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LETTERS TO THE EDITOR

DISCUSSION OF 'EFFECTS OF DRIVING AND SUBSEQUENT CONSOLIDATION ON BEHAVIOUR OF PILES'

(C. S. Desai, *Int. J. Numer. Anal. Methods Geomech.*, **2**, 283–301 (1978)

Desai has presented an interesting example analysis of the axisymmetric consolidation which occurs round a driven pile. The initial stress distribution at the start of consolidation is obtained by modelling the installation of the pile as the creation of a cylindrical cavity by radial soil movement in an ideal elastic perfectly plastic material. The analysis closely follows that described by Butterfield and Banerjee[1] and is believed to be in error, leading to incorrect expressions for the radius r_0 of the plastic zone and thus for the stress changes in the elastic and plastic zones.

The expression for the strains adopted by Desai and by Butterfield and Banerjee is valid provided that u is taken as the displacement of a point located *originally* at radius r. Thus the boundary conditions which they state are inconsistent. If a cylindrical cavity of radius a is created then $u = a$ when $r = 0$ and not when $r = a$ as suggested by Desai's equation (4a).

The strains round the created cavity are certainly far from small and so it is incorrect to use the equation $\varepsilon_r + \varepsilon_\theta = 0$ as a condition of no volume change. Using the expressions adopted in Desai's paper for the strains (where r is the original radius) the constant volume condition should be $(1-\varepsilon_r)(1-\varepsilon_\theta)-1=0$. The term $\varepsilon_r\varepsilon_\theta$ will not be negligible in this case.

In Desai's notation, the stress changes and soil movement in the elastic region around the cavity $(r \geqslant r_0)$ are given by (Hill,[4] Gibson and Anderson[3])

$$\Delta\sigma_r = -\Delta\sigma_\theta = c_u\left(\frac{r_0}{r}\right)^2 \tag{1}$$

$$u = \frac{c_u}{2G}\left(\frac{r_0^2}{r}\right) \Rightarrow u_0 = \frac{c_u}{2G}r_0 \tag{2}$$

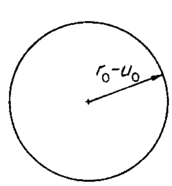
With this latter expression for u_0, the calculation of r_0, the radius of the inner plastic zone, is straightforward. Following Hill[4] (1950), the cavity creation is viewed macroscopically. For the case of zero shear stress on the pile–soil interface (i.e., $\alpha = 0$ in Desai's paper), Figure 1(b) shows the position of the plastic zone after creation of the cavity of radius a.

The initial position of the soil particles which are now on the elastic–plastic boundary ($r = r_0$) is shown in Figure 1(a). Equating the volume of soil inside the outer circle in each case (for undrained cavity expansion) yields

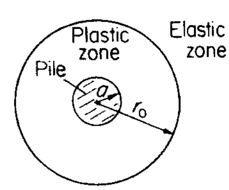
$$\pi(r_0^2 - a^2) = \pi(r_0 - u_0)^2$$
$$= \pi r_0^2\left(1 - \frac{c_u}{2G}\right)^2 \tag{3}$$

whence

$$r_0^2 = (G/c_u)a^2 \tag{4}$$



(a) Before creation of cavity



(b) After creation of cavity

Figure 1. Position of soil particles on the eventual plastic–elastic interface before and after creation of cavity

where terms in $(c_u/2G)^2$ have been ignored since $G \gg c_u$. This expression differs significantly from that given by Desai* and by Butterfield and Banerjee[1] by a factor of 4 (when $\alpha = 0$). This has consequences not only in the calculation of the width of the plastic zone around a driven pile, but also in the expressions for the stress changes in the elastic region (Desai's equation 5(â) is a factor of 4 too large) and in the plastic region.

Analytically, the creation of a cylindrical cavity may be simulated by expanding an existing cavity by a large amount, Carter *et al.*[2] describe a finite element procedure for this and give typical results for a variety of soil models including a work-hardening elasto-plastic model based on critical state concepts. In addition, it is shown that the finite element method gives values for r_0 and the stress changes around a cavity created in an ideal elastic, perfectly plastic material which agree with those deduced by Hill[4] and by Gibson and Anderson,[3] and thus disagree with the expressions given by Butterfield and Banerjee[1] and by Desai.

* There is also a typographical error in Desai's equation (8) for r_0 which should presumably read:

$$r_0^2 = \frac{(\alpha a)^2 + ((\alpha a)^4 + 64\beta^2 a^4)^{1/2}}{2}$$

M. F. RANDOLPH AND J. P. CARTER
University Engineering Department
Cambridge
England

REFERENCES

1. R. Butterfield and P. K. Banerjee, 'The effect of pore water pressures on the ultimate bearing capacity of driven piles', *Proc. 2nd South-East Asian Conf. on Soil Eng.*, Singapore (1970).
2. J. P. Carter, M. F. Randolph and C. P. Wroth, 'Stress and pore pressure changes in clay during and after the expansion of a cylindrical cavity', Univ. Tech. Report, *CUED/C-SOILS/TR 51*, to be published in *Int. J. Numer. Anal. Methods Geomech.*, **3** (1979).
3. R. E. Gibson and W. F. Anderson, 'In-situ measurement of soil properties with the pressure-meter', Civil Engineering and Public Works Review, **56**, 615–618 (1961).
4. R. Hill, *Mathematical Theory of Plasticity*, Oxford University Press, London, 1950.

AUTHOR'S RESPONSE TO RANDOLPH AND CARTER'S DISCUSSION†

The comments and discussion by Randolph and Carter are appreciated. The difference in the approaches used in the paper and that in their work by Randolph and Carter is worth noting and can be useful. However, at this stage of the understanding of the complex problem of consolidation of systems due to changes by driving or penetration, the differences in the final results caused by (two) different approaches cannot be ascertained or judged. Both are theoretical and essentially parametric. Any final judgement should await comprehensive verification of any approach.

In my opinion, what matters is the basic procedure that is capable of solving the boundary value problem with conditions created by driving. At this stage, the former is more important than the latter, because all current approaches for simulating driving and the resulting changes in stresses, including those of the author and of Randolph and Carter, are but (crude) approximations of reality.

The differences cited by Randolph and Carter in the results from the approach used in the paper and their procedure can be due to the different assumptions used in both. Although in view of the apparent simplifications used in the two approaches, it is difficult to be precise from the viewpoint of continuum mechanics, it can be stated that the approach used in the paper is based on an Eulerian formulation, whereas, if $u = a$ when $r = 0$ is used it will be a Lagrangian approach. Furthermore, the two results (for extents of plastic zones) are based on different yield criteria. In view of the assumption of small strain in the approach used by Desai, $\varepsilon_r + \varepsilon_\theta = 0$ is justified, although it may not be physically precise.

Regarding a general procedure, it may be appropriate to draw the attention of Randolph and Carter to the paragraph on General Procedure on page 284 and Figure 1 on page 285 of

† Preceding letter.

the paper. Here the general problem and its complexities are discussed, and a (qualitative) procedure is outlined for simulation of driving. Then, it is explained how, only as an approximation, the cavity expansion approach can be used. Cavity expansion is not used or recommended as the final answer!

For a consistent (Lagrangian) approach, it is necessary to use consistent definitions of stress, strain and properly transformed constitutive laws; such general three-dimensional procedures based on Eulerian and Lagrangian approaches for geomechanics problems have been recently developed and applied.[1,2] Also, a consistent large strain formulation should include relevant stiffness and load terms and changes in the geometry of the systems; simply revising the coordinates of nodal points as the cavity expands can be considered only a partial solution. Hence, although it is good that results of Carter *et al.*[3] compare favourably with previous theoretical solutions, they may not necessarily imply a general or a final solution to the problem. In this connection, it may be mentioned that recently Banerjee and co-workers[4] have shown that their finite element results based on an Eulerian approach compare well with those reported by Butterfield and Banerjee.[5]

In view of the foregoing, Randolph and Carter's belief that the approach used in the paper is in error may not be correct and may be premature.

ACKNOWLEDGEMENT

I wish to thank Dr. P. K. Banerjee, University of Wales, Cardiff, United Kingdom for useful comments on the draft of this response.

C. S. DESAI

REFERENCES

1. C. S. Desai, 'Three-dimensional geometric and material nonlinear analysis of moving structures', A Special Lecture, *Second Intl. Conf. on Computational Methods in Nonlinear Mechanics*, University of Texas, Austin, Texas (1978).
2. H. V. Phan, C. S. Desai, S. Sture and J. V. Perumpral, 'Three-dimensional soil–structure interaction analysis of moving structures', *U.S.–Japan Seminar on Interdisciplinary Finite Element Analysis*, Cornell University, Ithaca, New York (1978).
3. J. P. Carter, M. F. Randolph and C. P. Wroth, 'Stress and pore pressure changes in clay during and after the expansion of a cylindrical cavity', Univ. Tech. Report, *CUED/C-SOILS/TR 51*, University of Cambridge, England, to be published in *Int. J. Numer. Anal. Methods Geomech.*, **3** (1979).
4. R. C. Fathallah and P. K. Banerjee, 'Theoretical and experimental investigation of the behaviour of axially loaded single pile driven into saturated clays', University of Wales, Cardiff (1978).
5. R. Butterfield and P. K. Banerjee, 'The effect of pore water pressures on the ultimate bearing capacity of driven piles', *Proc. 2nd South-East Asian Conf. on Soil Engineering*, Singapore (1970).

CONFERENCE DIARY

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The main objective of the conference is to provide a forum for discussion between researchers and designers on relevant topics such as constitutive relations of soils, cyclic degradation, liquefaction shakedown, calibration of numerical models, field measurement and prediction of cyclic and dynamic response. Contributions relating to applications in the field of off-shore structures, nuclear engineering, dams, traffic engineering and other fields where soils are subject to cyclic or transient loading are welcome.

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Dr. G. N. Pande,
Department of Civil Engineering,
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