhu and Yin [5,6] also obtained analytical solutions for the same consolidation problem as Leo's; however, in contrast to Leo [3], a new normalized time factor was introduced in their study. Wang and Jiao [7] introduced the double porosity model into the analysis of vertical drain consolidation. In this approach, the variation of horizontal soil permeability can be depicted by an arbitrary function, which presents a relatively simple way to consider the gradual variation of soil permeability within the smear zone. Conte and Troncone [8] presented an analytical solution for the consolidation of vertical drain subjected to an arbitrary time-dependent loading. By incorporating surcharge and vacuum loading that vary with both depth and time, Walker and Indraratna [9] analyzed the consolidation behavior of a layered soil with vertical and horizontal drainage using spectral method.

The installation of vertical drains will inevitably disturb the soil adjacent to the drain and consequently reduce the soil permeability to a certain extent, called the smear effect. The smear effect was considered in many previous studies [3–6] by assuming a reduced, constant value of the horizontal permeability of soil through the smear zone. However, many laboratory studies [10–14] have shown the coefficient of permeability within the smear zone is highly variable. To reflect this variability, some researchers

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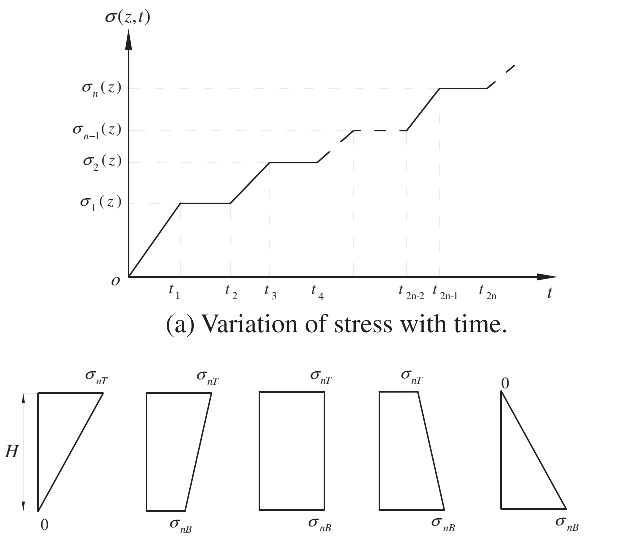

(b) Variations of the stress with depth by nth stage loading.

Fig. 1. Depth-varying stress by multi-stage loading.

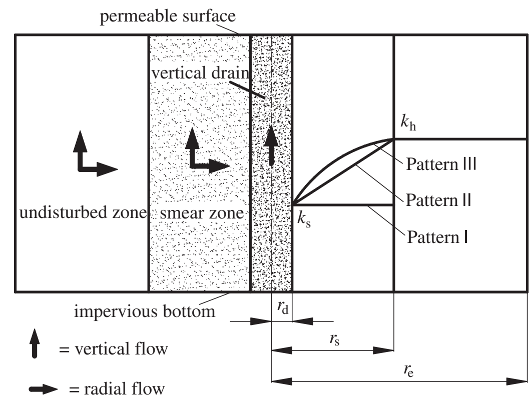

Fig. 2. A typical unit cell selected for modeling the consolidation of a vertical drain and three decay patterns of the horizontal permeability coefficient of the smeared soil toward the drain.

included the gradual decay of horizontal permeability of soil toward the drain, such as linear decay [15,16] and parabolic decay [17], in their analyses of vertical drain consolidation.

In practical engineering the construction loads of buildings or embankments on clayey soils are usually applied gradually to the surface of the foundation; this gradual process is often required because in many cases a rapid loading rate may lead to a ground failure. In these situations multi-stage loading is usually used to achieve higher soil strength by consolidating the soil layers to a certain degree before applying the next, larger load(s). In many previous studies of vertical drain consolidation, the stress caused by external loads was assumed to be uniform with depth. However, many factors can lead to variations in the stress applied along the vertical direction, and this variation usually has various forms as shown in Fig. 1. These various forms can be depicted by the top to bottom stress ratio (σ_{nT}/σ_{nB}) with various values, which correspond to various field situations. The field situations corresponding to various value of σ_{nT}/σ_{nB} are summarized in Table 1. Therefore, considering a more realistic linear variation of stress with depth seems more credible in the analysis of vertical drain consolidation. In addition, in many cases, the total stress may reduce with depth, and therefore, for a thick soil deposit, considering the well resistance in the analysis of the consolidation behavior of vertical drain might be important.

This paper presents an analytical solution for the consolidation of soils with vertical drains by incorporating a depth-varying stress induced by multi-stage loading. A variety of factors are considered, including both the radial and vertical drainage within the soil, well resistance and the smear effect. To reflect the unfavorable influence of the smear effect, the horizontal coefficient of the permeability of the soil within the smear zone is assumed to have three decay patterns toward the vertical drain, as shown in Fig. 2, including a reduced constant pattern (referred as Pattern I in this paper), a linear decay pattern (as Pattern II) and a parabolic decay pattern (as Pattern III).

2. Basic assumptions and governing equations

The consolidation model of the vertical drain in this study is idealized as shown in Fig. 2. The top surface is assumed to be fully permeable, whereas the bottom base is impervious. As shown in Fig. 2, to reflect the unfavorable influence during the installation of vertical drains, three decay patterns of the horizontal permeability of soil are incorporated in the analysis: a reduced constant pattern (Pattern I), a linear decay pattern (Pattern II) and a parabolic decay pattern (Pattern III). Similar to the study of Barron [2], some basic assumptions are made in this analysis as follows:

(1) The assumption of equal strain is adopted, i.e., the vertical drain and the surrounding soil only deform vertically, and they have equal strains at any depth and time.
(2) The water flow within both the soil and the vertical drain obeys Darcy's law.
(3) To consider the effect of well resistance in a simple way, an assumption was made by Xie [18] together with Wang and Jiao [7], in which the radial flow within the vertical drains was neglected. As demonstrated by Lu et al. [19], this assumption does not mean that no radial flow exits in the vertical drain but rather that the vertical drain was assumed to be infinitely permeable in the radial direction. In this

Table 1
Filed situations corresponding to various values of top to bottom total stress ratio.

Values of σ_{nT}/σ_{nB}	Corresponding field situations
$\sigma_{nT}/\sigma_{nB} = \infty$	The external loads are applied on a small area over a very thick soil layer, which may lead to variations in the stress over depth with a zero value at the bottom base of soil layer
$\sigma_{nT}/\sigma_{nB} > 1$	Using a relatively small loading area over a thick soil layer when the total stress at the bottom base does not decrease to zero
$\sigma_{nT}/\sigma_{nB} = 1$	The external load is applied on a relatively large area over a thin soil layer, and, moreover, the consolidation of soil layer by its self-weight stress is completed
$\sigma_{nT}/\sigma_{nB} < 1$	The external load is applied over the soil layer before the completion of consolidation by its self-weight stress
$\sigma_{nT}/\sigma_{nB} = 0$	Consolidation problems due to the gradual drawdown of a water table in the soil layer or the consolidation of a newly filling soil due to its self-weight stress

study, this revised assumption by Lu et al. [19] is used. Therefore, the excess pore water pressure within the vertical drain will be uniform at any depth. Additionally, the water flowing into the vertical drain is assumed to be equal to that flowing out of the vertical drain, i.e.,

$$\left[2\pi r \frac{k_r(r)}{\gamma_w}\frac{\partial u_s}{\partial r}\right]\bigg|_{r=r_d} = -\pi r_d^2 \frac{k_d}{\gamma_w}\frac{\partial^2 u_d}{\partial z^2} \tag{1}$$

where γ_w is the unit weight of water; u_s and u_d are the excess pore water pressures at any point and time in the soil and vertical drain, respectively; r_d, r_s, r_e are radii of the vertical drain, the smear zone and the influence zone, respectively; k_d is the vertical coefficient of permeability of drain well; and $k_r(r)$ is the horizontal coefficient of permeability of soil with respect to radial distance, as shown in Fig. 2. It ranges from k_s to k_h and can be expressed as

$$k_r(r) = k_h f(r) \tag{2}$$

The parameters k_h and k_s are the maximum and minimum horizontal coefficients of permeability of soil as shown in Fig. 2; $f(r)$ is a function to depict the decay pattern of horizontal coefficient of permeability of soil.

(4) The total stress caused by the multi-stage loading is a function depending on both time and depth, as shown in Fig. 1, and it is assumed to have the following form:

$$\sigma(z,t) = \begin{cases} \sigma_{n-1}(z) + \frac{t-t_{2n-2}}{t_{2n-1}-t_{2n-2}}[\sigma_n(z) - \sigma_{n-1}(z)], & t_{2n-2} \leqslant t \leqslant t_{2n-1} \\ \sigma_n(z), & t_{2n-1} \leqslant t \leqslant t_{2n} \end{cases} \tag{3}$$

where

$$\sigma_n(z) = \sigma_{nT} + (\sigma_{nB} - \sigma_{nT})\frac{z}{H} \tag{4}$$

where $\sigma_n(z)$is the final stress varying with depth caused by the nth stage loading, with values of σ_{nT} and σ_{nB} at the top surface and bottom base, respectively; t_{2n-2}, t_{2n} are the final times of the $(n-1)$th and nth stage loadings, respectively; t_{2n-1} corresponds to the time when the nth stage load is increased to the final value; and H is the thickness of the soil layer as well as the vertical drain length $(n = 1, 2, 3, \ldots;\ \sigma_0 = 0,\ t_0 = 0)$.Following the studies by Xie [18] and Wang and Jiao [7], the equation governing the equal strain consolidation of vertical drain with radial and vertical drainage can be obtained as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k_r(r)}{\gamma_w} r \frac{\partial u_s}{\partial r}\right] + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}_s}{\partial z^2} = -\frac{\partial \varepsilon_v}{\partial t} \tag{5}$$

where ε_v is the vertical strain of the surrounding soil and the vertical drain; k_v is the average vertical coefficient of permeability of soil; and $\bar{u}_s$ is the average excess pore water pressure in terms of radius at any depth and time, i.e.,

$$\bar{u}_s = \frac{1}{\pi(r_e^2 - r_d^2)}\int_{r_d}^{r_e} 2\pi r u_s(r,z,t)\mathrm{d}r \tag{6}$$

The vertical strain rate can be calculated as

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{1}{E_s}\frac{\partial}{\partial t}[\sigma(z,t) - \bar{u}_s(z,t)] \tag{7}$$

where E_s is the constrained compression modulus of soil and is assumed to be constant during consolidation.

The external radial boundary of the soil-drain unit cell is impervious. In addition, the excess pore water pressure is the same for the soil and the vertical drain at the soil–drain interface. Thus, the radial boundary conditions can be written as

$$\begin{cases} r = r_e: & \frac{\partial u_s}{\partial r} = 0 \\ r = r_d: & u_s = u_d \end{cases} \tag{8}$$

Integrating Eq. (5) twice with respect to r and combining with Eq. (8) leads to the following expression:

$$u_s = u_d + \frac{\gamma_w}{2k_h}\left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}_s}{\partial z^2}\right)[r_e^2 A'(r) - B'(r)] \tag{9}$$

where $A'(r) = \int_{r_d}^{r}\frac{\mathrm{d}\xi}{\xi f(\xi)},\ B'(r) = \int_{r_d}^{r}\frac{\xi \mathrm{d}\xi}{f(\xi)}$.

Substituting Eq. (9) into Eq. (6) yields

$$\bar{u}_s = u_d + \frac{\gamma_w r_e^2 F_c}{2k_h}\left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}_s}{\partial z^2}\right) \tag{10}$$

where $F_c = \frac{2(A'' r_e^2 - B'')}{r_e^2 r_d^2 (n^2-1)}$; $A'' = \int_{r_d}^{r_e} rA'(r)\mathrm{d}r$, $B'' = \int_{r_d}^{r_e} rB'(r)\mathrm{d}r$; and n is radius ratio of the influence zone to the vertical drain, $n = r_e/r_d$.F_c is a parameter to reflect the effect of the decay pattern of the horizontal permeability of the soil and the geometry of the soil-drain system. The detailed determination of F_c for these three decay patterns of this paper can be found in the study of Xie et al. [20].

Performing the partial differential derivative of Eq. (10) with respect to r and substituting it into Eq. (1) yields

$$\frac{k_d}{\gamma_w}\frac{\partial^2 u_d}{\partial z^2} = -(n^2-1)\left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}_s}{\partial z^2}\right) \tag{11}$$

Substituting Eq. (10) into Eq. (11) leads to

$$\bar{u}_s = u_d + G\frac{\partial^2 u_d}{\partial z^2} \tag{12}$$

By substituting Eqs. (7) and (12) into Eq. (10) and cancelling out the variable of $\bar{u}_s$, a partial differential equation, only containing the variable of u_d, can be obtained as

$$D\frac{\partial^4 u_d}{\partial z^4} + G\frac{\partial^3 u_d}{\partial t \partial z^2} + W\frac{\partial^2 u_d}{\partial z^2} + \frac{\partial u_d}{\partial t} = \frac{\partial \sigma(z,t)}{\partial t} \tag{13}$$

where $D = \frac{r_e^2 F_c k_d k_v E_s}{2(n^2-1)k_h \gamma_w}$, $G = -\frac{r_e^2 F_c k_d}{2k_h(n^2-1)}$, $W = -\left[\frac{k_v E_s}{\gamma_w} + \frac{k_d E_s}{(n^2-1)\gamma_w}\right]$

Eqs. (12) and (13) are the governing equations for the vertical drain consolidation. As shown in Fig. 2, the top surface of ground is fully permeable, and the bottom base is impervious. Therefore, the vertical boundary conditions can be written as

$$\begin{cases} z = 0: & \bar{u}_s(z,t) = 0,\ u_d(z,t) = 0 \\ z = H: & \frac{\partial \bar{u}_s(z,t)}{\partial z} = 0,\ \frac{\partial u_d(z,t)}{\partial z} = 0 \end{cases} \tag{14}$$

As shown in Fig. 1, initially, the stress within the surrounding soil equals zero, and hence the excess pore water pressure within the surrounding soil also equals zero, i.e.,

$$t = 0: \quad \bar{u}_s(z,t) = 0 \tag{15}$$

3. Solutions for the governing equations

3.1. Solution for excess pore water pressure

Referring to the study by Wang and Jiao [7], the solution for Eq. (13) can be assumed as the following form by introducing the Fourier series, $\sin(Mz/H)$:

$$u_d = \sum_{m=1}^{\infty} T_m(t)\sin\left(\frac{M}{H}z\right) \tag{16}$$

where $M = \frac{2m-1}{2}\pi$, $m = 1, 2, 3\ldots$

Then, the excess pore water pressure at any depth within the surrounding soil can be obtained by substituting Eq. (16) into Eq. (12) as

$$\bar{u}_s = \sum_{m=1}^{\infty}\left[1 - G\left(\frac{M}{H}\right)^2\right]T_m(t)\sin\left(\frac{M}{H}z\right) \tag{17}$$

Eqs. (16) and (17) satisfy the vertical boundary conditions in Eq. (14).

Substituting Eq. (16) into Eq. (13) yields

$$T'_m(t) + \beta_m T_m(t) = Q_m(t) \tag{18}$$

where

$$\beta_m = \frac{\frac{r_e^2 F_c k_d k_v E_s}{2(n^2-1)k_h\gamma_w}\left(\frac{M}{H}\right)^4 + \left[\frac{k_v E_s}{\gamma_w} + \frac{k_d E_s}{(n^2-1)\gamma_w}\right]\left(\frac{M}{H}\right)^2}{1 + \frac{r_e^2 F_c k_d}{2k_h(n^2-1)}\left(\frac{M}{H}\right)^2} \tag{19}$$

$$Q_m(t) = \frac{2}{H\left[1 - G\left(\frac{M}{H}\right)^2\right]}\int_0^H \frac{\partial\sigma(z,t)}{\partial t}\sin\left(\frac{M}{H}z\right)dz \tag{20}$$

Using the initial condition in Eq. (15), Eq. (18) can be solved as

$$T_m(t) = e^{-\beta_m t}\int_0^t Q_m(\tau)e^{-\beta_m\tau}d\tau \tag{21}$$

Substituting Eqs. (3) and (4) into Eq. (20) yields

$$Q_m(t) = \begin{cases} \frac{2\left[A_n - (-1)^m \frac{B_n}{M}\right]}{M\left[1 - G\left(\frac{M}{H}\right)^2\right](t_{2n-1} - t_{2n-2})}, & t_{2n-2} \leqslant t \leqslant t_{2n-1} \\ 0 & t_{2n-1} \leqslant t \leqslant t_{2n} \end{cases} \tag{22}$$

where $A_n = \sigma_{nT} - \sigma_{(n-1)T},\ B_n = \sigma_{nB} - \sigma_{nT} - (\sigma_{(n-1)B} - \sigma_{(n-1)T})$.

By substituting Eq. (22) into Eq. (21), the expression of $T_m(t)$ can be determined in a generalized form as below for both the loading periods ($t_{2n-2} \leqslant t \leqslant t_{2n-1}$) and the rest periods ($t_{2n-1} \leqslant t \leqslant t_{2n}$):

$$T_m(t) = \frac{2}{M\beta_m\left[1 - G\left(\frac{M}{H}\right)^2\right]}\sum_{i=1}^{n}\frac{\left[A_i - (-1)^m\frac{B_i}{M}\right]}{(t_{2i-1} - t_{2i-2})}\left(e^{-\beta_m(t-t_f)} - e^{-\beta_m(t-t_s)}\right) \tag{23}$$

where $t_s = \min[t, t_{2i-2}]$, $t_f = \min[t, t_{2i-1}]$.

Then the excess pore water pressures within the vertical drain and the surrounding soil can be finally obtained by substituting Eq. (23) into Eqs. (16) and (17).

3.2. Solution for average degree of consolidation

The average degree of consolidation for the vertical drain ground can be developed based on the solution for excess pore water pressure. The average degree of consolidation is defined as the ratio of the effective stress to the final total stress within the surrounding soil over the whole thickness of the ground. If the external loads are applied by j stages, the final total stress is equal to $\sigma_j(z)$ as described in Eq. (4).

$$U(t) = \frac{\int_0^H[\sigma(z,t) - u_s(z,t)]dz}{\int_0^H \sigma_j(z)dz} \tag{24}$$

The detailed expression for the average degree of consolidation can finally be determined as below by substituting Eqs. (3, 4, 17, and 23) into Eq. (24):

$$U(t) = \begin{cases} \frac{1}{(\sigma_{jB}+\sigma_{jT})}\left\{\left[(\sigma_{(n-1)B} + \sigma_{(n-1)T}) + \frac{t - t_{2n-2}}{t_{2n-1} - t_{2n-2}}C_n\right]\right. \\ \left. -2\sum_{m=1}^{\infty}\frac{T_m}{M}\left[1 - G\left(\frac{M}{H}\right)^2\right]\right\}, \\ t_{2n-2} \leqslant t \leqslant t_{2n-1} \\ \frac{1}{(\sigma_{jB}+\sigma_{jT})}\left\{(\sigma_{nB} + \sigma_{nT}) - 2\sum_{m=1}^{\infty}\frac{T_m}{M}\left[1 - G\left(\frac{M}{H}\right)^2\right]\right\}, \\ t_{2n-1} \leqslant t \leqslant t_{2n} \end{cases} \tag{25}$$

where $n = 1, 2, \ldots j$; $C_n = (\sigma_{nB} + \sigma_{nT}) - (\sigma_{(n-1)B} + \sigma_{(n-1)T})$. By letting $\sigma_{nB} = \sigma_{nT}$, this solution can be reduced to the particular case provided by Tang and Onitsuka [21], where the stress induced by the multi-stage loading is uniform with depth in the surrounding soil.

$j = 1$ and $\sigma_{1B} = \sigma_{1T}$ imply that the total stress induced by a single-stage loading is uniform with depth, a situation called ramp loading by Tang and Onitsuka [21] and Leo [3]. The solutions for the vertical drain consolidation subjected to such a ramp loading can be obtained from the present solution by letting $j = 1$ and $\sigma_{1B} = \sigma_{1T}$.

4. Parametric analysis

In this section, the consolidation behavior of a vertical drain was investigated using parametric analysis. The horizontal time factor of the undisturbed soil T_h ($T_h = c_h t/(4r_e^2) = k_h E_s t/(4r_e^2)$) instead of the real time t is selected as the horizontal axis. T_1, T_2, T_3 are the time factors corresponding to the real times t_1, t_2 and t_3, respectively.

Fig. 3 shows the dissipations of excess pore water pressure, predicted by the solutions above with the three decay patterns of the horizontal permeability coefficient for the surrounding soil. It can be seen that the excess pore water pressure always builds up during the loading periods but decreases during the rest periods. In addition, the dissipation of excess pore water pressure predicted by the solution of Pattern III is the most rapid; dissipation by the solution of Pattern I is the slowest, and that by Pattern II lies between those of Patterns I and III.

Fig. 4 shows the difference in the average degree of consolidation with various patterns of total stress over depth, as shown in Fig. 1. The average degree of consolidation increases with an

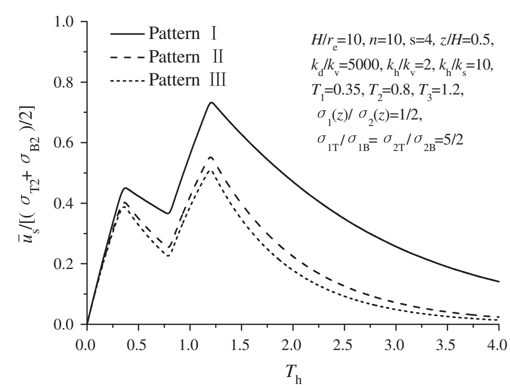

Fig. 3. Dissipations of the excess pore water pressure predicted with various decay patterns of the horizontal permeability of the surrounding soil.

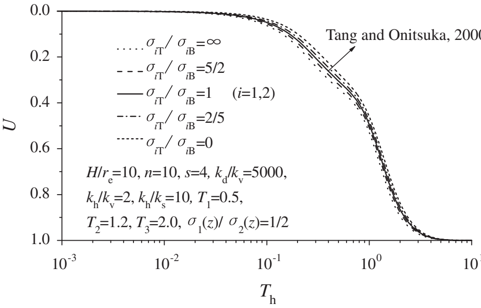

Fig. 4. Influence of the top to bottom stress ratio on the average degree of consolidation (Pattern III).

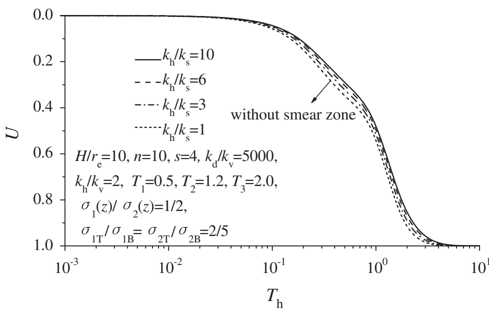

Fig. 5. Influence of reducing the horizontal permeability within the smear zone on the average degree of consolidation (Pattern III).

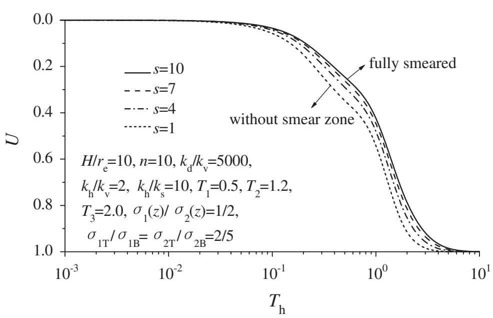

Fig. 6. Influence of the smear zone size on the average degree of consolidation (Pattern III).

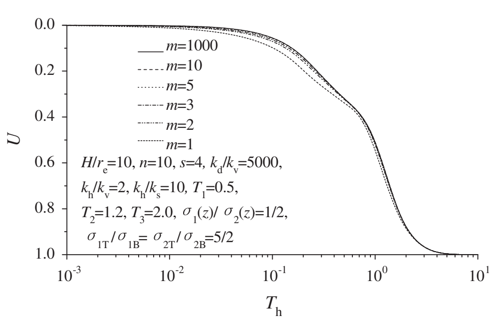

Fig. 7. Convergence of the proposed series solution (Pattern III).

increase in the top to bottom stress ratio value. As shown in Fig. 2, the vertical draining boundary of the ground is set at the top surface; hence, a large stress at close to the draining boundary will inevitably accelerate the consolidation rate of the soil ground. As a special case, the average degree of consolidation predicted by Tang and Onitsuka [21] (i.e., the case of $\sigma_{iT} = \sigma_{iB}$) lies in the middle of all the cases in Fig. 4.

The smear effect resulting from the installation of a vertical drain can be described by the reduction in the horizontal permeability coefficient within the smear zone and the size of the smear zone, the influence of which on the average degree of consolidation is investigated in Figs. 5 and 6. The average degree of consolidation increases with a reduction in the values of k_h/k_s and s. In other words, reducing the extent of the smear effect can accelerate the consolidation rate of the vertical drain ground.

To help engineers in determining the correct number of calculation terms for this series solution, the comparison of the average degree of consolidation calculated with various numbers of terms is presented in Fig. 7. This series solution converges rapidly, and at most, five calculation terms can produce sufficient accuracy for practical engineering.

5. Conclusions

This paper presented an analytical solution for the consolidation of a vertical drain with depth-varying stress induced by multi-stage loading. The smear effect and the well resistance were considered in the analysis. To reflect the unfavorable influence of the smear effect, three decay patterns of the horizontal permeability coefficient within the smear zone, a reduced constant pattern (Pattern I), a linear decay pattern (Pattern II) and a parabolic decay pattern (Pattern III), were incorporated. A parameter analysis was performed to investigate the consolidation behavior of a vertical drain. The results show that the excess pore water pressure always builds up during the loading periods but decreases during the rest periods. The consolidation rate increases with an increase in the value of the top to bottom stress ratio under the condition of PTIB (pervious top and impervious bottom). Reducing the smear effect will increase the consolidation rate. The present solution converges very rapidly, and at most five calculation terms will achieve sufficient accuracy in practice.

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