Conclusion: The dependence of electron mobility μ_n on sheet electron concentration in the 2DEG channel has been measured for the first time in AlGaN/GaN MOSHFETs. The $\mu_n(n_s)$ dependence is measured for n_s values up to $1.3 \times 10^{13}\,cm^{-2}$. The maximum value of $\mu_n \simeq 1400\,cm^2/Vs$ observed at $n_s = 7 \times 10^{12}\,cm^{-2}$, is very close to the value extracted from Hall measurements on the same wafer. It implies that 2DEG properties do not degrade with device processing. As the gate-bias voltage V_G reaches the threshold-voltage V_T ($V_G \rightarrow V_T$) the measured value of μ_n approaches ~$100\,cm^2/Vs$. This value is very close to the value of electron-mobility μ_n in bulk GaN layers.

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Comment

Electro-thermal device and circuit simulation with thermal nonlinearity due to temperature dependent diffusivity

K. Krabbenhoft and L. Damkilde

Introduction: Recently, a semi-analytical method for dealing with the transient diffusion equation with temperature dependent thermal properties has been proposed by Batty and Snowden [1]. However, in the derivation of the method a mistake is made which may lead to inaccurate results, especially in the case where the thermal properties vary strongly with the temperature.

For the sake of convenience we shall consider the one dimensional diffusion equation without sources

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) \tag{1}$$

where T is temperature, t is time, ρ is density, c is specific heat, and $k(T)$ is temperature dependent conductivity. This equation is nonlinear by virtue of the conductivity being temperature dependent. The nonlinearity becomes even more apparent when the equation is rewritten as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial k(T)}{\partial T}\left(\frac{\partial T}{\partial x}\right)^2 + k(t)\frac{\partial^2 T}{\partial x^2} \tag{2}$$

Kirchhoff transformation: To reduce the nonlinearity of the governing eqn. 1 the well known Kirchhoff transformation can be used. We define a new variable ψ by

$$\psi = \int_0^T k(T)dt \tag{3}$$

Since $d\psi/dT = k(T)$ we have

$$k(T)\frac{\partial T}{\partial t} = \frac{\partial \psi}{\partial x} \tag{4}$$

which inserted into eqn. 1 gives

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} \tag{5}$$

and by again using $d\psi/dT = k(T)$ we obtain

$$\frac{1}{\kappa(\psi)}\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} \tag{6}$$

where

$$\kappa = \frac{k(T)}{\rho c} \tag{7}$$

Since κ is a function of ψ (or T) the governing equation is still nonlinear if $\partial T/\partial t$ is different from zero. However in the steady state case with $\partial T/\partial t = 0$ the Kirchhoff transformation results in a linear equation which can be solved exactly as done by Batty *et al.* [2].

Time transformation: To further reduce the nonlinearity of eqn. 6 Batty and Snowden [1] use a transformation of time by defining a new time variable t' by

$$t' = \int_0^t \kappa(\psi)dt \tag{8}$$

The derivative of ψ with respect to t can be expressed as

$$\frac{\partial \psi}{\partial t} = \frac{\partial t'}{\partial t}\frac{\partial \psi}{\partial t'} = \kappa(\psi)\frac{\partial \psi}{\partial t'} \tag{9}$$

and the left-hand side of eqn. 6 can then be reduced to

$$\frac{1}{\kappa}\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial t'} \tag{10}$$

However, the authors overlook the fact that since t' is a function of both t and x the right-hand side of eqn. 6 is also influenced by the variable transformation. Formally, we introduce a new variable $x' = x$, and using the chain rule we then obtain

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial \psi}{\partial t'}\frac{\partial t'}{\partial x} = \frac{\partial \psi}{\partial x'} + \frac{\partial \psi}{\partial t'}\frac{\partial t'}{\partial x} \tag{11}$$

where $\partial t'/\partial x$ can be written as

$$\frac{\partial t'}{\partial x} = \int_0^t \frac{\partial \kappa}{\partial x}dt \tag{12}$$

Inserting eqns. 10 and 11 into eqn. 6 we have

$$\frac{\partial \psi}{\partial t'} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x'} + \frac{\partial \psi}{\partial t'}\frac{\partial t'}{\partial x}\right) \tag{13}$$

This can be expanded to

$$-\left(\frac{\partial t'}{\partial x}\right)^2 \frac{\partial^2 \psi}{\partial t'^2} + \left(1 - \frac{\partial^2 t'}{\partial x^2}\right)\frac{\partial \psi}{\partial t'} = \frac{\partial^2 \psi}{\partial x'^2} + 2\frac{\partial t'}{\partial x}\frac{\partial^2 \psi}{\partial x' \partial t'} \tag{14}$$

which obviously offers no improvement in terms of complexity over the original eqn. 6.

Example: To illustrate the effect of neglecting the dependence between t' and x we consider the conduction of heat into a semi-infinite one dimensional slab. The boundary and initial conditions are

$$T(0,t) = 1.0 \qquad T(x,0) = 0.0 \tag{15}$$

The conductivity varies with the temperature as

$$k = \frac{1}{1 - \lambda T} \tag{16}$$

and we set $\rho c = 1.0$.

For this problem Fujita [3] has obtained an exact solution for $\lambda = 0.965$. This solution is shown in Fig. 1 together with the solution obtained by the Batty-Snowden method. As seen, there is a significant deviation between the computed result and the exact solution. Only in the beginning and towards the end do the two solutions match since we here have a constant temperature field. In this case κ is also constant and the term $\partial t'/\partial x$ in eqn. 14 vanishes.

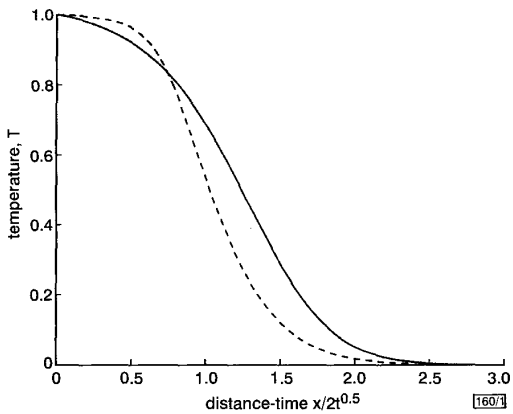

Fig. 1 *Temperature distribution*

——— exact (Fujita [3])
– – – – Batty-Snowden method [1]

Conclusion: The recently proposed method for solving the nonlinear diffusion problem by using a transformation of time has been shown to contain a mistake which may lead to inaccurate results. Whereas a Kirchhoff transformation in the steady state case is advantageous, nothing is gained by using a transformation of time. Thus, in the general case with an arbitrary temperature dependence of the thermal properties we are left with numerical methods such as the finite difference method (see e.g. Crank [4]) or the finite element method (see Lewis *et al.* [5]) as the only way of establishing satisfactory solutions.

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Reply to Comment on 'Electro-thermal device and circuit simulation with thermal nonlinearity due to temperature dependent diffusivity'

W. Batty, S. David and C.M. Snowden

Introduction: The comments made by Krabbenhoft and Damkilde [1] help clarify the approximation involved in the thermal impedance matrix implementation of the time variable transformation [2]. Under successive Kirchhoff transformation, $T \rightarrow \theta$, and time variable transformation, $t \rightarrow r$, the heat diffusion equation

$$\nabla \cdot [\kappa(T)\nabla T] + g(x,y,z,t) = \rho C \frac{\partial T}{\partial t} \tag{1}$$

becomes [2]

$$\nabla^2\theta + \frac{g}{\kappa_S} = \frac{1}{\kappa_S}\frac{\partial \theta}{\partial \tau} \tag{2}$$

This transformation is exact and eqn. 2 is expressed in the same form as eqn. 38 of [3]. The Laplacian, $\nabla^2_{\tau\tau}$, with τ held constant, takes the form described in [1]. (Note, however, that the term $\partial^2 t'/\partial x^2$ in eqn. (14) of [1] is identically zero for smooth functions, as it can be written more explicitly, $\partial^2 t'/\partial x^2)_{t't}$, where the subscripts denote t' and t held constant in successive differentiations.)

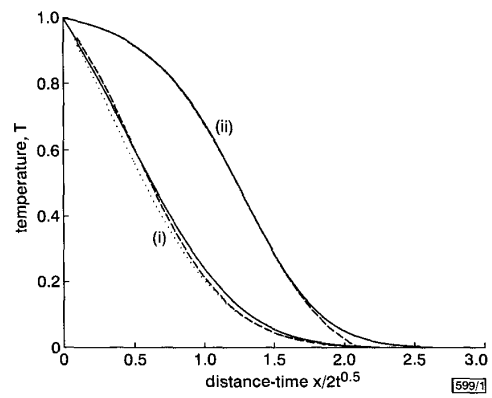

Fig. 1 *Exact solution, time variable transformation and extended transformation*

(i) $\lambda = 0.5$
——— exact solution (Fujita [4])
– – – – time variable transformation
····· Kirchhoff transformation
(ii) $\lambda = 0.965$
——— exact solution (Fujita [4])
– – – – extended transformation

The advantage of the transformed equation, eqn. 2, is that for moderate temperature dependences, $\nabla^2_{\tau\tau}$ can be reasonably approximated by its conventional rectangular Cartesian form, and eqn. 2 solved exactly analytically in regular volumes with general linear boundary conditions. To illustrate this approximation, consider the one-dimensional system treated analytically by Fujita [1, 4], with $\lambda = 0.5$ in $k(T) = 1/1 - \lambda T$, for $0 \leq T \leq 1$. This degree of temperature dependence is comparable with that of $k(T)$ for semiconductors, over temperature ranges of interest, as shown in Fig. 1 of [2]. In Fig. 1 curve (i), the solid line shows the exact Fujita solution for $\lambda = 0.5$; curve (i), dashed line shows the corresponding time variable transformation solution; and curve (i), dotted line, shows the result of the Kirchhoff transformation alone. The time variable transformation is seen to improve agreement beyond that of the lone Kirchhoff transformation, and offers an alternative to use of a mean diffusivity, k_{mean}, the choice of which is not uniquely defined. The curves look closely spaced, but correspond to errors in rise times of ~25%.

The temperature dependence considered by Krabbenhoft and Damkilde in [1] is extreme. In terms of Kirchhoff transformed temperature θ, $k(\theta) = \exp(\lambda\theta)$, with $\lambda = 0.965$ and $0 \leq \theta \leq 3.47$. To describe such strong nonlinearity, the position dependence of the transformed time, τ, must be fully treated. The linearisation