International Journal of Geomechanics. Submitted August 1, 2010; accepted August 25, 2011; posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

Analytical solution for the consolidation of a composite foundation reinforced by an impervious column with an arbitrary stress increment

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Abstract: Based on the axisymmetric consolidation model, the governing equation and the corresponding solution were developed for the consolidation of a composite foundation with an impervious column by incorporating an arbitrary stress increment. Then, the consolidation behavior was investigated as part of the parameter analysis for a composite foundation with an impervious column. The results show that the consolidation rate for a composite foundation with an impervious column was slower than that for a composite foundation with a granular column but was more rapid than that for a natural soil foundation. The consolidation rate accelerated with increasing values of the column-soil constrained modulus ratio or the top-to-bottom stress increment ratio and with decreasing values of the loading period or the radius ratio of the influence zone to the column. The column-soil total stress ratio increased with consolidation and approached the value of the column-soil constrained modulus ratio.

CE Database subject headings: Consolidation; Foundations; Impervious Columns; Soft soils.

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Introduction

In terms of material properties, the columns commonly used in composite foundations can be divided into two types: the granular material column and the cohesive material column. The former type of column usually has a higher permeability than the surrounding soil and hence is utilized worldwide to accelerate the consolidation rate of soft soils. Typical examples are the stone column and the sand column (or sand drain well). In contrast, the cohesive material column often has a much lower permeability than the surrounding soil and even behaves as an impervious inclusion. These types of columns include the cement-soil column generated by the mechanism of deep mixing or by jet grouting methods. For this reason, this type of column was assumed to be an impervious column in this study. Compared with the consolidation theory for a composite foundation with a granular column, few studies have been reported in the literature on the consolidation of a composite foundation with an impervious column. In the following section, some representative research works will be reviewed to demonstrate the development of the consolidation theory for composite foundations.

In the past decades, significant strides have been made in developing the consolidation theory for a composite foundation with a granular column, including the theory for a sand drain well foundation. Han and Ye (2002) developed an analytical solution for computing the consolidation rates of a stone column–reinforced foundation by accounting for the smear and well resistance effects. By considering both the radial and vertical drainage in the soil, Leo (2004) obtained a series of closed-form solutions for the consolidation of a vertical drain foundation subjected to step- or ramp-loading. The same problem was also studied by Zhu and Yin (2004). Different from Leo (2004), a normalized time factor was defined in their study to simplify the solution for use in actual design work. Walker and Indraratna (2006, 2007 and 2009) presented a series of solutions for the consolidation of a vertical drain foundation by considering a wide range of engineering problems, such as the parabolic distribution of the horizontal permeability coefficient of the surrounding soil and the overlapping of the smear zone and the multilayered soil, which involve a novel use of the spectral method. Indraratna et al. (2008) investigated the radial consolidation of a vertical drain system beneath a circular embankment load and obtained an equivalent axisymmetric solution for the consolidation of concentric rings of vertical drains. Castro and Sagaseta (2009) considered the lateral deformation of the stone column and presented an analytical solution for the consolidation of a composite foundation in either the elastic or the elastoplastic regime. Xie et al. (2009) derived a general analytical solution for the consolidation of a composite foundation by accounting for the linear variation of the horizontal permeability coefficient within the disturbed soil zone along with a time- and depth-dependent stress increment caused by the external loads. In addition, Xie et al. (2009) also developed an analytical solution by modifying the traditional flow continuity condition at the soil-column interface to eliminate the contradiction between the equal strain assumption and the flow continuity assumption. Lu et al. (2010) considered the combined flows in the vertical and radial directions within the column and obtained an analytical solution to account for the column deformation and the column consolidation in a coupled fashion. Miao et al. (2008) investigated the consolidation behavior of a double-layered soil foundation partially penetrated by a deep mixed column with the single- and double-drainage conditions under a multistaged loading. However, this study did not include the influence of the edge effect when the external loads are applied to a limited area.

As stated in the aforementioned reviews, many achievements have been reported in the literature for the consolidation of a composite foundation. However, most of these valuable research studies were mainly focused on a composite foundation reinforced by a granular column; few research studies have been reported in the literature for the consolidation of a composite foundation with an impervious column. In fact, compared with granular columns, impervious columns usually have a much lower permeability, and in many cases, the permeability of an impervious column is even lower than that of the surrounding soil. As revealed by Lu et al. (2010), the assumption of traditional flow continuity at the soil-column interface, which has commonly been used in the consolidation theory for a granular column foundation, implies that the column is assumed to be infinitely permeable in the radial direction. Obviously, this assumption is not applicable to an impervious column because the impervious column usually has a very small permeability and even behaves as an impervious body. Therefore, the consolidation theory for a granular column composite foundation is not suitable for an impervious column foundation.

In practice, instead of considering the uniform distribution of the stress increment along the vertical direction, the variation should be considered for the consolidation of a composite foundation in many cases, such as during the consolidation analysis of newly filled soil because of its self-weight stress, the gradual drawdown of the water table in the soil layer and the situations in which a relatively small loading area is used over a very thick soil layer. In a strict sense, considering the variation of the vertical stress is a three-dimensional or axisymmetrical (if the loading is symmetrical) problem, but for simplification, it is approximated as a one-dimensional problem in this paper. In addition, the external loads are usually applied gradually, but not instantaneously, over the surface of a soil layer. Therefore, it is more rational to assume the stress increment is a function of both time and depth. Based on an approximately one-dimensional

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problem, this paper develops an analytical solution by considering a time- and depth-dependent stress increment for the consolidation of a composite foundation stabilized by impervious columns,

Basic Equation and Solution

Governing Equation

The present analysis is based on the axisymmetric consolidation model commonly used in the previous consolidation theories for a vertical drain foundation. As shown in Fig. 1, a cylindrical unit cell, including an impervious column surrounded by the disturbed soil zone, which is in turn surrounded by an undisturbed soil zone, is selected for the analysis presented in this paper. The column-soil interface was assumed to be an impervious boundary because no drainage path is provided by the impervious column for water flow in the radial direction. In addition, the stress increment caused by the external load was assumed to be linearly varying along the column depth and increasing with time up to the final value, then remaining constant, as shown in Fig. 2. Therefore, the relationship can be expressed as follows:

$$\sigma(z,t)=\begin{cases}\left[\sigma_{\mathrm{T}}+(\sigma_{\mathrm{B}}-\sigma_{\mathrm{T}})\dfrac{z}{H}\right]\dfrac{t}{t_{\mathrm{c}}}, & t<t_{\mathrm{c}}\\ \sigma_{\mathrm{T}}+(\sigma_{\mathrm{B}}-\sigma_{\mathrm{T}})\dfrac{z}{H}, & t\geq t_{\mathrm{c}}\end{cases} \tag{1}$$

where $\sigma_{\mathrm{T}}, \sigma_{\mathrm{B}}$ are the final stress increments at the top and bottom of the foundation, respectively; t_{c} is the loading period; and H is the thickness of the composite foundation.

In addition, the following two assumptions were also made during the analysis:

① The assumption of equal strains was adopted; that is, the column and the surrounding soil only deform vertically and have an equal vertical strain at any depth. It should be noted that this assumption does not mean a constant vertical strain with depth in the

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consolidating soil. In fact, from Eq. (3) below, it is evident that the vertical strain is a function dependent on both depth and time.

② Darcy's law is obeyed.

From the equilibrium equation, the following relation can be obtained:

$$\pi\left(r_{\mathrm{e}}^2-r_{\mathrm{c}}^2\right)\overline{\sigma}_{\mathrm{s}}(z,t)+\pi r_{\mathrm{c}}^2\overline{\sigma}_{\mathrm{c}}(z,t)=\pi r_{\mathrm{e}}^2\sigma(z,t) \tag{2}$$

where $\overline{\sigma}_{\mathrm{s}}, \overline{\sigma}_{\mathrm{c}}$ are the total average vertical stress at any depth within the soil and the column, respectively; and $r_{\mathrm{e}}, r_{\mathrm{c}}$ are the radii of the influence zone and the column, respectively.

No water exchange occurs between the surrounding soil and the column because the column is impervious. Therefore, no water exists in the column, and the excess pore water pressure within the column equals zero at all times. The equal strain assumption produces the following equation:

$$\frac{\overline{\sigma}_{\mathrm{s}}(z,t)-\overline{u}_{\mathrm{s}}(z,t)}{E_{\mathrm{s}}}=\frac{\overline{\sigma}_{\mathrm{c}}(z,t)}{E_{\mathrm{c}}}=\varepsilon_{\mathrm{v}}(z,t) \tag{3}$$

where $E_{\mathrm{s}}, E_{\mathrm{c}}$ are the constrained moduli of the surrounding soil and the column, respectively; ε_{v} is the vertical strain of the surrounding soil as well as the column; and $\overline{u}_{\mathrm{s}}$ is the average excess pore water pressure at any depth within the surrounding soil.

From Eqs. (2) and (3), the following can be derived:

$$\varepsilon_{\mathrm{v}}=\frac{n^2\sigma(z,t)-\left(n^2-1\right)\overline{u}_{\mathrm{s}}}{E_{\mathrm{s}}\left(n^2-1+Y\right)} \tag{4}$$

where $n=r_{\mathrm{e}}/r_{\mathrm{c}}$ is the radius ratio of the influence zone to the column; and $Y=E_{\mathrm{c}}/E_{\mathrm{s}}$ is the constrained modulus ratio of the column to the soil.

From the above equation, the rate of the vertical strain can be obtained:

$$\frac{\partial\varepsilon_{\mathrm{v}}}{\partial t}=-\frac{1}{E_{\mathrm{s}}\left(n^2-1+Y\right)}\left[\left(n^2-1\right)\frac{\partial\overline{u}_{\mathrm{s}}}{\partial t}-n^2\frac{\partial\sigma(z,t)}{\partial t}\right] \tag{5}$$

Because no radial flow will occur in the composite foundation with impervious columns, the consolidation equation for the composite foundation with an impervious column can be written as shown below by following the principle of mass conservation:

$$\frac{\partial \varepsilon_{\mathrm{v}}}{\partial t}+\frac{k_{\mathrm{v}}}{\gamma_{\mathrm{w}}}\frac{\partial^2 \bar{u}_{\mathrm{s}}}{\partial z^2}=0 \tag{6}$$

where k_{v} is the vertical permeability coefficient of the surrounding soil; and γ_{w} is the unit weight of water.

Substituting Eq. (5) into Eq. (6) yields

$$\frac{\partial \bar{u}_{\mathrm{s}}}{\partial t}-c_{\mathrm{vf}}\frac{\partial^2 \bar{u}_{\mathrm{s}}}{\partial z^2}=\frac{n^2}{n^2-1}\frac{\partial \sigma(z,t)}{\partial t} \tag{7}$$

where c_{vf} is a positive constant that can be defined as the composite coefficient of consolidation for a composite foundation, $c_{\mathrm{vf}}=c_{\mathrm{v}}\left(n^2-1+Y\right)/\left(n^2-1\right)$, in which c_{v} is the vertical consolidation coefficient of soil, $c_{\mathrm{v}}=k_{\mathrm{v}}E_{\mathrm{s}}/\gamma_{\mathrm{w}}$.

Equation (7) is the governing equation for the consolidation problem of this paper. As shown in Fig. 1, the top surface of the composite foundation is pervious and the bottom surface is impervious. Therefore, the vertical boundary condition can be written as

$$\begin{cases} z=0: & \bar{u}_{\mathrm{s}}(z,t)=0 \\ z=H: & \dfrac{\partial \bar{u}_{\mathrm{s}}(z,t)}{\partial z}=0 \end{cases} \tag{8}$$

At the initial moment, no vertical deformation occurs in either the column or the surrounding soil. Therefore, the following can be derived from Eq. (3):

$$t=0: \ \bar{\sigma}_{\mathrm{s}}=\bar{u}_{\mathrm{s}}, \quad \bar{\sigma}_{\mathrm{c}}=0 \tag{9}$$

Substituting Eq. (9) into Eq. (2) and combining with Eq. (1) gives

$$\bar{u}_{\mathrm{s}}=0 \tag{10}$$

This is the initial condition for the consolidation problem in this paper. So far, the governing

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equation and the initial and boundary conditions have been obtained. The next procedure is to obtain the solution for the governing equation to satisfy the initial and boundary conditions.

Analytical Solution

Equation (7) is an inhomogeneous partial differential equation. To solve it, the corresponding homogeneous case is considered first:

$$\frac{\partial \bar{u}_{\mathrm{s}}}{\partial t} - c_{\mathrm{vf}} \frac{\partial^2 \bar{u}_{\mathrm{s}}}{\partial z^2} = 0 \tag{11}$$

In fact, Eq. (11) corresponds to the case in which the external load is applied instantaneously. In addition, it can be seen that this equation is the same as that provided by Terzaghi (1943) in format, except that the consolidation coefficient for soil c_{v} was replaced by the composite one c_{vf}, to reflect the effects from both the soil and the column. Therefore, referring to the study by Terzaghi (1943), the solution for Eq. (11) can be readily obtained as:

$$\bar{u}_{\mathrm{s}} = \sum_{m=1}^{\infty} A_m \sin\left(\frac{M}{H} z\right) e^{-\beta_m t} \tag{12}$$

where β_m is a series of positive constants and A_m is an unknown parameter to be determined from the initial condition. It is noted that under the instant loading, the initial condition is no longer the same as Eq. (10). In fact, it is not necessary to determine A_m at this stage. The purpose of solving Eq. (11) is only to obtain the Fourier series $\sin\left(Mz/H\right)$ in Eq. (12).

Because the solution for the homogeneous partial differential equation (11) has been obtained as Eq. (12), it is convenient to assume that the solution for the corresponding inhomogeneous case in Eq. (7) will have the following form in terms of the Fourier series $\sin\left(Mz/H\right)$:

$$\bar{u}_{\mathrm{s}} = \sum_{m=1}^{\infty} T_m\left(t\right) \sin\left(\frac{M}{H} z\right) \tag{13}$$

This equation has satisfied the vertical boundary conditions in Eq. (8). The next procedure is

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to determine the expression of $T_m(t)$ to satisfy the initial condition in Eq. (10).

By substituting Eq. (13) into Eq. (7), the following can be obtained:

$$\sum_{m=1}^{\infty} T_m'(t)\sin\left(\frac{M}{H}z\right)+c_{\mathrm{vf}}\sum_{m=1}^{\infty}\frac{M^2}{H^2}T_m(t)\sin\left(\frac{M}{H}z\right)=\frac{n^2}{n^2-1}\frac{\partial\sigma(z,t)}{\partial t} \tag{14}$$

For the orthogonality of the Fourier series, multiplying $\sin\left(\frac{M}{H}z\right)$ on both sides of Eq. (14) yields

$$T_m'(t)+\beta_m T_m(t)=Q_m(t) \tag{15}$$

where

$$\beta_m=\frac{M^2 c_{\mathrm{vf}}}{H^2},\ M=(2m-1)\pi/2,\ m=1,2,3... \tag{16}$$

$$Q_m(t)=\frac{n^2}{n^2-1}\frac{2}{H}\int_0^H\frac{\partial\sigma(z,t)}{\partial t}\sin\left(\frac{M}{H}z\right)\mathrm{d}z \tag{17}$$

Substituting Eq. (1) into Eq. (17) gives

$$Q_m(t)=\begin{cases}\dfrac{2n^2}{Mt_{\mathrm{c}}(n^2-1)}\left[\sigma_{\mathrm{T}}-(-1)^m\left(\dfrac{\sigma_{\mathrm{B}}-\sigma_{\mathrm{T}}}{M}\right)\right], & t<t_{\mathrm{c}}\\ 0, & t\ge t_{\mathrm{c}}\end{cases} \tag{18}$$

Combined with Eq. (13), the initial condition in Eq. (10) can be rewritten as

$$T_m(0)=0 \tag{19}$$

Equations (15) and (19) are an ordinary differential equation and the corresponding initial condition, respectively. The solution for Eq. (15) satisfying Eq. (19) can be readily obtained as

$$T_m(t)=e^{-\beta_m t}\int_0^t Q_m(\tau)e^{\beta_m\tau}\mathrm{d}\tau \tag{20}$$

The detailed expression for $T_m(t)$ can be derived by substituting Eq. (18) into Eq. (20):

$$T_m = \begin{cases} \dfrac{2n^2\left(1-e^{-\beta_m t}\right)}{Mt_c\beta_m\left(n^2-1\right)}\left[\sigma_T-(-1)^m\left(\dfrac{\sigma_B-\sigma_T}{M}\right)\right], & t < t_c \\ \dfrac{2n^2\left[e^{-\beta_m(t-t_c)}-e^{-\beta_m t}\right]}{Mt_c\beta_m\left(n^2-1\right)}\left[\sigma_T-(-1)^m\left(\dfrac{\sigma_B-\sigma_T}{M}\right)\right], & t \ge t_c \end{cases} \tag{21}$$

Finally, the solution for the average excess pore water pressure within the surroundings can be obtained as follows by substituting Eq. (21) into Eq. (13):

$$\bar{u}_s = \begin{cases} \dfrac{n^2}{n^2-1}\displaystyle\sum_{m=1}^{\infty}\dfrac{2\left[\sigma_T-(-1)^m\left(\dfrac{\sigma_B-\sigma_T}{M}\right)\right]}{Mt_c\beta_m}\left(1-e^{-\beta_m t}\right)\sin\left(\dfrac{M}{H}z\right), & t < t_c \\ \dfrac{n^2}{n^2-1}\displaystyle\sum_{m=1}^{\infty}\dfrac{2\left[\sigma_T-(-1)^m\left(\dfrac{\sigma_B-\sigma_T}{M}\right)\right]}{Mt_c\beta_m}\left[e^{-\beta_m(t-t_c)}-e^{-\beta_m t}\right]\sin\left(\dfrac{M}{H}z\right), & t \ge t_c \end{cases} \tag{22}$$

Based on the solution for the excess pore water pressure within the surrounding soil, the average degree of consolidation of the composite foundation with an impervious column is defined as the ratio of the average effective stress to the final average total stress within the surrounding soil at any depth, i.e.,

$$U(t) = \frac{\int_0^H \left(\bar{\sigma}_s - \bar{u}_s\right)\mathrm{d}z}{\int_0^H \bar{\sigma}_s(\mathrm{z},\infty)\,\mathrm{d}z} \tag{23}$$

From Eqs. (2) and (3), the average effective stress within the surrounding soil can be derived as

$$\bar{\sigma}_s = \frac{n^2\sigma(z,t) + Y\bar{u}_s}{n^2-1+Y} \tag{24}$$

Letting $t \to \infty$ and $\bar{u}_s \to 0$, the final average total stress within the surrounding soil can be obtained from Eq. (24) as

$$\bar{\sigma}_s(\mathrm{z},\infty) = \frac{n^2\sigma(z,\infty)}{n^2-1+Y} = \frac{n^2\sigma(z,t_c)}{n^2-1+Y} \tag{25}$$

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Substituting Eqs. (24) and (25) into Eq. (23) yields

$$U(t)=\frac{\int_0^H \sigma(z,t)\mathrm{d}z}{\int_0^H \sigma(z,t_{\mathrm{c}})\mathrm{d}z}-\frac{n^2-1}{n^2}\frac{\int_0^H \bar{u}_{\mathrm{s}}\mathrm{d}z}{\int_0^H \sigma(z,t_{\mathrm{c}})\mathrm{d}z} \tag{26}$$

The final solution for the average degree of consolidation can be determined as follows by substituting Eqs. (1) and (22) into Eq. (26):

$$U(t)=\begin{cases}\dfrac{t}{t_{\mathrm{c}}}-\dfrac{4}{\sigma_{\mathrm{T}}+\sigma_{\mathrm{B}}}\displaystyle\sum_{m=1}^{\infty}\frac{\left[\sigma_{\mathrm{T}}-(-1)^m\left(\dfrac{\sigma_{\mathrm{B}}-\sigma_{\mathrm{T}}}{M}\right)\right]}{M^2 t_{\mathrm{c}}\beta_m}\left(1-e^{-\beta_m t}\right), & t<t_{\mathrm{c}}\\ 1-\dfrac{4}{\sigma_{\mathrm{T}}+\sigma_{\mathrm{B}}}\displaystyle\sum_{m=1}^{\infty}\frac{\left[\sigma_{\mathrm{T}}-(-1)^m\left(\dfrac{\sigma_{\mathrm{B}}-\sigma_{\mathrm{T}}}{M}\right)\right]}{M^2 t_{\mathrm{c}}\beta_m}\left[e^{-\beta_m(t-t_{\mathrm{c}})}-e^{-\beta_m t}\right], & t\ge t_{\mathrm{c}}\end{cases} \tag{27}$$

Degenerations of the Obtained Solution

In this section, the present solution was degenerated to several particular cases that are commonly encountered in practice.

① Letting $\sigma_{\mathrm{B}}=\sigma_{\mathrm{T}}$, Eq. (27) was degenerated to

$$U(t)=\begin{cases}\dfrac{t}{t_{\mathrm{c}}}-\displaystyle\sum_{m=1}^{\infty}\frac{2}{M^2 t_{\mathrm{c}}\beta_m}\left(1-e^{-\beta_m t}\right), & t<t_{\mathrm{c}}\\ 1-\displaystyle\sum_{m=1}^{\infty}\frac{2}{M^2 t_{\mathrm{c}}\beta_m}\left[e^{-\beta_m(t-t_{\mathrm{c}})}-e^{-\beta_m t}\right], & t\ge t_{\mathrm{c}}\end{cases} \tag{28}$$

This is the solution for the consolidation of a composite foundation with an impervious column subjected to ramp loading, that is, the stress increment caused by the external load is kept constant along the column depth while it linearly increases with time up to the final value and then remains constant.

② By letting $t_{\mathrm{c}}\to 0$, the solution for the case of $t\ge t_{\mathrm{c}}$ in Eq. (28) is reduced to

$$U(t)=1-\sum_{m=1}^{\infty}\frac{2}{M^2}e^{-\beta_m t} \tag{29}$$

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Equation (29) is the solution for the average degree of consolidation of a composite foundation with an impervious column under an instant loading. In this situation, the stress increment caused by the external load is kept constant both with time and depth.

③ By letting $t_c \to 0$, the solution for the case of $t \geq t_c$ in Eq. (27) can be degenerated to

$$U(t) = 1 - \frac{4}{\sigma_T + \sigma_B} \sum_{m=1}^{\infty} \left[\sigma_T - (-1)^m \left(\frac{\sigma_B - \sigma_T}{M} \right) \right] \frac{e^{-\beta_m t}}{M^2} \tag{30}$$

This solution corresponds to the case in which the external load is applied instantaneously and the stress increment varies linearly with depth in the composite foundation.

④ When $n \to \infty$ (i.e., $r_c \to 0$), the solution presented in Eq. (29) can be reduced as follows:

$$U(t) = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2} e^{-M^2 T_v} \tag{31}$$

where T_v is the time factor for the soil in the vertical direction, $T_v = c_v t / H^2$.

This is the one-dimensional consolidation solution provided by Terzaghi (1943). In fact, $n \to \infty$ implies that the radius of the column in the composite foundation approaches zero, which means no column exists in the foundation. Therefore, the composite foundation becomes a natural soil foundation, and hence, the consolidation solution for the former becomes the solution for the latter.

Parametric Analysis of Consolidation Behavior

In this section, a parametric analysis is performed to investigate the consolidation behavior of a composite foundation with an impervious column. In the analysis, a vertical time factor is used in lieu of real time as the time axis for general significance. In addition, $T_c \left(= c_v t_c / H^2\right)$ is the vertical time factor for soil corresponding to the loading period t_c.

First, a comparison was made to show the difference of the consolidation rates predicted by the Xie et al. solution (2009) for a composite foundation with a granular column, the Terzaghi solution (1943) for a natural soil foundation and the present solution for a composite foundation with an impervious column. As shown in Fig. 3, the average degree of consolidation for a composite foundation with a granular column predicted by Xie et al. (2009) is much larger than that for an impervious column foundation predicted by this paper. The reason is that for a composite foundation with a granular column, the high permeability of the column allows water to dissipate primarily in the radial direction. The water flow also has a much shorter drainage path than along the vertical direction. However, no radial drainage path is available for the composite foundation with an impervious column, and hence, the water is dissipated only in the vertical direction, which leads to a much longer drainage path than the radial drainage path. Nevertheless, compared with a natural soil foundation, the column in a composite foundation will take a much higher proportion of the external load for its much higher constrained modulus than the surrounding soil; therefore, the average degree of consolidation for the composite foundation with an impervious column predicted by the present solution is much larger than that for a natural soil foundation predicted by Terzaghi (1943).

Figure 5 shows the influence of the distribution of the stress increment along the vertical direction on the average degree of consolidation for a composite foundation with an impervious column. It can be seen that the average degree of consolidation increases with increasing values of σ_T/σ_B.

As shown in Fig. 5, the loading period exerts a great influence on the average degree of consolidation. The longer the loading period, the slower the consolidation rate. $T_c = 0$ implies that the external load is applied instantly and that in this situation, the average degree of consolidation is at its maximum.

The influence of the constrained modulus ratio of the column to soil is investigated in Fig. 6. It can be seen that with the increase in the value of Y, the average degree of consolidation is enhanced. In other words, the stiffer the column, the larger the average degree of consolidation.

The value of n can reflect the relative size of the radius of the influence zone compared with the radius of the column. The larger the value of n, the larger the size of the influence zone or the smaller the column radius. From Fig. 7, it can be seen that a reduction in the value of n leads to an increase in the average degree of consolidation. However, there is an optimal value of n for reducing the consolidation rate. Beyond this value, the consolidation rate cannot be reduced significantly. As shown in Fig. 7, when $n > 15$, the average degree of consolidation shows little sensitivity to the variation in the value of n.

Figure 8 shows the distributions of the excess pore water pressure within the soil along the vertical direction at a given time. It can be seen that with a reduction in the value of σ_T/σ_B, the excess pore water pressure increases within the lower half of the soil, whereas it decreases within the upper half of the soil. In other words, the excess pore water pressure within the lower half of the soil dissipates more and more slowly with a reduction in the value of σ_T/σ_B.

Figure 9 shows a series of dissipation curves for the excess pore water pressure with different loading periods at a depth of $z = H/2$. It is evident that the excess pore water pressure dissipates more and more rapidly with a reduction of the loading period. When the load is applied instantly ($T_c = 0$), the corresponding excess pore pressure dissipates more rapidly than the other cases.

Figure 10 contains the development curves of the column-soil total stress ratio with time at a given depth. It can be seen the column-soil total stress ratio increases with increasing values of $\sigma_{\mathrm{T}}/\sigma_{\mathrm{B}}$ because the consolidation rate accelerates with an increase in the value of $\sigma_{\mathrm{T}}/\sigma_{\mathrm{B}}$. In addition, for different values of $\sigma_{\mathrm{T}}/\sigma_{\mathrm{B}}$, the column-soil total stress ratios all increase with time and approach the final value, which is equal to the column-soil constrained modulus ratio. On the other hand, inflexion points exist in each curve at $T_{\mathrm{v}} = T_{\mathrm{c}}$ because the external load is not applied continuously and the loading rate changes sharply at $T_{\mathrm{v}} = T_{\mathrm{c}}$, as shown in Fig. 2.

Conclusions

Based on the axisymmetric consolidation model, an analytical solution was developed for the consolidation of a composite foundation with an impervious column by considering several important factors in practice, such as an imperious boundary condition at the column-soil interface and the time- and depth-dependent characteristics of the stress increment. A comparison was made between three types of foundation, including two types of composite foundations—a granular column and with an impervious column—along with a nature soil foundation. Finally, a parametric study was performed to investigate the consolidation behavior of the composite foundation. Several conclusions were drawn from the analysis:

①The consolidation rate of a composite foundation with a granular column is larger than that for a composite foundation with an impervious column, which is in turn larger than that for a natural soil foundation.

②The consolidation rate accelerates with an increase in the value of Y and $\sigma_{\mathrm{T}}/\sigma_{\mathrm{B}}$ and with a reduction in the value of T_{c} and n.

③The column-soil total stress ratio increases with time and finally approaches the value of column-soil constrained modulus ratio. In addition, the larger the value of $\sigma_{\mathrm{T}}/\sigma_{\mathrm{B}}$, the larger the column-soil total stress ratio.

Acknowledgements

This research was supported by the National Nature Science Foundation of China (NSFC) (No. 51009135), the National Science Foundation for Postdoctoral Scientists of China (Grant No. 20100481183), the Science Foundation for Young Scholars of China University of Mining and Technology (No. 2009A008) and the Science Foundation for Postdoctoral Researchers from the State Key Laboratory for Geomechanics and Deep Underground Engineering (No. PD1003). All of this support is gratefully acknowledged.

Notation

c_{v}, vertical consolidation coefficient for the surrounding soil, $c_{\mathrm{v}} = \dfrac{k_{\mathrm{v}} E_{\mathrm{s}}}{\gamma_{\mathrm{w}}}$

c_{vf}, vertical consolidation coefficient for the composite foundation, $c_{\mathrm{vf}} = \dfrac{c_{\mathrm{v}}\left(n^2 - 1 + Y\right)}{n^2 - 1}$

$E_{\mathrm{s}}, E_{\mathrm{c}}$, compression moduli of the surrounding soil and the column, respectively

H, thickness of the soil as well as the column length

k_{v}, vertical permeability coefficient of soil.

M, parameter, $M = \dfrac{2m-1}{2}\pi$, $m = 1, 2, 3, \ldots$

n, parameter, $n = r_{\mathrm{e}}/r_{\mathrm{c}}$.

$r_{\mathrm{c}}, r_{\mathrm{e}}$, radii of the column and the influence zone, respectively

t_{c}, loading period, i.e., the time consumed by the total stress increasing up to the final value

T_{c}, vertical time factor corresponding to the loading period t_{c}, $T_{\mathrm{c}} = \dfrac{c_{\mathrm{v}} t_{\mathrm{c}}}{H^2}$

T_{v}, vertical time factor for the soil, $T_{\mathrm{v}} = \dfrac{c_{\mathrm{v}} t}{H^2}$

U, average degree of consolidation for the composite foundation

$\bar{u}_{\mathrm{s}}$, average excess pore water pressure in the soil at any depth

Y, parameter, $Y = E_{\mathrm{c}} / E_{\mathrm{s}}$

β_m, a series of positive constant

$\sigma(z,t)$, total stress increment caused by an external load in the composite foundation

$\bar{\sigma}_{\mathrm{s}}, \bar{\sigma}_{\mathrm{c}}$, average total stress increment in the surrounding soil and the column, respectively

$\sigma_{\mathrm{T}}, \sigma_{\mathrm{B}}$, average total stress increment at the top and bottom of the composite foundation, respectively

ε_{v}, vertical strain of the soil and the column at any depth

γ_{w}, unit weight of water

References

Castro, J., and Sagaseta, C. (2009). "Consolidation around stone columns. Influence of column deformation." International Journal for Numerical and Analytical Methods in Geomechanics, 33(7): 851-877.

Han, J., and Ye, S. L. (2002). "A theoretical solution for consolidation rates of stone column-reinforced foundations accounting for smear and well resistance effects." International Journal of Geomechanics, 2(2):135-151.

Indraratna, B., Aljorany, A., and Rujikiatkamjorn, C. (2008). "Analytical and numerical modeling of consolidation by vertical drain beneath a circular embankment." International Journal of Geomechanics, 8(3):199-206.

Leo, C. J. (2004). "Equal strain consolidation by vertical drains." Journal of Geotechnical and Geoenvironmental Engineering, 130(3):316–327.

International Journal of Geomechanics. Submitted August 1, 2010; accepted August 25, 2011;
posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

Lu, M. M., Xie, K. H., and Guo, B. (2010). "Consolidation theory for a composite foundation considering radial and vertical flows within the column and the variation of soil permeability within the disturbed soil zone." Canadian Geotechnical Journal, 47(2): 207-217.

Miao, L., Wang, X., and Kavazanjian, E. Jr. (2008). "Consolidation of a Double-Layered Compressible Foundation Partially Penetrated by Deep Mixed Columns." Journal of Geotechnical and Geoenvironmental Engineering, 134(8):1210–1214.

Terzaghi, K. (1943). "Theoretical Soil Mechanics." John Wiley ＆ Sons, New York.

Walker, R., and Indraratna, B. (2006). "Vertical drain consolidation with parabolic distribution of permeability in smear zone." Journal of Geotechnical and Geoenvironmental Engineering, 132(7):937-941.

Walker, R., and Indraratna, B. (2007). "Vertical drain consolidation with overlapping smear zones." Geotechnique, 57(5):463-467.

Walker, R., and Indraratna, B. (2009). "Consolidation analysis of a stratified soil with vertical and horizontal drainage using the spectral method." Geotechnique, 59(5): 439–449.

Xie, K. H., Lu, M. M., Hu, A. F., Chen, G. H. (2009). "A general theoretical solution for the consolidation of a composite foundation." Computers and Geotechnics, 36(1-2): 24-30.

Xie, K. H., Lu, M. M., and Liu, G. B. (2009). "Equal strain consolidation for stone-column reinforced foundation." International Journal for Numerical and Analytical Methods in Geomechanics, 33(15): 1721-1735.

Zhu, G., and Yin, J. H. (1998). "Consolidation of soil under depth- dependent ramp load." Canadian Geotechnical Journal, 35(2): 344-350.

Zhu, G. and Yin, J. H. (2004). "Consolidation analysis of soil with vertical and horizontal drainage under ramp loading considering smear effects." Geotextiles and Geomembranes, 22(1-2):63-74.

International Journal of Geomechanics. Submitted August 1, 2010; accepted August 25, 2011; posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

Figure Captions

Fig. 1. Schematic diagram for the consolidation of a composite foundation with an undrained column

Fig. 2. Stress increment caused by the external load varying with (a) time and (b) depth

Fig. 3. Comparison of three types of foundations

Fig. 4. Influence of the distribution of the stress increment along the vertical direction on the average degree of consolidation

Fig. 5. Influence of the loading period on the average degree of consolidation

Fig. 6. Influence of the column-soil constrained modulus ratio on the average degree of consolidation

Fig. 7. Influence of the radius ratio of the influence zone to the column on the average degree of consolidation

Fig. 8. Distribution of the excess pore water pressures along the vertical direction at a given time

Fig. 9. Dissipation of the excess pore water pressure with time at various loading periods

Fig. 10. Development of the total column-to-soil stress ratio with time at a given depth

Figure 01

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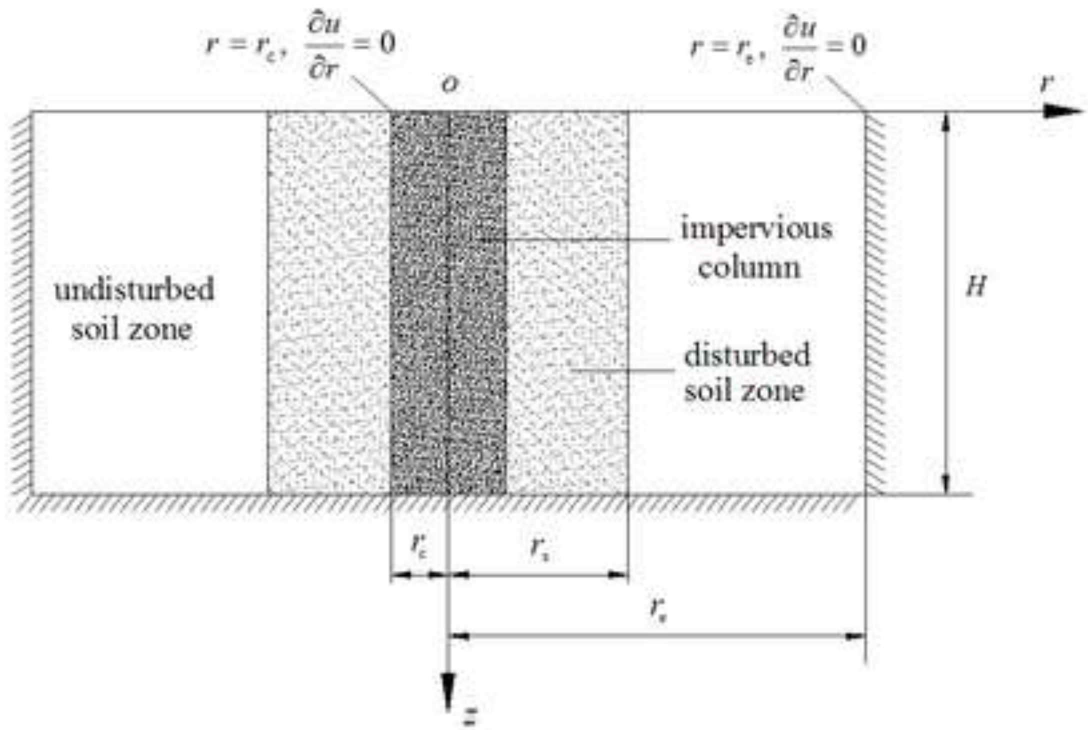

Figure 02

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posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

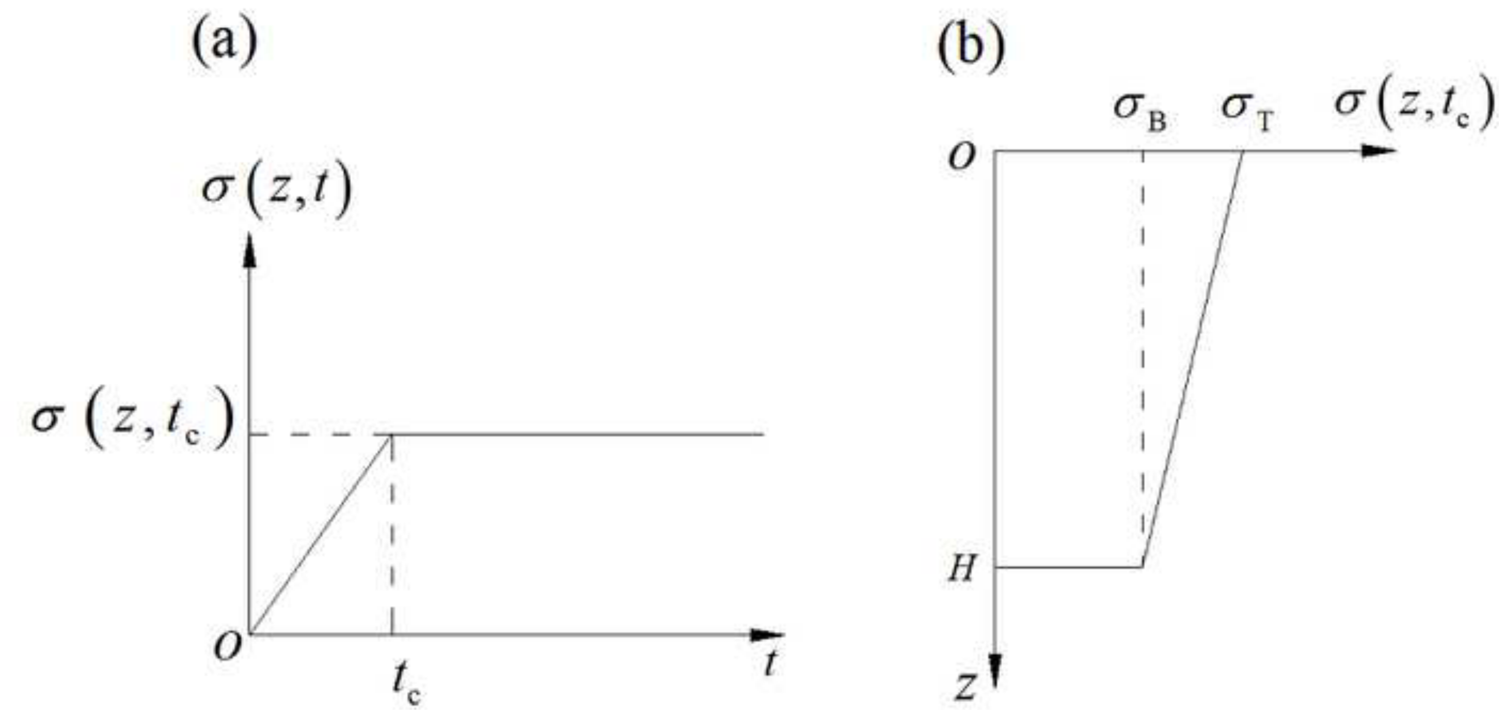

Figure 03

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posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

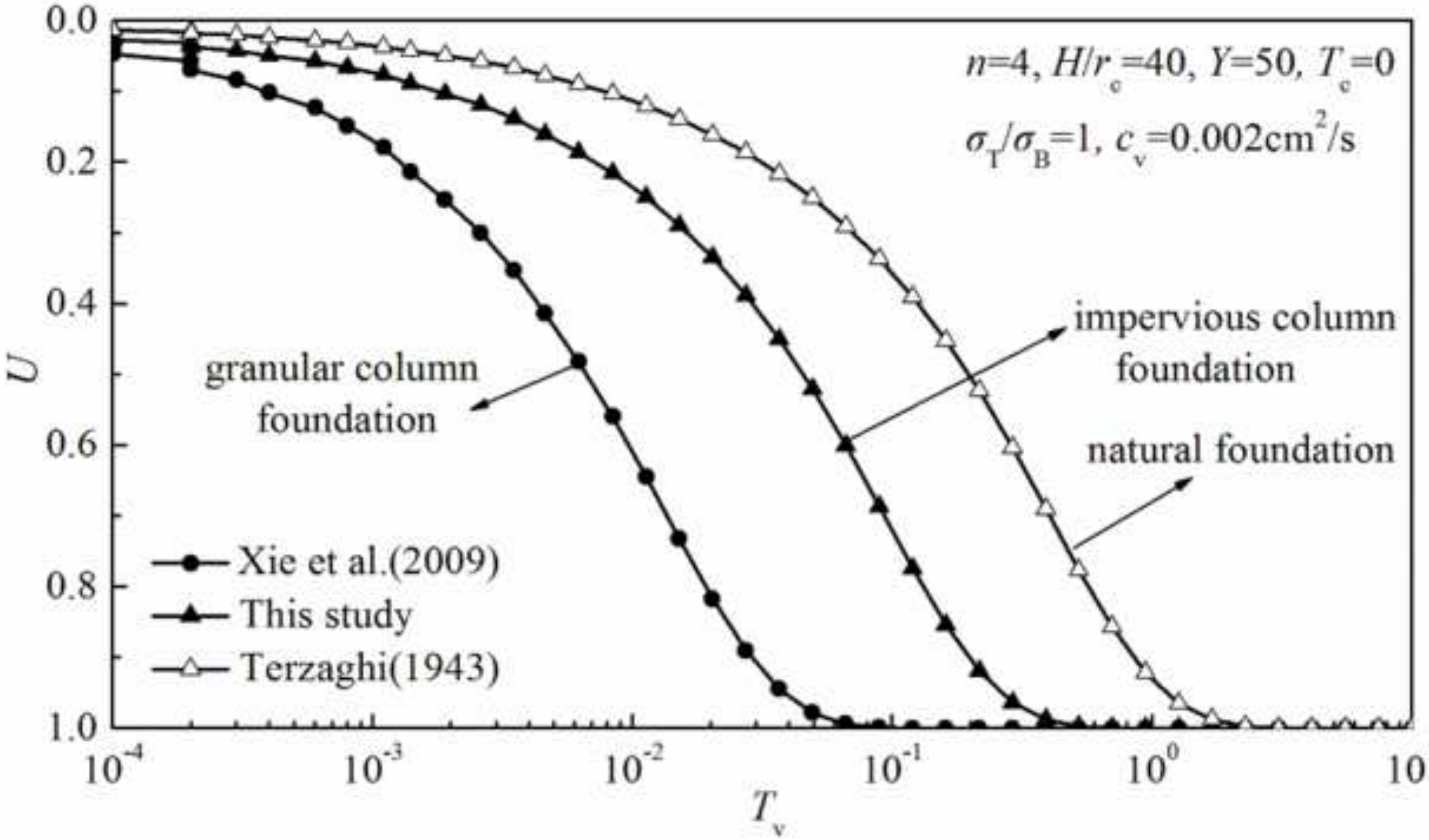

Figure 04

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posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

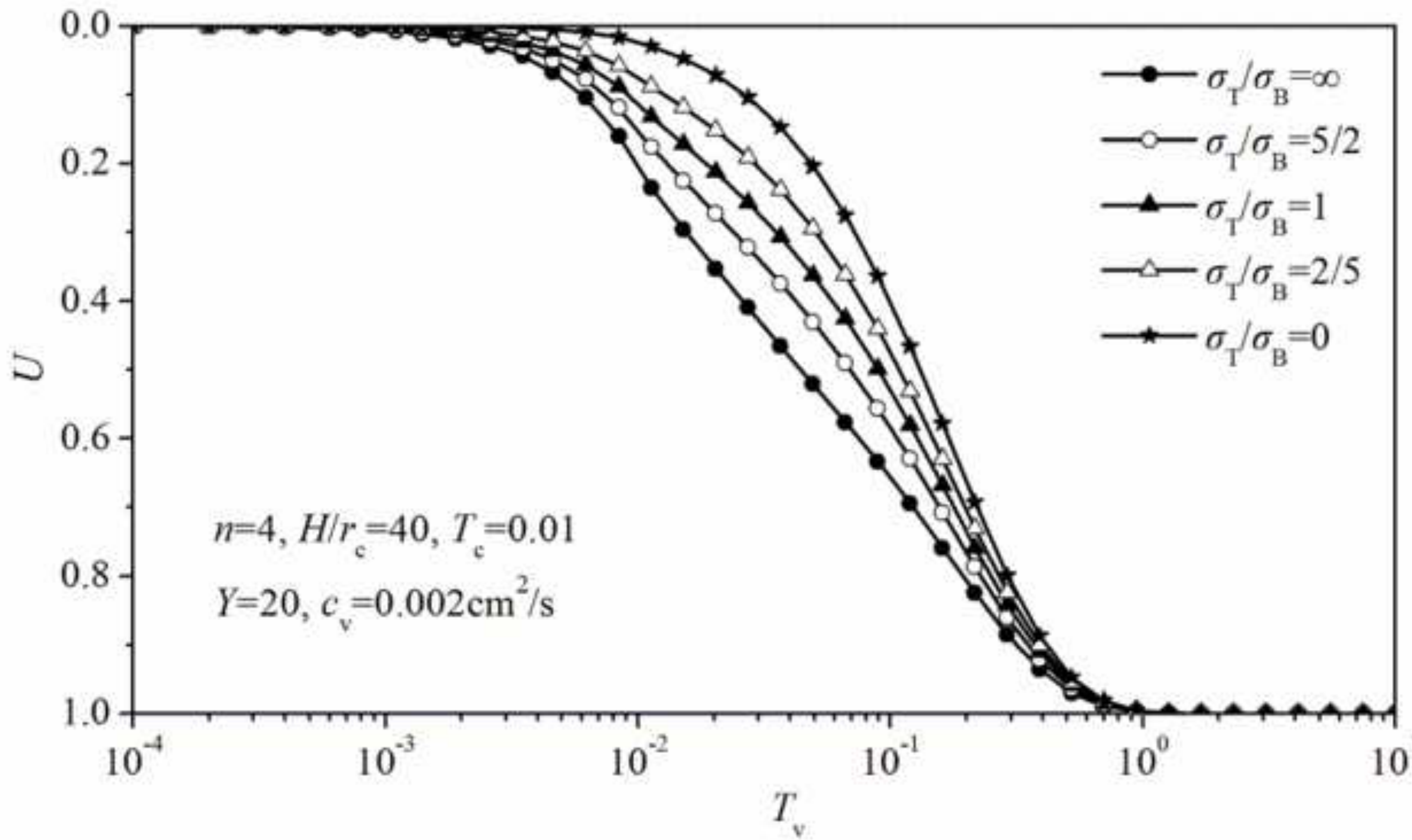

Figure 05

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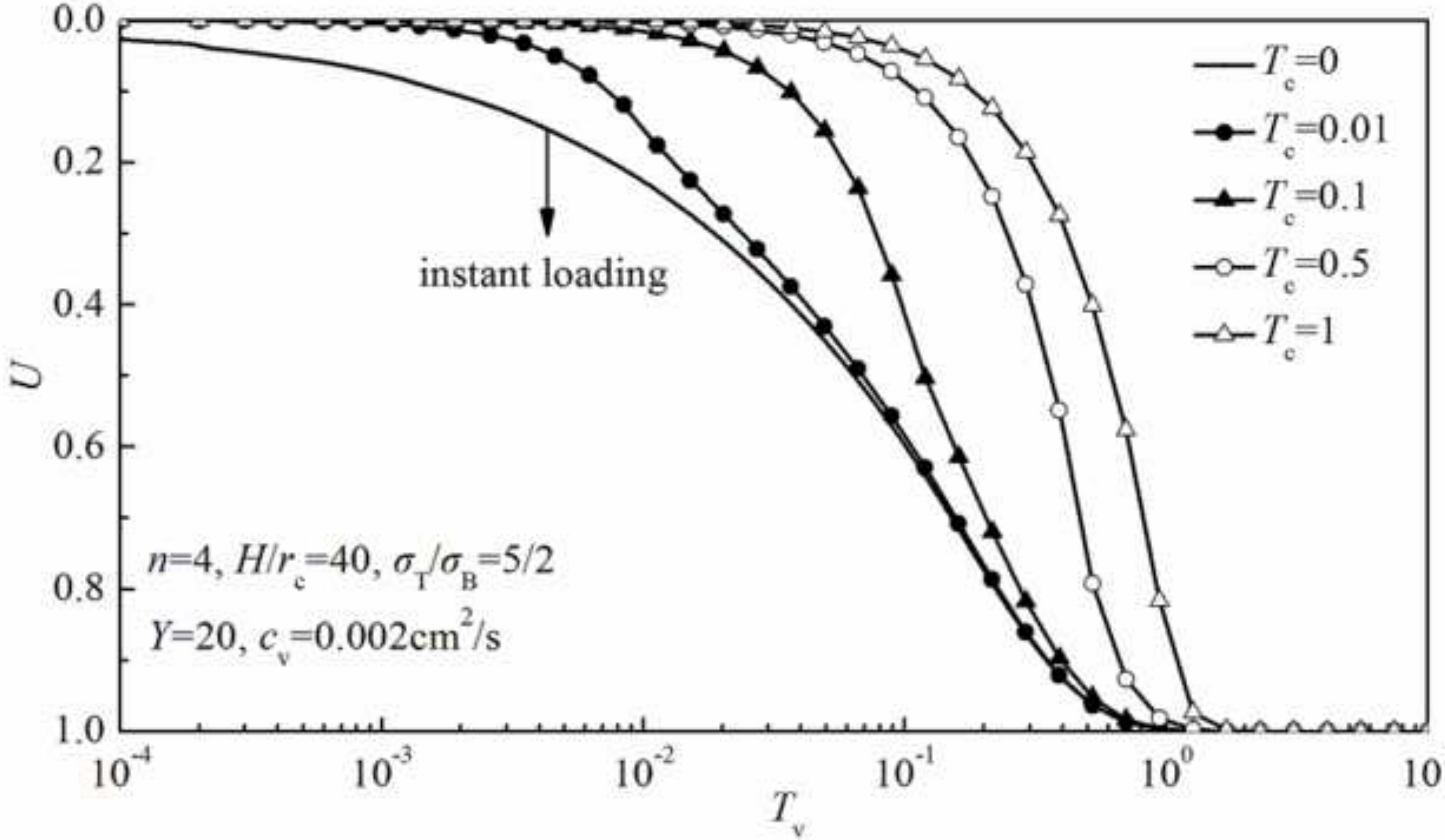

Figure 06

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posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

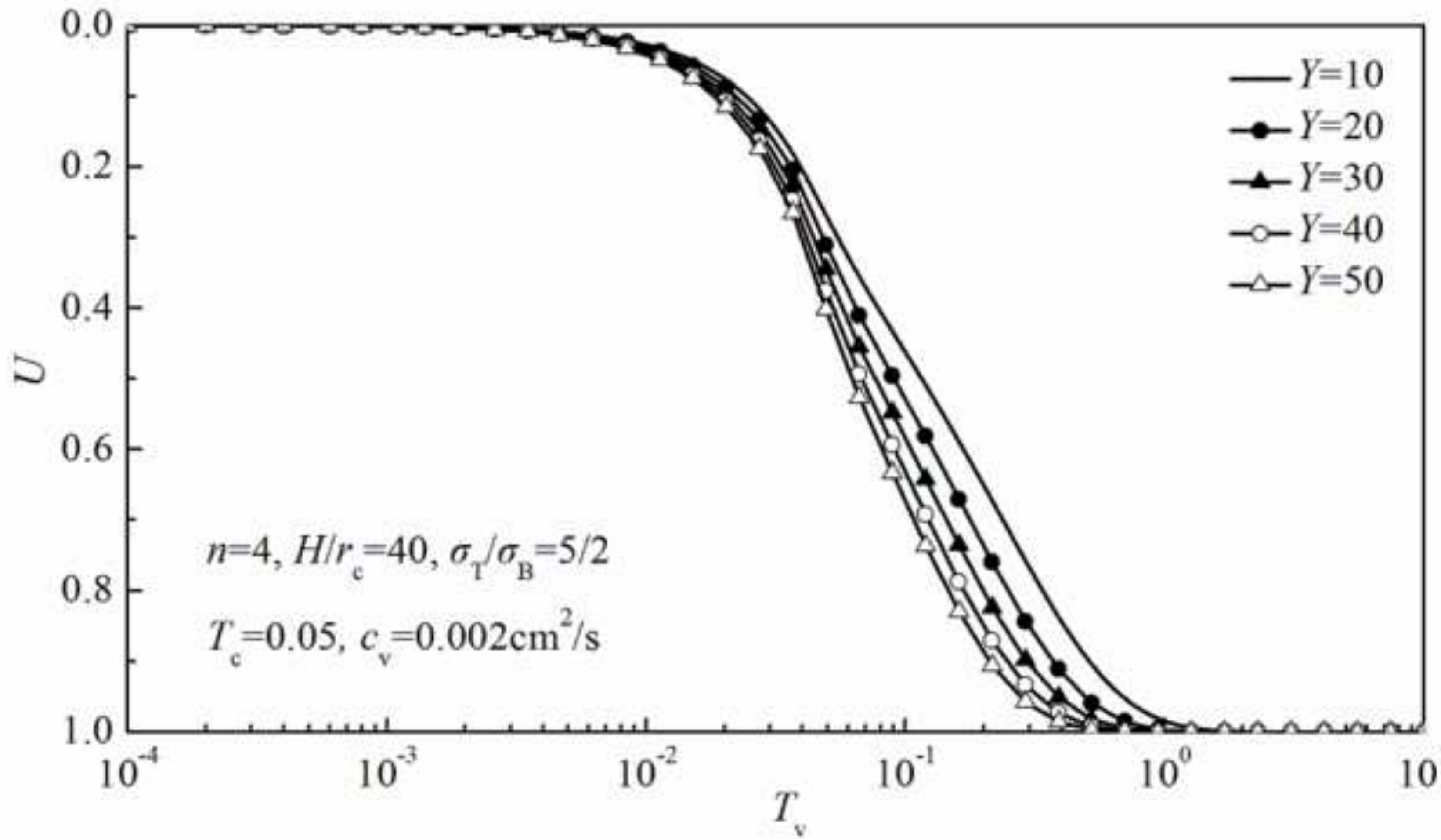

Figure 07

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posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

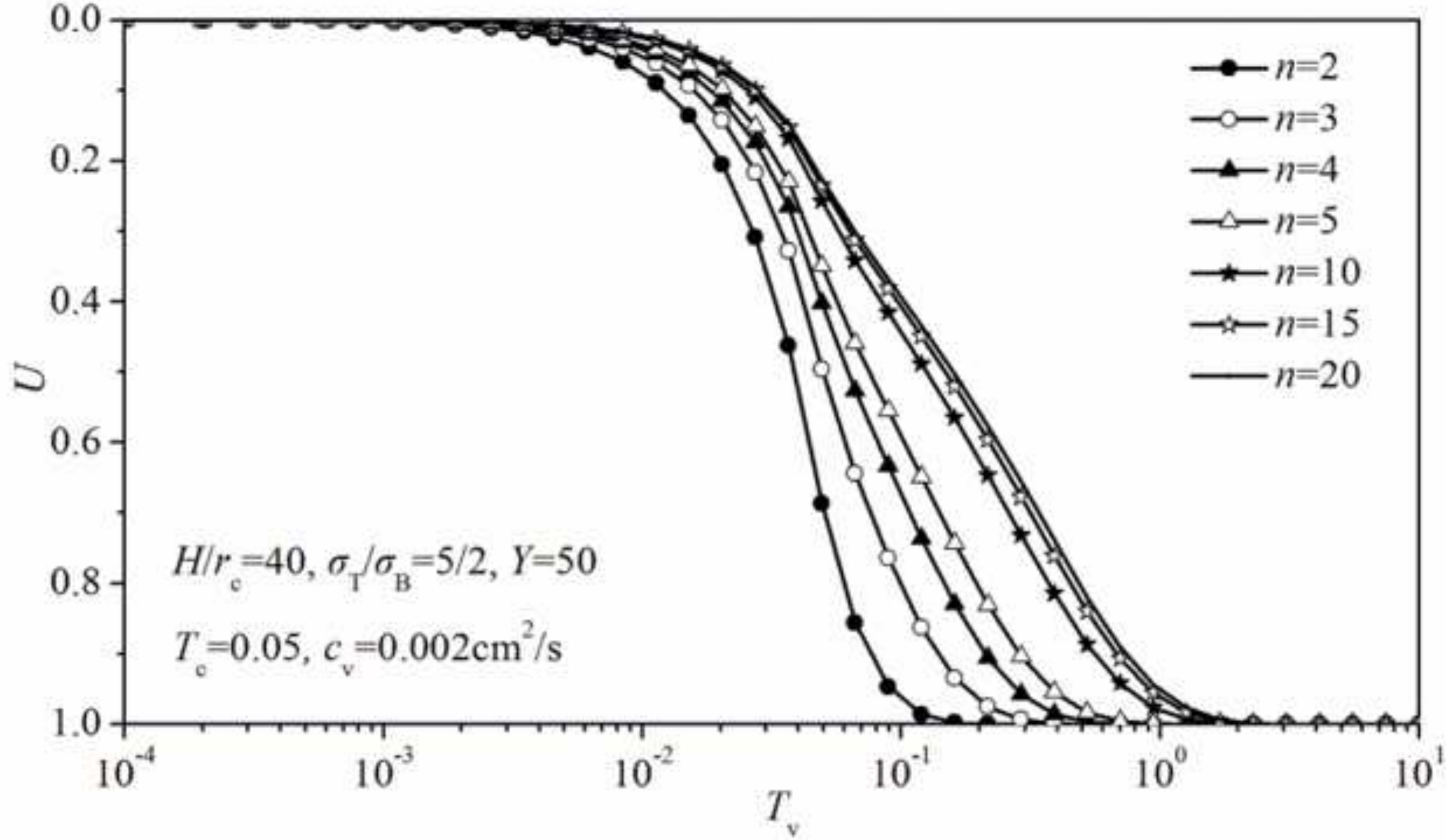

Figure 08

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posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

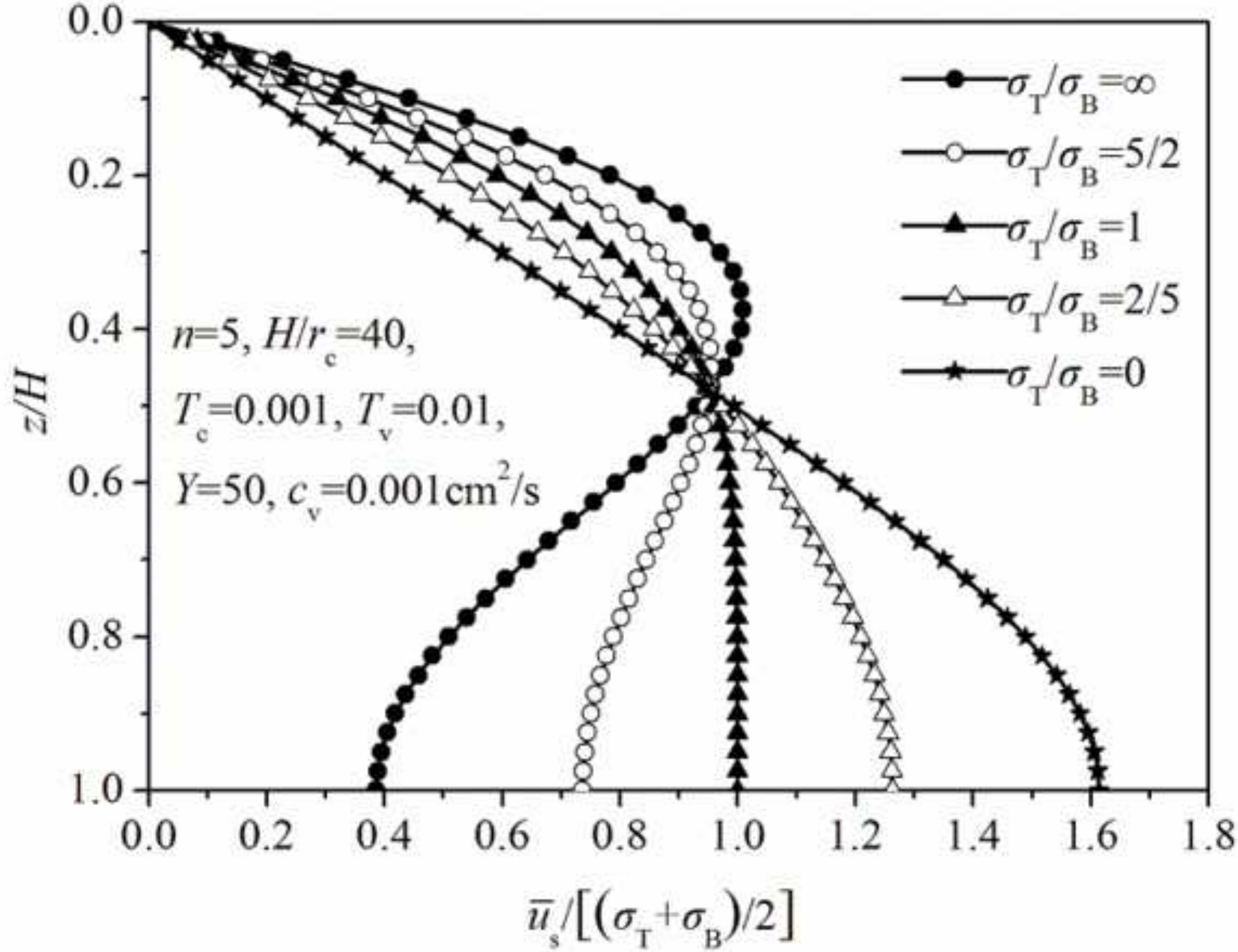

Figure 09

International Journal of Geomechanics. Submitted August 1, 2010; accepted August 25, 2011;
posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176

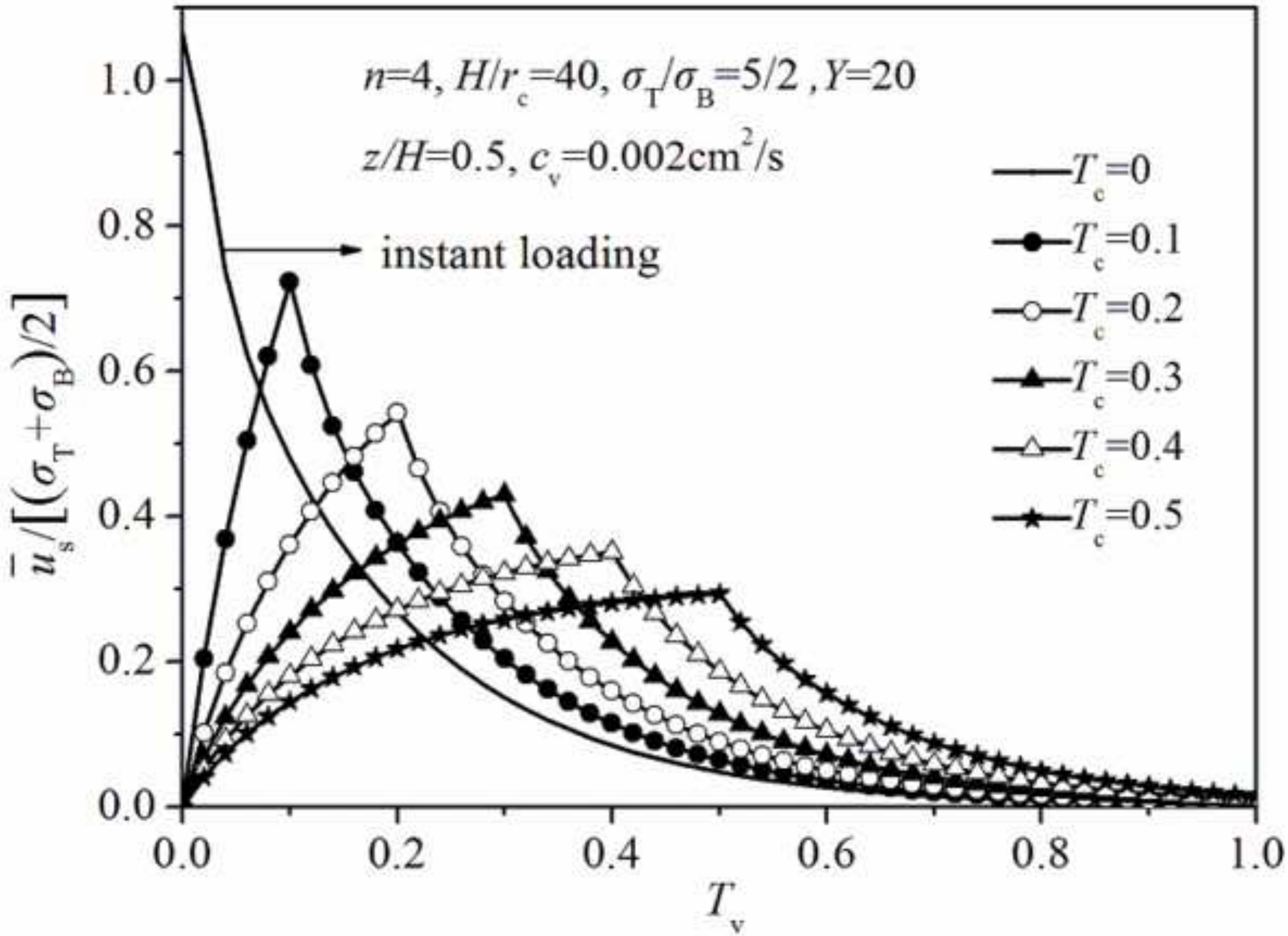

Figure 10

International Journal of Geomechanics. Submitted August 1, 2010; accepted August 25, 2011;
posted ahead of print August 27, 2011. doi:10.1061/(ASCE)GM.1943-5622.0000176