Design Method for Stabilization of Slopes with Piles[a]

Discussion by Robert W. Day,[4] Fellow, ASCE

The authors have prepared an interesting paper on the use of piles to stablize slopes. Rather than using piles, the discusser prefers to use drilled piers to stabilize slopes. The advantages of drilled piers are as follows:

- There are no vibrations, because the piers are not driven into the ground.
- The drilled shafts can be down-hole logged to confirm embedment depths.
- Tieback anchors can be directly installed to the top of the pier without the need of a beam to connect all of the pile heads.

DESIGN OF PIER WALLS

A common approach for the design of pier walls is to use slope stability analysis. The factor of safety of the slope, when stabilized by the pier wall, is selected first. Depending on such factors as the size of the slope failure, the proximity of critical facilities, or the nature of the pier wall stabilization (temporary versus permanent), a factor of safety of 1.2–1.5 is routinely selected. The next step is to determine the lateral design force (P_L) that each pier must resist to increase the factor of safety of the slope up to the selected value.

Fig. 13 shows an unstable slope having a planar slip surface inclined at an angle (α) to the horizontal. This mode of failure is similar to the Portuguese Bend landslide (Ehlig 1986, 1992) and the eventual wedge failure of the Desert View Drive fill embankment (Day 1996). The shear strength of the slip surface can be defined by the effective friction angle (ϕ') and the effective cohesion (c'). The factor of safety (FS) of the slope with the pier wall is derived by summing forces parallel to the slip surface and is written as

$$\text{FS} = \frac{\text{Resisting Forces}}{\text{Driving Force}} = \frac{c'L + (W\cos\alpha - uL)\tan\phi' + P_i}{W\sin\alpha} \tag{15}$$

where L = length of the slip surface; W = total weight of the failure wedge; u = average pore water pressure along the slip surface; and P_i = required pier wall force that is inclined at an angle (α), as shown in Fig. 13.

The elements of (15) are determined as follows:

- α and L: The angle of inclination (α) and the length of the slip surface (L) are based on the geometry of the failure wedge.
- W: Samples of the failure wedge material can be used to obtain the unit weight. By knowing the unit weight, the total weight (W) of the failure wedge (Fig. 13) can be calculated.
- ϕ' and c': The shear strength parameters (ϕ' and c') can be determined from laboratory shear testing of slip surface specimens.
- u: By installing piezometers in the slope, the pore water pressure (u) can be measured.
- FS: As previously mentioned, a factor of safety (FS) of 1.2–1.5 is routinely selected for the slope stabilized with a pier wall.

The only unknown in (15) is P_i (inclined pier wall force; Fig. 13). The following equation can be used to calculate the lateral design force (P_L) for each pier having an on-center spacing of S:

$$P_L = SP_i\cos\alpha \tag{16}$$

The location of the lateral design force (P_L) is ordinarily assumed to be at a distance of 1/3 H above the slip surface, where H is defined in Fig. 13. The lateral design force (P_L) would be resisted by passive pressure exerted on that portion of the pier below the slip surface.

ARCHING BETWEEN ADJACENT PIERS

An important consideration in the design of pier walls is the pier spacing. Table 1 presents recommendations for maximum spacing of piers versus soil or rock type. This table was based on the performance of existing pier walls. In general, the pier spacing should be decreased as the rock becomes more fractured or the soil becomes more plastic.

SUMMARY

Rather than driven piles, the discusser believes that pier walls are a better method to stabilize slopes. For the design of each pier, the lateral design force (P_L) must be calculated. If there is a distinct slip surface with a wedge type failure (Fig.

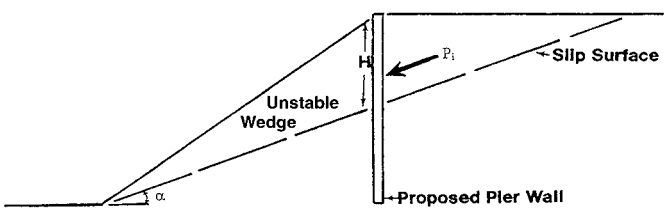

FIG. 13. Design of Pier Wall for Wedge Slope Failure

TABLE 1. Allowable Pier Spacing

Material type (1)	Maximum on-center pier spacing (2)
Intact rock	No limit
Fractured rock	4D
Clean sand or gravel	3D
Clayey sand or silt	2D
Plastic clay	1.5D

Note: D = diameter of pier.

FIG. 14. Construction of Concrete Piers at Desert View Drive

[a]April 1997, Vol. 123, No. 4, by S. Hassiotis, J. L. Chameau, and M. Gunaratne (Paper 10896).

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FIG. 15. Construction of Pier Wall (Arrow Points to Tieback Anchor)

13), then the lateral design force (P_L) can be calculated from slope stability analysis by using (15) and (16).

Table 1 presents the maximum on-center spacing of piers. The pier spacing should be decreased as the rock becomes more fractured or the soil becomes more plastic.

Figs. 14 and 15 show pictures of the installation of pier walls to stabilize slopes. In Fig. 14, the piers were designed as simple cantilevered members, whereas in Fig. 15, tie-back anchors were installed at the top of the piers to reduce the bending moments and shear forces in the piers.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

c' = effective cohesion;
D = diameter of pier;
FS = factor of safety;
H = distance from slip surface to ground surface, as defined in Fig. 13;
L = length of slip surface;
P_i = required pier wall force that is inclined at an angle (α);
P_L = lateral design force for each pier;
S = on-center spacing of piers;
u = average pore water pressure along slip surface;
W = total weight of failure wedge;
α = angle of inclination of slip surface;
γ = total unit weight; and
ϕ' = effective friction angle.

Discussion by T. S. Hull[5] and H. G. Poulos,[6] Fellow, ASCE

Analysis of the influence of piles on the stability of slopes with marginal factors of safety has attracted the interest of engineers for many years, but it still remains a problem, with no definitive approach that has found universal approval. The contribution of the authors is welcomed and represents a paper that the discussers feel sure will attract much attention from practitioners and theorists.

In this vein, the discussers would like to draw attention to some points of interest.

The paper has focused upon the friction circle method (Scott 1969), which perhaps warrants some comment from the authors. The friction circle method requires that the forces due to the cohesive and frictional soil resistance and the force due to the self-weight of the slipping soil are coincident at a point. As pointed out in the paper, in the original friction circle analysis, the resultant of the cohesive component of soil strength is assumed to act at a radius larger than the radius of the slip circle in order that the moment induced by the resultant force equals that developed from a uniform shear stress around the slip circle. The ratio of the larger radius to the slip circle radius is given by the ratio of the arc length of the slip circle to its chord length. It is not clear in the analysis presented in the paper how the resultant pile-soil interaction force F_p is treated to ensure the moments in the analysis are in equilibrium.

Using Bishop's modified slip circle analysis for the slope that is depicted in Fig. 3 of the paper, the discussers obtain a similar circle but a somewhat different result (FS = 1.12) to that of the authors (FS = 1.08). The discussers' use of the tabulated values of Taylor's friction circle analysis also gives a slightly higher factor of safety for the slope, with no piles, of 1.11. This discrepancy in itself is not important, but it does set the scale of what might be taken as commonly experienced differences in measures of a factor of safety from two methods.

The concept of the piled slope having a different critical circle to that of a slope without piles is presumably well known. However, it seems to have been presented in parts of the paper in a manner that does not clearly indicate the inapplicability of considering the unpiled critical circle when piles are incorporated, unless it also happens to be the critical circle of the slope with piles installed. It is worth mentioning that the analysis of the slope without the piles present will give the range of positions within which to place piles. Any piles outside the region of soil affected by the critical circle that was found without piles in the slope can theoretically have no influence on slope stability.

The use of piles towards the top of a slope must be viewed cautiously in light of the possibility of tension failure leading to a small mass of soil restrained above the piles while the lower region of soil fails. It is also in this region near the top of the slope that the axial response of the piles may become more important than is normally considered, since the soil movement is now predominantly down the piles and not laterally across the piles. This feature of pile response leads to consideration of the distributed soil-pile load used by the authors.

The chosen distribution of soil load on the piles depends upon the early work of Ito and Matsui (1975), which generated some discussion from De Beer and Carpentier (1977) and a closure in reply from Ito and Matsui (1978). It may be assumed that the undefined symbol A_c appearing in (1) of the authors' paper is c multiplied by A and a later appearance of A in (8) is not related to the A appearing in the equation from Ito and Matsui. The arguments about the applicability of the stress assumptions that were presented in the discussion and reply referenced above suggest that the chosen distribution is only one of many that are possible, and some comment from the authors concerning their choice would be appreciated.

In particular, in Fig. 3, the distribution of pile load due to the movement of soil in the region of the slip circle is depicted

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as having an inclined resultant force. The equation of Ito and Matsui is based upon an analysis of the extrusion of metal through dies and assumes plane strain conditions. In the current application by the authors, some component of skin friction has been assumed to act down the pile, and that raises questions about the ability of the solution of Ito and Matsui to model such a situation. The assumption of pile rigidity made by Ito and Matsui is due to the need to assume plane strain conditions in order to solve the plasticity problem and does not in itself influence the solution that is based upon limiting equilibrium of the pile and soil.

It appears that the only possibility for the pile response that is considered by the analysis of the authors is that of a pile that has sufficient embedment in a strong underlying stratum, and further is strong enough, in both bending and shear resistance, so that the loads arising from the slipping soil acting on the pile can be sustained. As indicated by Hull et al. (1991) and Lee et al. (1991) this is only one of a number of modes of failure that may occur in a pile loaded by sliding soil acting over the upper length of the pile. Indeed, if the pile does not extend deep enough below the most critical circle, a situation may occur in which the pile lengths extending below the slip circle are dragged through the soil. Between the two cases of the pile remaining fixed in the underlying soil and the pile being rotated by the failing circle of soil, an intermediate case when the pile rotates excessively can be experienced. In any case, the possibility of developing the full strength of the pile will introduce another limiting pile response.

Eqs. (7) and (8) presented in the paper for the piled slope present some difficulties, especially when a case is considered where the pile force F_p becomes nearly zero. The equations do not provide the answer given by those of Taylor [(5) and (6)] for a slope without piles. It seems logical that such an answer should be available from the presented equations. The possibility of a misrepresentation of the equations in the paper is increased when it is realized that a negative stability number results from use of them.

From Fig. 6 of the paper, the increase in the factor of safety for the steeper slope from just above 1 to values up to 3, seems to be an excessive improvement, and a check on the pile integrity and the adequacy of the pile length below the circle should be carried out to verify that such an improvement is possible. The high value of factor of safety that is predicted by the authors might be explained by the definition that has been adopted for the factor of safety. The available cohesive component in the numerator will remain unchanged while the mobilized cohesive component in the denominator is likely to be decreased when piles are introduced, whereas the available and mobilized frictional components are both likely to be reduced, since the normal stress on the slip circle might be reduced by the inclusion of piles. This raises the issue of whether it is sensible to base the factor of safety upon the mobilized shear stresses around a slip circle, which are not actually computed with any degree of confidence. If the presence of the piles in the slope causes the cohesive and frictional mobilized (required) components of force to be reduced, then the ratio of the available to mobilized soil shear stress may increase, although the disturbing force of the soil remains unchanged.

The discussers have carried out an alternative analysis of the piled slope in which the equilibrium of moments is used through a modified Bishop slip circle analysis, and the influence of the piles is introduced by considering a bending moment and shear force multiplied by the appropriate distance (Lee et al. 1995). The bending moment and shear force are those acting at the intersection of the slip circle with the pile and are generated in a boundary element analysis of the pile by applying sufficient soil movement to the upper portion of a pile to generate the pile-soil limiting pressures, as suggested by Ito and Matsui (1975). The factor of safety is then defined by the sum of the resisting moments divided by the overturning moment due to the self weight of the slope. One major difference from the authors' approach is that the denominator of the factor of safety remains constant; another difference is that the failure mechanism of a slip circle remains consistent with equilibrium of moments.

Fig. 16 shows the slope with the centers of the critical circles for a piled slope and a slope with no piles. The factor of safety without piles is found to be 1.12, and the critical circle from the Bishop analysis essentially agrees with that from the authors' analysis. The introduction of piles raises the factor of safety to 1.45 in the discussers' analysis, whereas the authors' analysis predicts a value of 1.82 and the two circles are now different. These two answers are sufficiently different to raise considerable concern over the validity of the authors' less con-

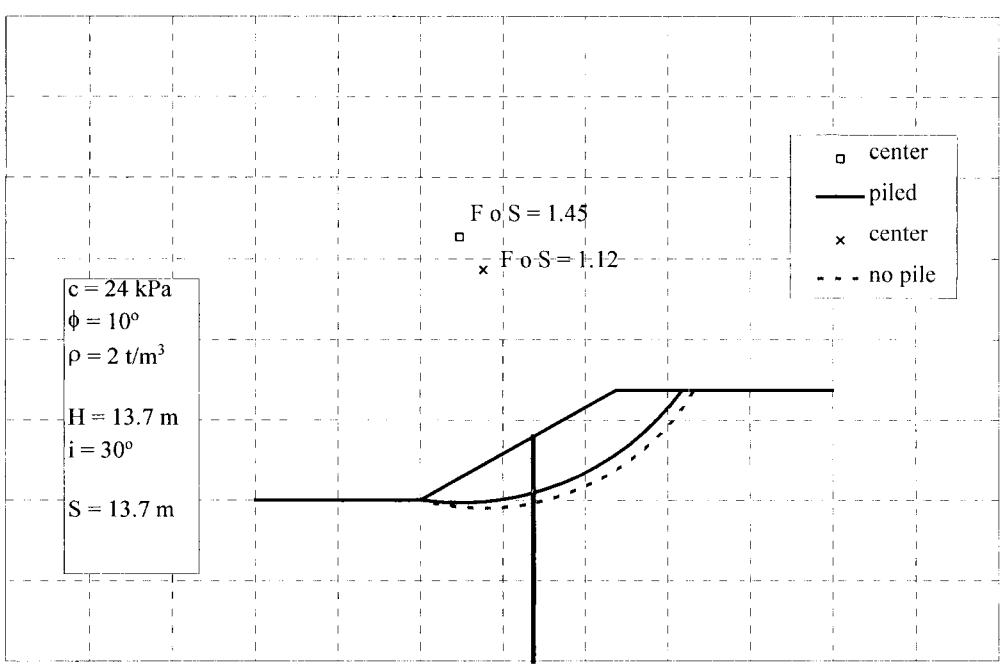

FIG. 16. Bishop Modified Analysis of Slope with Piles

servative answer. In order to make a comparison with the authors' analysis, the possibility of structural failure of the pile has not been considered by the discussers. However, our analysis indicates that the maximum bending moment sustained by a row of 400 mm diameter concrete piles at a spacing of 1 m is in excess of 2.0 MNm, which far exceeds the structural capacity of the piles.

Some discussion by the authors of the validity of the comparison between the two factors of safety, and the question of the structural capacity of the piles, would be appreciated.

APPENDIX. REFERENCES

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Closure by S. Hassiotis,[7] J. L. Chameau,[8] and N. Gunaratne[9]

The writers thank the discussers for their valuable comments. Regarding the comments on the moment equilibrium, the writers wish to point out that in deriving (7) and (8), the moment equilibrium was considered as well. However, the writers understand the concern in light of the graphical error in Fig. 3, where the three forces P, C_r, and W are mistakenly drawn as being concurrent, as in the case of unpiled slopes. Accordingly, the corrected free-body diagram is provided in Fig. 17. It must be reiterated that the concurrency was by no means assumed in the derivation of the above equations.

On page 317 the authors state that ''the assumption that the critical surface does not change with the addition of piles would lead to non-conservative answers for the factor of safety.'' The writers do agree with the discussers that this is a known fact and that hence such a statement is superfluous. They are also in agreement with the discussers in that the only use of knowing the unpiled critical surface is to locate the area where piles can be placed for effective stabilization.

As for the chosen soil pressure distribution on the pile, at the time the writers were aware of only the one developed by Ito and Matsui (1975) for the case of passive piles; therefore, this was the obvious choice. The writers appreciate the references provided by the discussion group on alternative methods.

The writers apologize for the following omissions and once again thank the discussers for highlighting them:

1. In (1), A_c should be $A \cdot c$ where A is as defined. On the other hand, as the discussers correctly recognized, A in (8), which is different from the above A, can be defined as

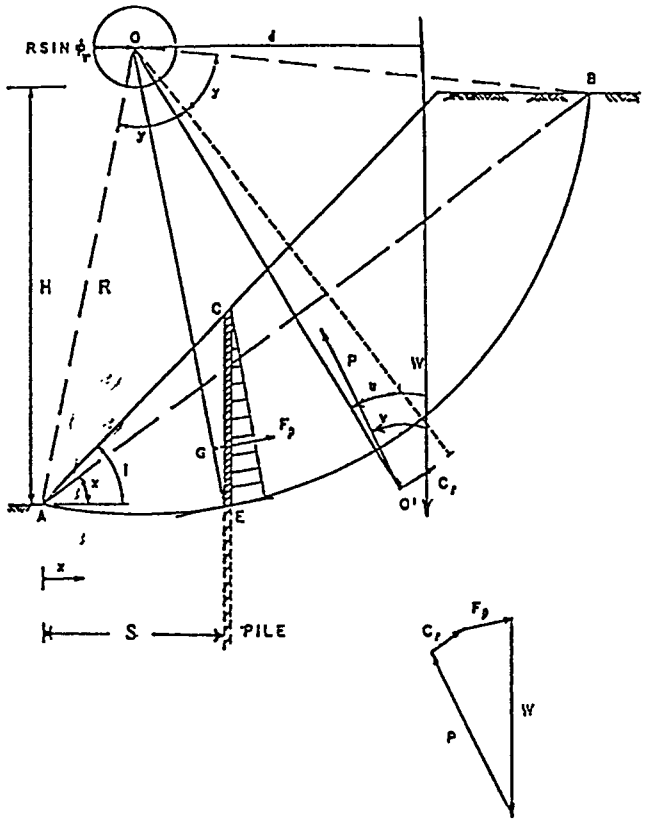

FIG. 17. Forces on Slope Reinforced with Piles

$$A = \frac{\cos(CEO)}{\sin(v)} \cdot \frac{H}{2} \cdot csc(x) \cdot csc(y) \cdot \sin(\phi) + OG$$

2. The definition of E in (8) includes a typographical mistake. The correct definition should be

$$E = 1 - 2\cot^2 i + 3\cot i \cot x - 3\cot i \cot y + 3\cot x \cot$$

3. Eq. (7) also contains a minor typographical error; i.e. the $(x - v)$ in the denominator must be $(x - u)$. The correct (7) is as follows:

$$\frac{C_u}{F_c \gamma H} = \frac{E - \dfrac{12F_p}{\gamma H^3}\left[\dfrac{\cos(CEO)}{\sin v}\dfrac{H}{2}\operatorname{cosec} x \operatorname{cosec} y \sin\phi + OG\right]}{6\operatorname{cosec}^2 x \operatorname{cosec} y \sin\phi\left[\dfrac{\cos x}{\sin v} + \operatorname{cosec}(u - v)\cos(x - u)\right]}$$

The discussers' concern regarding the reduction of (7) to (3) in the absence of piles (i.e., $F_p = 0$), is addressed as follows.

When the slope is not stabilized, since $F_p = 0$, (7) reduces to

$$\frac{C_u}{F_c \gamma H} = \frac{E}{6\operatorname{cosec}^2 x \operatorname{cosec} y \sin\phi\left[\dfrac{\cos x}{\sin v} + \operatorname{cosec}(u - v)\cos(x - u)\right]} \tag{17}$$

It is shown in Hassiotis and Chameau (1984) that

$$E\frac{\gamma H^3}{12} = Wd \tag{18}$$

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[9]Assoc. Prof., Dept. of Civ. and Envir. Engrg., Univ. of South Florida, Tampa, FL.

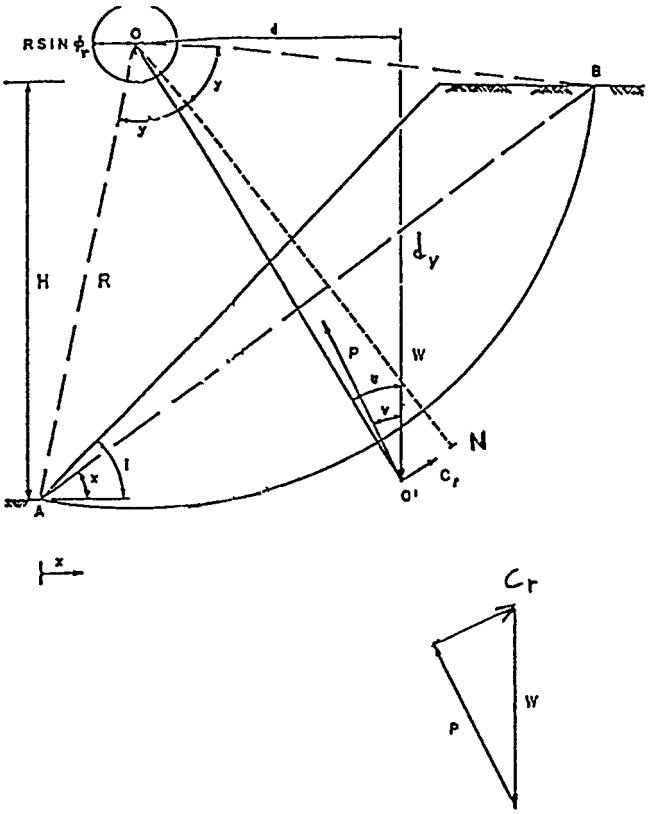

FIG. 18. Forces on Free Slope

where the weight of the sliding soil mass, W, is given by

$$W = \gamma \frac{H^2}{4} \operatorname{cosec}^2 x \operatorname{cosec}^2 y(y - \sin y \cos y) + \frac{\gamma H^2}{2} (\cot x - \cot i) \tag{19}$$

and d = moment arm of W about O.

In the absence of piles, one can use Fig. 18 to geometrically determine the perpendicular distance from O to the line of action of C_r

$$ON = \frac{R \sin \phi}{\sin(u - v)} \cos(x - u) \tag{20}$$

and the vertical distance from O to O'

$$d_v = d \cot v - \frac{R \sin \phi}{\sin v} \tag{21}$$

Furthermore:

$$AB = H \operatorname{cosec} x \tag{22}$$

and

$$R = \frac{H}{2} \operatorname{cosec} x \operatorname{cosec} y \tag{23}$$

Since the moment of C_r about O is equal to the sum of moments of the components of C_r

$$\left(d \cot v - \frac{R \sin \phi}{\sin v}\right) (C_r \cos x) + d(C_r \sin x) = \frac{R \sin \phi \cos(x - u)}{\sin(u - v)} (C_r) \tag{24}$$

Then, using the results in (18), (19), and (24), (17) can be written as

$$\frac{C_u}{F_c \gamma H} = \left\{ \frac{12}{\gamma H^3} \left[\gamma \frac{H^2}{4} \operatorname{cosec}^2 x \operatorname{cosec}^2 y(y - \sin y \cos y) + \gamma \frac{H^2}{2} (\cot x - \cot i) \right] d \right\} \Big/ \left\{ 6 \operatorname{cosec}^2 x \operatorname{cosec} y \left[\frac{\cos x \sin \phi}{\sin v} + \frac{d}{R} (\sin x + \cot v \cos x) \sin \phi - \frac{\cos x \sin \phi}{\sin v} \right] \right\} \tag{25}$$

and

$$\frac{C_u}{F_c \gamma H} = \frac{\left[\frac{1}{2} \operatorname{cosec}^2 x \operatorname{cosec}^2 y(y - \sin y \cos y) + (\cot x - \cot i) \right]}{\operatorname{cosec} x \operatorname{cosec} y[1 + \cot v \cot x]} \cdot \frac{R}{H} \tag{26}$$

Finally, using (23), (26) can be simplified to

$$\frac{C_u}{F_c \gamma H} = \frac{\frac{1}{2} \operatorname{cosec}^2 x(y \operatorname{cosec}^2 y - \cot y) + (\cot x - \cot i)}{2(1 + \cot v \cot x)}$$

Similarly, it can be shown that (8) modifies to (6) as well. These results generalize the applicability of (7) and (8).

With regard to the stresses in the pile, the writers have attempted to show that restricting the pile top would minimize the moment and the shear forces. This is clearly shown in Figs. 11 and 12 with respect to moment and shear in the pile, respectively.

Limit Analysis versus Limit Equilibrium for Slope Stability[a]

Discussion by Dov Leshchinsky,[5] Member, ASCE

The authors present lower and upper bound results for simple slopes obtained from limit analysis of plasticity. They suggest using the lower and upper bound results as a ''yardstick'' in assessing the accuracy of limit equilibrium analyses. The authors compared the results of their limit analysis with results obtained from Bishop's simplified analysis as applied to simple slopes that are comprised of cohesive soil and of c-ϕ soil. The following comments are made:

1. The extremization in the authors' limit analysis (i.e., maximization in the lower bound and minimization in the upper bound) is done with respect to the unit weight of the soil. Extremization in limit equilibrium, however, is conducted with respect to the shear strength of the soil(s) (i.e., minimum factor of safety or maximum utilization of the available shear strength). It is not clear whether the extremization used in the two types of analyses is physically equivalent. In limit equilibrium, for example, extremization with respect to unit weight (or

[a]January 1998, Vol. 124, No. 1, by H. S. Yu, R. Salgado, S. W. Sloan, and J. M. Kim (Paper 12469).

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weight of the potentially ''sliding'' mass) may not be consistent or entirely rational. That is, when a deep-seated failure is considered, a portion of the weight of the sliding mass may drive failure while, simultaneously, another portion may resist failure.

2. The authors present an extensive example problem using simple slopes comprised of cohesive soil (i.e., $\phi = 0$). For cohesive slopes combined with circular slip surface, the limit equilibrium problem is statically determinate, and nearly all methods, including methods of slices, produce the same critical results (i.e., same *Fs* and critical circular surface). These results satisfy the global limit equilibrium equations for the sliding body either explicitly (e.g., moment) or implicitly (e.g., force). Consequently, the reference to Bishop's method only (see Figs. 5–10) narrows the scope of the paper's conclusions.
3. For *cohesive* ($\phi = 0$) slopes, it can be shown (e.g., Chen 1975) that a circular surface constitutes a kinematically admissible mechanism in upper bound computations (considering a rigid body rotation). This mechanism, combined with rigorous upper bound, must also satisfy global limit equilibrium [using the linear Mohr-Coulomb criterion; e.g., Leshchinsky et al. (1985); Michalowski (1989)]. If fact, for circular surfaces in cohesive slopes, the equations of global limit equilibrium produce identical critical results to the equations of external work and energy dissipation in the upper bound (e.g., Leshchinsky et al. 1985). Item 2 states that, for cohesive slopes combined with circular failure surfaces, most limit equilibrium methods [including Bishop's and the one presented by Leshchinsky et al. (1985)] will produce the same critical results. Hence, one would expect the authors' upper bound results to be equal to those of limit equilibrium for the particular case of cohesive slopes. If at all, the limit equilibrium results may be even less critical than the upper bound results, in which case the authors' numerical procedure has produced a more critical mechanism than simply a rigid single-body rotation. Close examination of Figs. 5 and 6 indeed shows the upper bound and the limit equilibrium results to be nearly identical. However, as the slope flattens and as the rate of undrained shear strength increases with depth, the limit equilibrium results become significantly more critical than the authors' upper bound results (or even their lower bound), as can be seen in Figs. 7–9. The discusser wonders whether this behavior is not the consequence of a numerical inaccuracy in the computational process. The deviations in results are especially important when considering conclusion 3. The discusser thinks that figures showing calculated velocity fields superimposed on Bishop's critical circles could be instructive in the comparative study presented by the authors.
4. It is recognized that limit analysis is different from limit equilibrium analysis. However, items 2 and 3 imply that a problem using simple slopes comprised of *cohesive* soil could be useful for initial verification of numerical accuracy of different computational schemes and computer codes (i.e., could serve as a benchmark). Such a problem may not produce much insight in a comparative study.

APPENDIX. REFERENCE

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Closure by H. S. Yu,[6] Member, ASCE, R. Salgado,[7] Associate Member, ASCE, S. W. Sloan,[8] and J. M. Kim[9]

The writers would like to thank the discusser for his interest in the above paper. It is noted that the issues raised in the discussion are purely concerned with the stability of cohesive slopes. The following comments are made to clarify some of the main issues:

1. Limit analysis and limit equilibrium methods are two very different methods, and in general they give different solutions for slope stability problems. As presented in the paper, both of these methods have been used to optimize the stability number $N_F = \gamma H/s_{uo}^m$ for soil slopes. In limit analysis, optimization of the stability number is done in terms of the unit weight for a given soil strength. On the other hand, the limit equilibrium method is conducted in terms of soil strength for a given unit weight. As both the limit analysis and limit equilibrium methods are used to optimize the same parameter, N_F, it is therefore perfectly reasonable to compare the results of the stability numbers N_F obtained from these two different methods. The writers believe that the point may have been missed on the discusser that, in the paper, the optimization is done for the stability number $N_F = \gamma H/s_{uo}^m$ and not really on unit weight or soil strength alone.
2. The main purpose of the paper was to present numerical upper and lower bound solutions for stability of simple earth slopes. The chief conclusion drawn from the results presented in the paper is that, for cohesive slopes, the exact stability numbers can be bracketed from above and below to within 5% by the newly derived lower and upper bound solutions.

   A secondary aim of the paper was to compare our lower and upper bound solutions with limit equilibrium solutions. The discusser points out that, for some special cases (e.g., cohesive slopes with circular failure surfaces), limit equilibrium results may also be regarded as a valid upper bound. For more general cases (e.g., cohesive slopes with noncircular failure surfaces, cohesive-frictional slopes, etc.), however, the limit equilibrium results are neither an upper bound nor a lower bound. It is also generally true that different limit equilibrium methods or commercial limit equilibrium codes may give very different solutions for the same problem. Two main reasons that may cause such a difference are: (1) different assumptions and failure surfaces may be used in different limit equilibrium methods; and (2) different search strategies may be used to locate the most critical failure surface. It is therefore necessary that we make it clear that STABL was used with the option of Bishop's simplified method for our limit equilibrium analyses.
3. With respect to the results presented in Figs. 5–9, it should be pointed out that there was a mistake made in the paper in processing and plotting the limit equilibrium solutions. The corrected versions of these figures are given in the Errata at the end of this closure. The issues raised in item 3 of the discussion would largely disappear after considering the corrected version of Figs. 5–9. As

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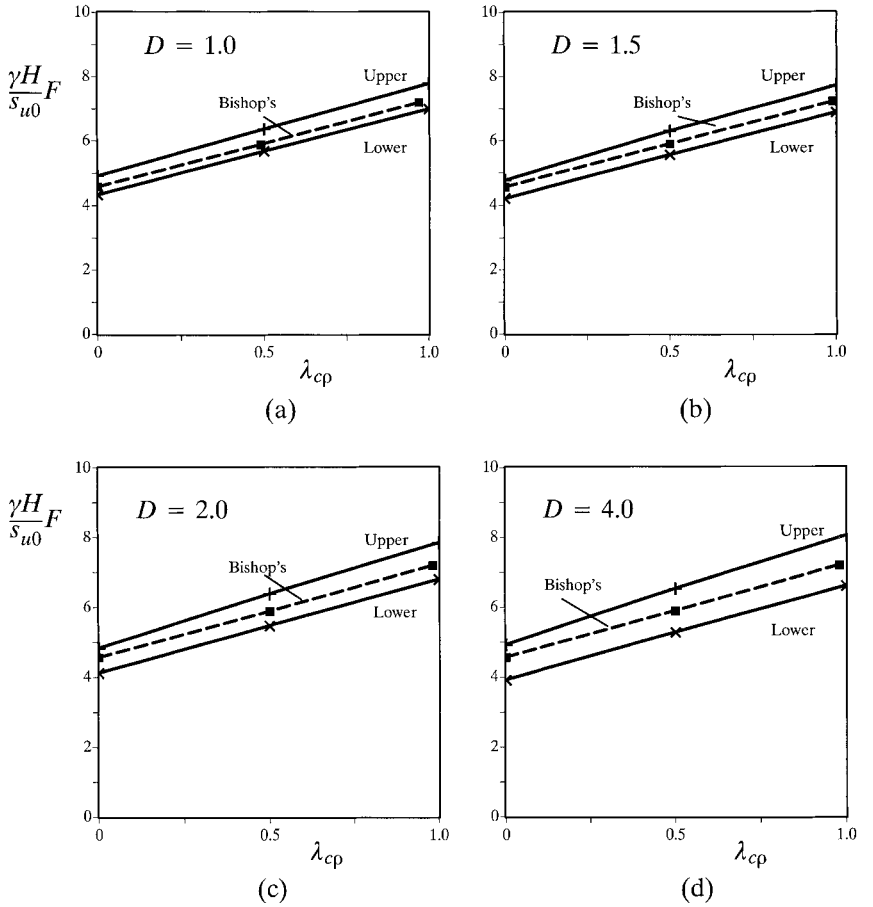

FIG. 5. Effect of Increasing Strength with Depth on Stability Number for Slope Angle of 75°

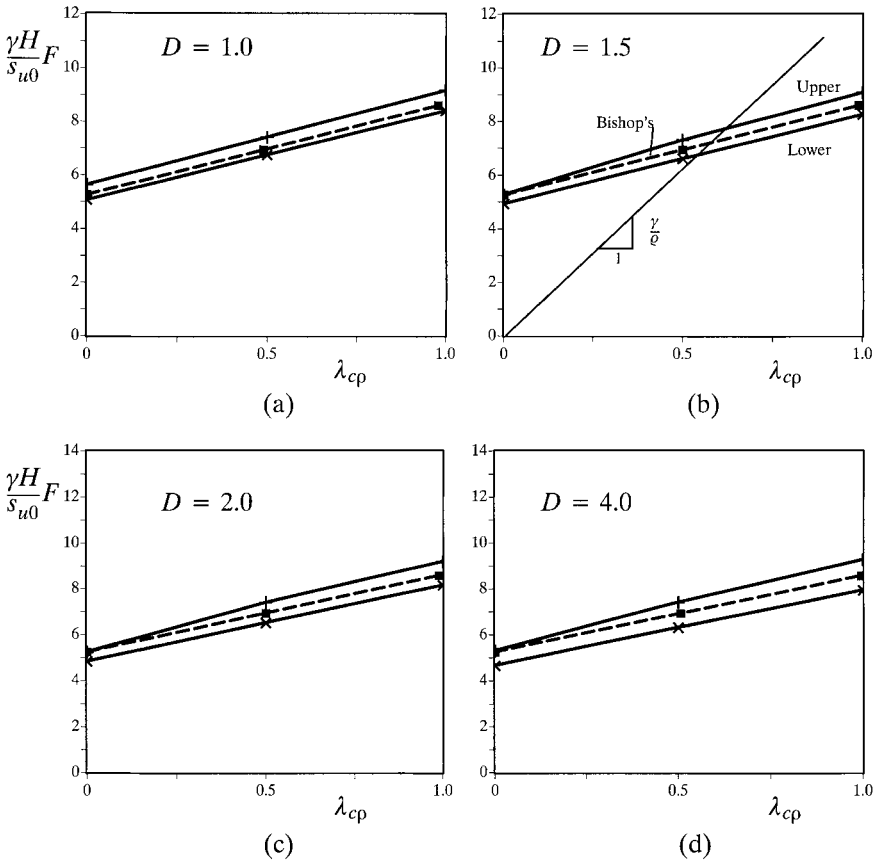

FIG. 6. Effect of Increasing Strength with Depth on Stability Number for Slope Angle of 60°

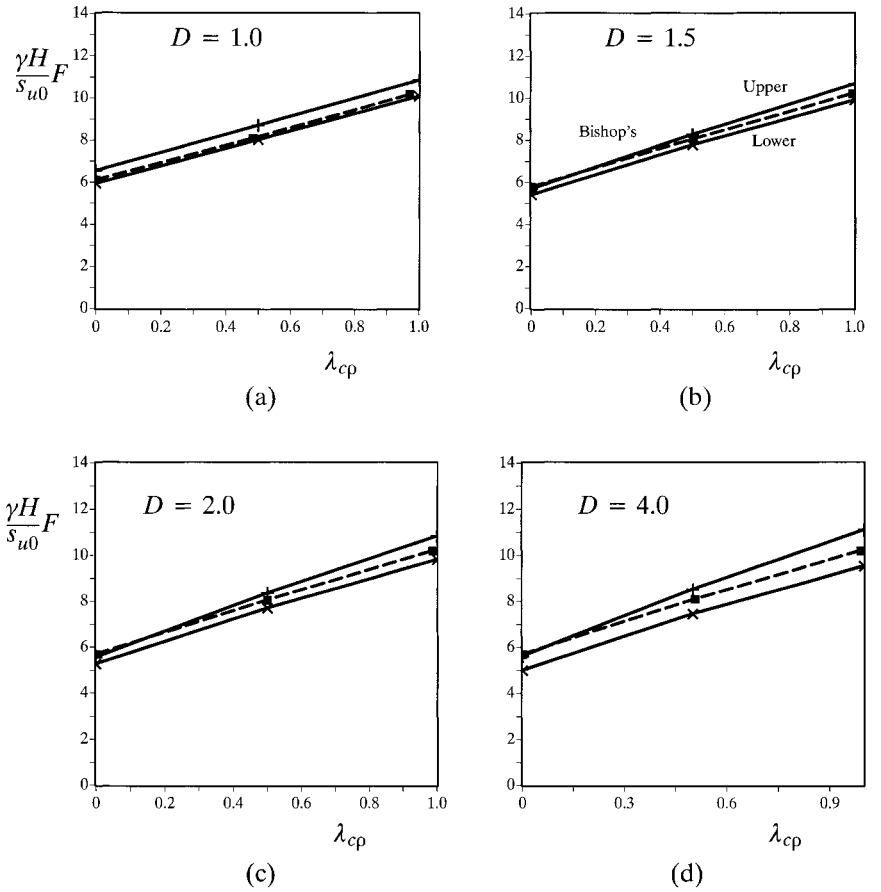

FIG. 7. Effect of Increasing Strength with Depth on Stability Number for Slope Angle of 45°

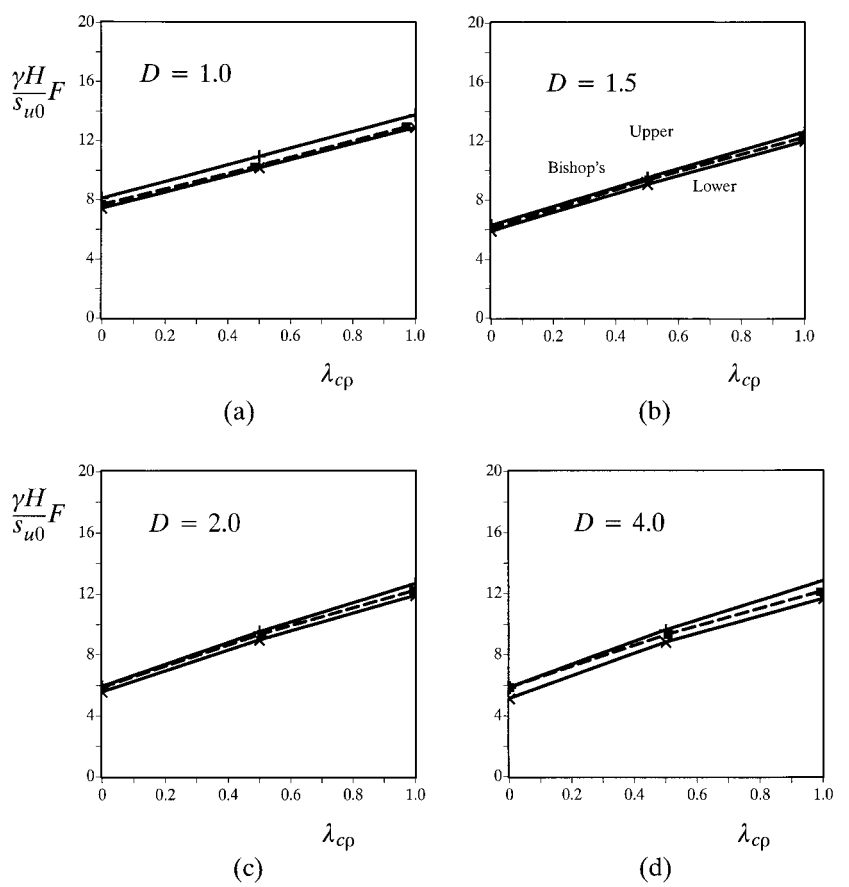

FIG. 8. Effect of Increasing Strength with Depth on Stability Number for Slope Angle of 30°

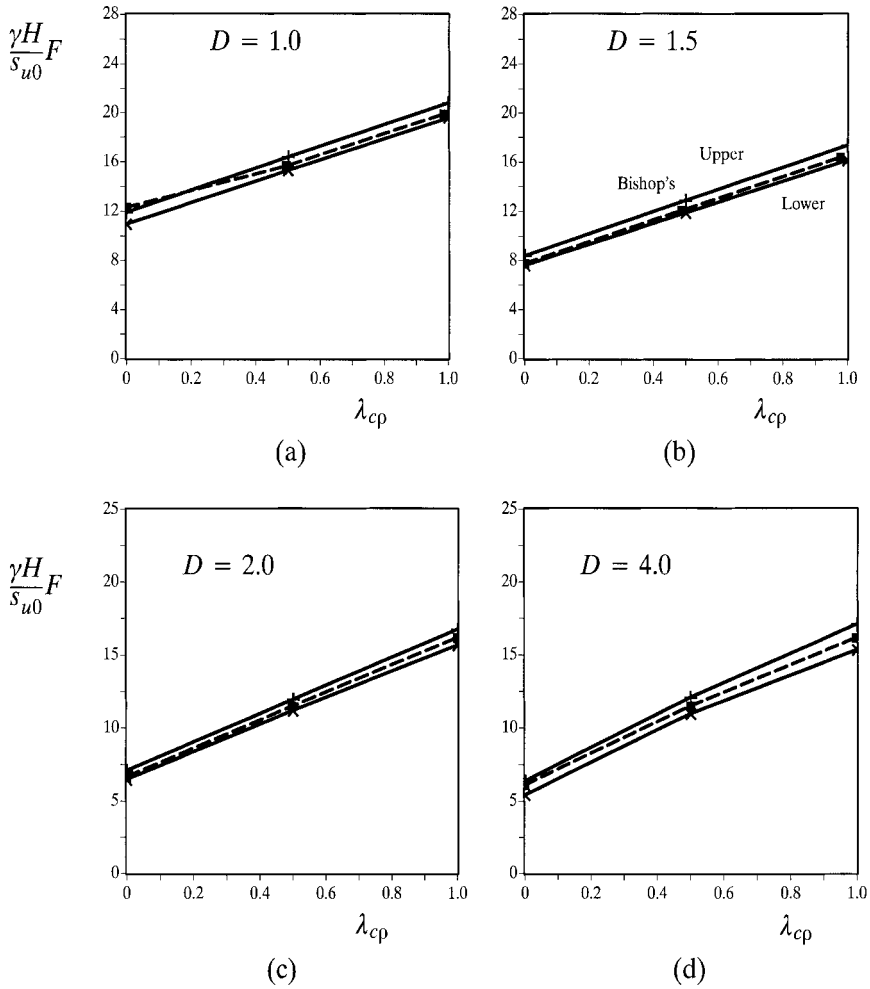

FIG. 9. Effect of Increasing Strength with Depth on Stability Number for Slope Angle of 15°

shown in the corrected Figs. 5–9, the limit equilibrium results agree well with the numerical lower and upper bound solutions for all slope angles and depth factors considered. While higher than the numerical lower bounds, the limit equilibrium solutions are slightly lower than the numerical upper bounds. This is in agreement with the earlier results presented in Chen and Liu (1990), who find that for cohesive slopes their upper bounds are also slightly higher than the limit equilibrium results.

4. It should also be pointed out that care must be exercised in using the plot N_F versus $\lambda_{c\rho}$ to compare the results of limit analysis and limit equilibrium methods. This is because the limit equilibrium solutions are not unique in this plot; in fact, they are dependent on the unit weight used in the limit equilibrium calculation. The reason for this is that, while the stability number $N_F = \gamma HF/s_{uo}$ is independent of the unit weight (e.g., doubling the unit weight would reduce the safety factor by 50% if other parameters remain the same), the dimensional parameter $\lambda_{c\rho} = \rho HF/s_{u0}$ depends on the safety factor and will therefore be a function of the actual unit weight. To make a fair comparison, the limit equilibrium results plotted in the corrected version of Figs. 5–9 are obtained using the average unit weight of the optimized unit weights obtained from both the upper and lower bound analyses.

Errata. Mistakes were made in processing and plotting the limit equilibrium solutions for cohesive slopes. The corrected versions of Figs. 5–9 are given here.

METHOD TO ESTIMATE WATER STORAGE CAPACITY OF CAPILLARY BARRIERS[a]

Discussion by Glendon W. Gee,[3] Andy L. Ward,[4] and Philip D. Meyer[5]

The paper by Stormont and Morris provides an interesting approach to computing water storage capacity of capillary barriers used as landfill covers. They correctly show that available water storage capacity can be increased up to a factor of two for a silt loam soil when it is used in a capillary barrier, as compared with existing as a deep soil profile. For this very reason such a capillary barrier, utilizing silt loam soil, was constructed and successfully tested at the U.S. Department of Energy's Hanford Site in southeastern Washington State (Ward and Gee 1997). Silt loam soil provides optimal water storage for capillary barriers and ensures minimal drainage. Less ben-

[a]April 1998, Vol. 124, No. 4, by John C. Stormont and Carl E. Morris (Paper 15758).

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efits are obtained when capillary barriers use more sandy soils (Warren et al. 1997).

However, some caution should be used in applying the method of Stormont and Morris if cover design critically depends on water storage. For example, there are so-called ET caps or covers that rely on evapotranspiration to remove water from the soil cover and prevent drainage. In these cases water storage capacity must be known accurately to estimate the cover thickness needed to prevent deep percolation and subsequent leaching of the underlying waste. Uncertainty of the estimate may result in increased costs and risks.

Stormont and Morris demonstrated a methodology that is based on laboratory-estimated soil properties (i.e., field capacity, wilting point, and water retention characteristics). Field capacity was assumed equal to a soil suction of 3,500 mm for all soils. This assumption, while often acceptable for soils of medium texture (e.g., silts and loams), grossly overestimates the suction values of sands and underestimates the suction values of clayey soils. In a 1-m-thick soil profile, the errors in estimates could be 30% or more for the extreme textural classes (sands and clays). The impact of this uncertainty was not addressed.

Another cause for uncertainty is the spatial variability of the field capacity and other soil hydraulic properties for any soil texture. Parameters used from such references as Carsel and Parrish (1988) have large uncertainties associated with them, and when such values are used, the uncertainties should be accounted for in a systematic way. Recent approaches using Bayesian updating (Meyer et al. 1997) or the use of neural networks (Schaap et al. 1998) offer some hope in this regard.

An additional cause for uncertainty is the hysteretic behavior of soil-water characteristics. Water retention data from the laboratory are typically obtained from primary drying curves. That is, samples are wetted to saturation and then drained at increasing pressures on hanging water columns or pressure plates. In the field, surface soils are seldom saturated, and the water retention characteristics needed to predict drainage from the capillary barrier can best be derived from wetting curves, since wetting events typically cause drainage (Fayer and Gee 1997). Such wetting curves or characteristics are seldom available from the laboratory or from typical databases. Wetting and drainage characteristics for a given suction are significantly different for most soils. As an example, from data of Fayer et al. (1992), we calculate that the available water capacity for a 1 m silt loam soil at 3,500 mm suction, as determined from the primary drying curve, is 180 mm, versus only 80 mm from the wetting curve. The wetting curve estimate of available water capacity is less than half of the drying curve estimate. This uncertainty is larger than the capillary barrier effect on the available water capacity, as predicted by the method of Stormont and Morris.

Finally, there is uncertainty in the estimate of the wilting point. Vegetated soil covers, particularly in the arid west, withdraw water to a lower limit of water extraction that is significantly less than that predicted using a 150 m suction value. Often at arid sites the wilting point can exceed 600 m suction. The water content at this suction is always less than at 150 m suction, for all soil types, and the available water will always be underestimated when the 150 m suction value is used. For a silt loam soil, this amounts to an underestimate of available water of more than 30 mm in a 1 m soil cover.

The discussers would endorse a limited application of Stormont and Morris' method. We suggest that there will be large uncertainties in field capacity, wilting point, and water retention characteristics, and only when these uncertainties are accounted for can such a method be used to provide sound engineering judgement for cover design. A recommended procedure for using this method would include actual field measurements of the soil hydraulic properties of the cover materials.

APPENDIX. REFERENCES

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Closure by John C. Stormont[6] and Carl E. Morris[7]

The writers appreciate the discussers' interest in our paper and are pleased that conclusions we reached in the paper are consistent with their experience. The discussers commented on uncertainties associated with parameters used in our method. Their first comment concerns the uncertainties associated with the field capacity. The writers fully recognized and stated that the assumption that field capacity is defined at a suction head of 3,500 mm is arbitrary and that the field capacity definition will vary with soil type. However, this assumption is commonly used. The consequence of using this definition of field capacity was also addressed in the paper. Further, field capacity is *not* used in our method to predict the water storage capacity of capillary barriers. Rather, field capacity is used to provide a first-order estimate of water storage in soils without a capillary break to compare with soils with a capillary break.

The second uncertainty raised by the discussers is related to the spatial variability of the set of hydraulic properties the writers used to exercise our method. These properties were used to provide estimates of capillary barrier performance as a function of the overlying soil texture, not to evaluate a design for a specific site. While the writers agree that spatial variability can have a significant impact on all systems and processes involving water flow through soils, including unsaturated flow applicable to capillary barriers, a meaningful analysis of the effects of spatial variability requires site-specific data.

The effect of hysteresis of the soil's moisture characteristic curve is mentioned as a further uncertainty by the discussers. This issue was discussed in the paper. The writers clearly stated that it is preferable to use parameters that have been derived from wetting curves, because they will provide the most conservative estimate of storage capacity. Further, re-

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gardless of whether wetting or drying curves are used, our method provides a useful comparison between capillary barriers and simple soil covers. Uncertainty due to hysteresis does not change the conclusion that the capillary barrier effect will always result in additional water storage compared with a simple soil cover.

The discussers' final comment is that there is uncertainty related to the permanent wilting point. In particular, some soils may be reduced to a lower water content than that conventionally associated with the wilting point. If a lower water content is appropriate for a particular soil and location, the water storage capacity will increase in either a simple soil cover or a capillary barrier configuration. Thus, the writers' assumption that the wilting point is calculated at 150 m of suction head is conservative with respect to net soil water storage. Further, it is a simple matter to apply a different estimate for the lower limit of the water content to the method given in the paper.

In summary, the method proposed in the paper is quite simple, and estimates of storage capacities can be revised as soil moisture characteristic data is refined. The writers agree that, for best results, field measurements of soil properties to be used in the cover system should be used in the calculations. This practice is widely acknowledged.