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A REVIEW OF MODELS FOR PREDICTING THE THERMOMECHANICAL BEHAVIOUR OF SOFT CLAYS

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SUMMARY

This paper critically examines the use of the modified Cam clay stress–strain model in predicting the thermomechanical behaviour of soft clays. The equations governing the thermomechanical behaviour of a saturated soil are summarized and their methods of solution are briefly discussed. The observed thermomechanical soil behaviour reported in the literature has been compared with the predictions made using the modified Cam clay model. In making these comparisons, two extensions of the well-known modified Cam clay model have been considered: one proposed by Britto *et al.*[1] in which heating induces thermal stresses and strains in the soil but has no direct effect on the work hardening, and the other proposed by Hueckel and Borsetto[2] in which a change in temperature also affects the yield surface. The comparisons are confined to the behaviour of normally and lightly overconsolidated clays, where the modified Cam clay is known to perform well. Apart from the effect of a single heating–cooling loop, cyclic behaviour is not considered. It is concluded that both models provide reasonable predictions under isotropic stress conditions. Although exhaustive comparisons have not been made for deviatoric stress excursions (because of the lack of experimental data), it appears from preliminary studies that neither model performs particularly well for this form of loading.

1. INTRODUCTION

The thermomechanical behaviour of soils has not been under the close scrutiny of researchers until recently, with most analyses in the past being confined to isothermal conditions. However, research interest has grown as a result of an increasing number of problems involving thermal effects. Some of the important cases of thermomechanical behaviour are: the disposal of high-level radioactive waste, extraction of oil or geothermal energy, road subgrade or furnace foundations which are usually subjected to cyclic changes of temperature, and sample disturbance due to temperature changes during sampling, storage and testing. The thermal behaviour of soils has many aspects, but only the stress–strain behaviour of saturated soft clays subjected to moderate temperature changes without freezing or boiling of the pore water is considered here.

Heating can influence the stress–strain behaviour of a soil in at least three ways. Firstly, it may change the mechanical properties of a soil; for example, the permeability has been observed to decrease with temperature. Secondly, it can affect the mechanical response of a soil; an increase in the temperature may produce expansive or compressive volumetric strains and generate excess pore pressures under undrained conditions. Finally, it may alter the microstructure of the soil, affecting its plastic behaviour; softening or hardening can occur due to heating. It is not always possible to separate the above three effects.

This paper critically examines the use of the modified Cam clay stress–strain model[3] in predicting the thermomechanical behaviour of soft clays. The observed soil behaviour found in

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the literature has been compared with the predictions made using the modified Cam clay model. The comparison is basically confined to the behaviour of normally and lightly overconsolidated clays, where the modified Cam clay is known to perform well. Apart from the effect of a single heating–cooling loop, cyclic behaviour is not considered.

2. LITERATURE REVIEW

2.1. Experimental observations

The volumetric behaviour of clays subjected to temperature changes has been observed by a number of researchers (e.g. References 4 and 5). A saturated soil under undrained conditions expands and develops positive pore pressures during heating. According to Campanella and Mitchell,[5] a temperature change of 1°C may produce an excess pore pressure of about 1·5 per cent of the mean effective pressure. The drained volumetric strains due to heating are usually observed to be compressive. However, the magnitude of volumetric strain per unit increase in the temperature decreases with the overconsolidation ratio, and may become negative in heavily overconsolidated clays.[6]

There is a lack of reliable evidence on the shear behaviour of soils subjected to temperature changes, and in particular the effect of heating on the shear strength of a soil is a subject of much controversy (e.g. see the review by Mitchell[7]). The shear deformation behaviour under thermal conditions appears to be complex; this may partly be due to creep. For example, undrained heating of a triaxial specimen under constant total pressure and deviator stress has been observed to produce compressive axial strains (e.g. Reference 8). Observations of both the shear and volumetric behaviour of soils are discussed critically later in this paper.

2.2. Analytical models

Campanella and Mitchell[5] developed a theoretical relationship to predict the excess pore pressures and volumetric strains associated with temperature changes. It was assumed that the compressibility characteristics of the soil are unaffected by the temperature. The excess pore pressure is generated due to the differential thermal expansion between the soil grains and the pore water, and also due to the collapse of the soil structure and physicochemical effects.

Britto *et al.*[1] presented a fully coupled linear elastic finite element formulation which was later extended by them to accommodate the modified Cam clay stress–strain model. The effect of temperature was assumed to be isotropic as in metals, and the thermal volumetric strains were calculated using a coefficient of drained thermal volume expansion, i.e. the magnitude of expansive volumetric strain per unit increase in the temperature. Hueckel and Borsetto[2] proposed an alternative thermoplastic model to describe the soil behaviour. The plastic behaviour was described by the modified Cam clay model with added thermal softening. This model of Hueckel and Borsetto[2] was developed specifically to account for irreversible strains that occur as a result of temperature changes, as have been observed in the experiments reported by Baldi *et al.*[6,9] These two models are considered in detail later in this paper.

2.3. Boundary value problems

Traditionally, steady-state and transient thermal boundary value problems in soils and rocks have been analysed using linear elasticity with uncoupled thermal diffusion and consolidation behaviour. Recently, however, fully coupled solutions have been found in some cases using transform techniques, and boundary element or finite element methods (e.g. References 10–12). Britto *et al.*[1,13] compared the test results from a geotechnical centrifuge, obtained during the heating of canisters buried in clay, with the finite element predictions obtained using the modified Cam clay soil model. They found good agreement for pore pressures in normally consolidated clay but not in overconsolidated clay.

3. GOVERNING EQUATIONS AND FORMULATION

The derivation given below is similar to that used by Britto *et al.*[1] for the fully coupled finite element formulation, but unlike the latter the symmetry of the governing equations is ensured.

3.1. Equilibrium

Let σ be the vector of stress increments of a body due to the applied loading and temperature change. σ is defined with the usual notation of compression positive. In the absence of body forces, equilibrium of the body can be expressed in a Cartesian co-ordinate system as

$$\partial^{\mathrm{T}}\sigma = 0 \tag{1}$$

where

$$\sigma = (\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx})$$

and ∂ is a differential operator, defined by

$$\partial^{\mathrm{T}} = \begin{bmatrix} \partial/\partial x & 0 & 0 & \partial/\partial y & 0 & \partial/\partial z \\ 0 & \partial/\partial y & 0 & \partial/\partial x & \partial/\partial z & 0 \\ 0 & 0 & \partial/\partial z & 0 & \partial/\partial y & \partial/\partial x \end{bmatrix} \tag{2}$$

3.2. Strain–displacement relations

Using the convention of positive compressive strains, these relations in matrix form are given by

$$\varepsilon = -\partial u \tag{3}$$

where $\varepsilon^{\mathrm{T}} = (\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})$ is the vector of strain components and $u^{\mathrm{T}} = (u_x, u_y, u_z)$ is the vector of displacement components.

3.3. Principle of effective stress

There is no evidence to indicate that the principle of effective stress is affected by temperature changes. Hence it is convenient to use

$$\sigma = \sigma' + ap \tag{4}$$

where $\sigma' = (\sigma'_x, \sigma'_y, \sigma'_z, \tau_{xy}, \tau_{yz}, \tau_{zx})^{\mathrm{T}}$, $a^{\mathrm{T}} = (1, 1, 1, 0, 0, 0)$ and p is the pore water pressure.

3.4. Constitutive relationship

In the absence of thermal effects the incremental stress–strain relationship may be expressed as

$$\mathrm{d}\sigma' = D\,\mathrm{d}\varepsilon \tag{5a}$$

The derivation of D, the matrix of elastoplastic 'constants' for the modified Cam clay model, has been described in detail elsewhere, e.g. in Reference 14. In the present treatment compressive stresses are considered to be positive.

In the presence of temperature changes the above relationship can be modified as

$$d\sigma' = K'\beta \, d\theta a + \boldsymbol{D} \, d\varepsilon \tag{5b}$$

where, $d\theta$ is the incremental temperature change. K' is the effective bulk modulus of the soil given by

$$K' = \frac{1}{a^{\mathrm{T}} D^{-1} a} \tag{6}$$

Britto *et al.*[1] referred to the quantity β as the coefficient of drained thermal volume expansion. However, a better definition of β may be the coefficient of effective thermal volume expansion of the soil. β is given by the difference between α_{st}, the coefficient of thermal volume expansion of the soil mass (due to physicochemical effects), and α_{s}, the coefficient of thermal volume expansion of the solid grains, i.e. $\beta = \alpha_{\mathrm{st}} - \alpha_{\mathrm{s}}$. Usually, β is negative, as an increase in the temperature causes a volumetric compression, i.e. $\alpha_{\mathrm{s}} > \alpha_{\mathrm{st}}$. It is implicitly assumed in deriving equation (5b) that thermal stresses, but not necessarily the thermal strains, are isotropic. There is no experimental evidence either to support or contradict this assumption. However, the advantage here is that a displacement finite element formulation with the modified Cam clay model gives rise to a symmetric stiffness matrix, unlike the case where isotropic thermal strains are assumed.

3.5. Continuity condition

Assuming that the pore water and soil grains are unaffected by stress changes, the continuity condition can be expressed as

$$\int_0^t \nabla^{\mathrm{T}} v \, dt = \varepsilon_{\mathrm{v}} + \alpha\theta \tag{7}$$

where $\nabla^{\mathrm{T}} = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the gradient operator, $v = (v_x, v_y, v_z)$ the seepage velocity, $\varepsilon_{\mathrm{v}} = a^{\mathrm{T}}\varepsilon$ the volumetric strain, and $\alpha = (1-n)\alpha_{\mathrm{s}} + n\alpha_{\mathrm{w}}$ the coefficient of undrained thermal volume expansion.

Under undrained conditions, there is no seepage water flow throughout the soil, i.e. the left-hand side of equation (7) equals zero. Under fully drained conditions there is no generation of excess pore pressure anywhere.

3.6. Conservation of energy

The complete coupling of mechanical and thermal processes has been investigated by several researchers using different methods. The equation adopted here has been derived by Smith and Booker.[15] Convective terms are not included in this derivation as the work of Hickox and Watts[16] indicates that they can safely be ignored in relatively impermeable clayey soils. Consequently, the conservation of energy is described by

$$\int_0^t -\nabla^{\mathrm{T}}\left(\frac{h}{T}\right) dt = \left(\frac{m}{T} - \beta^2 K'\right)\theta - \alpha p - \beta K' \varepsilon_{\mathrm{v}} \tag{8}$$

where $h^{\mathrm{T}} = (h_x, h_y, h_z)$ is the heat flux and $m = (1-n)\rho_{\mathrm{s}} c_{\mathrm{s}} + n\rho_{\mathrm{w}} c_{\mathrm{w}}$ the heat capacity of the soil.

t is the time interval considered, during which the absolute temperature increases by the amount θ to the value T. ρ_{s}, ρ_{w} and c_{s}, c_{w} are the densities and the specific heats of the soil grains and the pore water, respectively. The last three terms on the right-hand side of this equation represent the contribution from mechanical effects, which are usually very small and may be neglected. Britto *et al.*[1] used only the first term ($m\theta/T$) in their formulation. However, the inclusion of the last two terms preserves the symmetry of the formulation. When the last three terms are small, T can be replaced by the absolute ambient temperature, T_0, without any significant loss of accuracy.

3.7. *Water flow*

It is assumed that the water flow is governed by Darcy's law, i.e.

$$v = -\frac{k}{\gamma_w}\nabla p \tag{9}$$

where γ_w is the unit weight of water and k the permeability of the soil, which is assumed to be isotropic. However, k is also a function of temperature, as discussed later.

3.8. *Heat flow*

It is assumed that the heat conduction through the soil is governed by Fourier's law:

$$h = -f\nabla\theta \tag{10}$$

where f is the thermal conductivity of the soil.

3.9. *Formulation*

A complete analysis of fully coupled thermomechanical behaviour should include all ten equations given above. This is necessary in solving transient boundary value problems with heat and water flow. Finite element formulations for such cases have been given by Britto *et al.*[1] and Lewis *et al.*[17] Problems that do not involve transient behvaiour, e.g. the undrained and fully drained responses, require equations (1)–(8) only. The predictions of the drained and undrained behaviour of triaxial samples reported in this paper were made using a computer program developed specifically to determine the response along any axisymmetric stress or strain path. The program has the capability of determining three parameters out of axial stress, axial strain, radial stress, radial strain, excess pore pressure and temperature change when the others are given.

4. MATERIAL PROPERTIES

The predictions here were made using data on the experimental behaviour of kaolin and illite presented by Campanella and Mitchell,[5] and of Boom clay reported by Baldi *et al.*[6] and Hueckel and Pellegrini.[18] The material properties used in these analyses are given in Table I. The selection of soil parameters is decribed in this section.

4.1. *Modified Cam clay parameters*

The modified Cam clay stress–strain model is fully described by five material constants, namely, the strength parameter, M, the gradients of the normal consolidation and swelling lines, λ and κ, the voids ratio at unit pressure on the critical state line, e_{cs}, and another elastic constant, usually the effective Poisson's ratio, v', or the shear modulus, G. Since only certain aspects of the thermal behaviour have been observed in each soil, all five parameters were required only for the analysis of Boom clay. No modified Cam clay parameters were required for kaolin and only three parameters, λ, κ and e_{cs}, were needed for illite.

The soil parameters required for illite were determined from the data at ambient temperature presented by Campanella and Mitchell.[5] In the case of Boom clay, the values given by Hueckel and Pellegrini[18] were adopted for M and λ. A constant value of ν' is used in this analysis; it was selected to give a shear modulus that varies with stress level but which is identical to that used by Hueckel and Pellegrini[18] at a mean effective pressure of 5·7 MPa. The other two parameters, κ and e_{cs}, were determined from the data presented by Baldi *et al.*[6]

4.2. Effect of temperature on modified Cam clay parameters

Several researchers (e.g. Reference 5) have observed that the effect of temperature on λ and κ is not significant. However, a parallel shift in the normal consolidation line appears to occur due to the thermal volumetric strain. Assuming that the modified Cam clay yield locus is not affected by temperature changes, the critical state line in voids ratio versus effective pressure space should also shift by the same amount as the normal consolidation line and therefore e_{cs} will be affected by temperature. This effect has been included in the present analysis. The available evidence is insufficient to determine whether the soil parameters, M, ν' and G, are affected by temperature changes. Hence, they have been assumed to be insensitive to temperature.

4.3. Thermal parameters

The thermal properties required for the analysis are α_s, α_w and β. Direct measurements of α_s are not available and Campanella and Mitchell[5] have recommended the value given in Table I which is based on the data for similar substances. α_w has been observed to vary substantially with temperature but to be insensitive to variations in water pressure in the moderate range of water pressure considered here.[19] Figure 1 illustrates the variation of the coefficient of volume expansion of water with temperature, taken from a table of properties given by Chapman[19]. In the analyses, the value of α_w at a particular temperature has been linearly interpolated from this table.

β is usually observed to be negative in normally or lightly overconsolidated clays as compression occurs due to drained heating. However, the experimental evidence available is insufficient to determine the dependency of β on effective pressure and temperature. β is observed to be highly

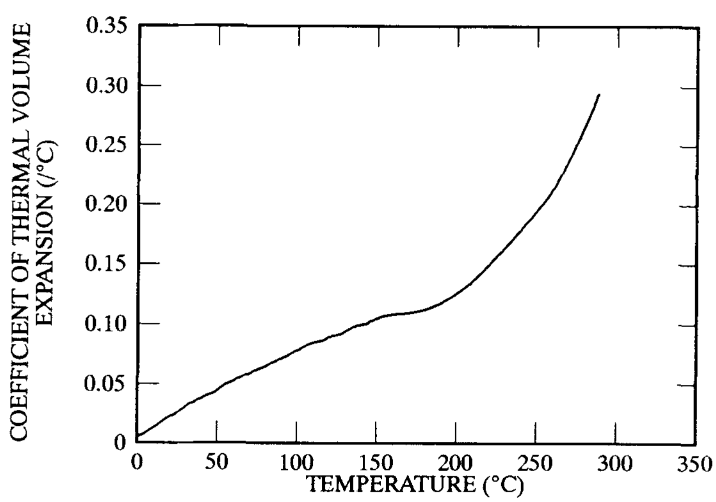

Figure 1. Variation of coefficient of thermal volume expansion of water with temperature

dependent on the overconsolidation ratio, OCR. The experiments by Plum and Esrig[20] have shown the absolute value of β decreasing with OCR, and becoming very small close to an OCR of 2·0. Baldi *et al.*[6] have observed β decreasing in magnitude with OCR and becoming positive (expansive volume changes during heating) in heavily overconsolidated samples.

The results of two types of analyses are presented in this paper: analysis A, using a constant value of β, and analysis B, using a linearly decreasing value of β with the stress ratio, η. In analysis B, as illustrated in Figure 2, β is assumed to decrease in magnitude with η, to be zero at η equal to 2, and to remain at zero at higher values of η. The stress ratio, η, is defined as p_c'/p' where p_c' is the intercept of the current modified Cam clay yield locus with the p' axis; η is identical to OCR during isotropic consolidation. This assumed variation of β with η is simple and in general agrees with the observations of Plum and Esrig.[20] It was not possible to obtain a more precise variation of β with η due to lack of complete experimental data. In both analyses, the magnitude of β (Table I)

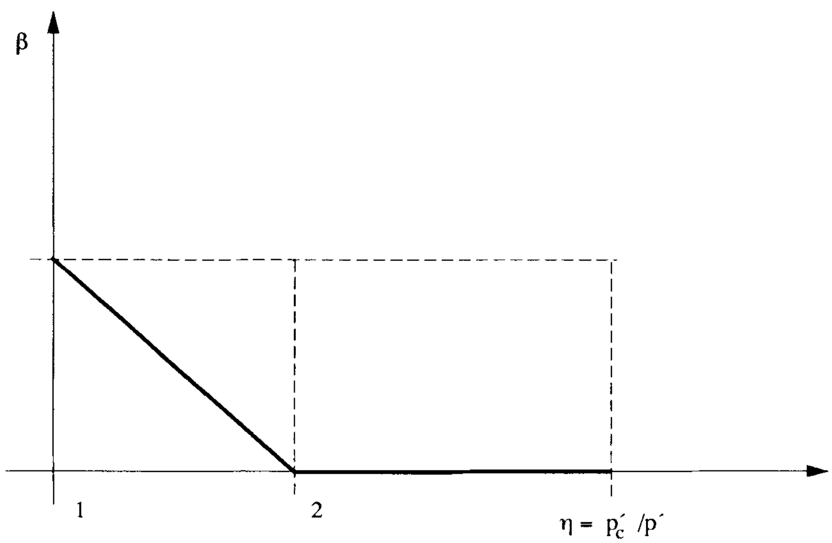

Figure 2. Variation of β with the stress ratio assumed in analysis B

Table I. Material properties

Modified Cam clay parameters					
Soil Type	κ	λ	M	e_{cs} at 1 kPa	ν'
Illite	0·047	0·171	—	1·799	0·25
Boom clay	0·020	0·129	0·87	1·330	0·38
Thermal properties					
Soil type	α_s	α_w	β*A	β*B at OCR = 1	All thermal coefficient are/°C.
Illite	0·35E-04	See Figure 1	−0·24E-03	−0·24E-03	
Boom clay	0·35E-04		−0·30E-03	−0·30E-03	*A and *B refer to
Kaolinite	0·35E-04		—	—	analyses A and B.

was selected to give the best prediction of drained volumetric behaviour due to temperature changes.

4.4. Additional data required for analysis of transient problems

The additional soil parameters required for the analysis of transient problems are ρ_s, ρ_w, c_s, c_w, γ_w, k and f. Except for the permeability k, the effect of temperature on the other parameters is generally accepted to be insignificant. These properties can be directly measured or selected from the data available in the literature. The permeability of a soil increases substatially with the temperature as the viscosity of pore water decreases. Houston and Lin[21] have reported good agreement between the measured value of permeability at a higher temperature and the value predicted from observations at the ambient temperature, correcting for the change in the viscosity and the voids ratio using the Kozeny–Carmen equation. This has not been necessary here because transient problems are not considered. They will be the subject of a future paper.

5. COMPARISON BETWEEN PREDICTIONS AND OBSERVATIONS

The experimentally observed volumetric and shear behaviour of triaxial samples is compared here with the predictions made using the modified Cam clay model. The experimental results for kaolin and illite were presented by Campanella and Mitchell,[5] whilst those for Boom clay were reported by Baldi *et al.*[6] and Hueckel and Pellegrini.[18] In all experiments the volumetric strains were determined using the measured axial strains assuming isotropic deformation behaviour.

5.1. Undrained volumetric behaviour

Figure 3 shows the volumetric strains observed during the undrained heating and cooling of a saturated kaolinite sample. The effective cell pressure on the sample was 180 kPa at the beginning of the undrained heating, but its stress history was not reported. The temperature of the specimen was initially decreased from 15·5 to 4·5°C, and then increased in steps to 60°C before returning to the original value. The predictions are presented on the same graph, and in this case

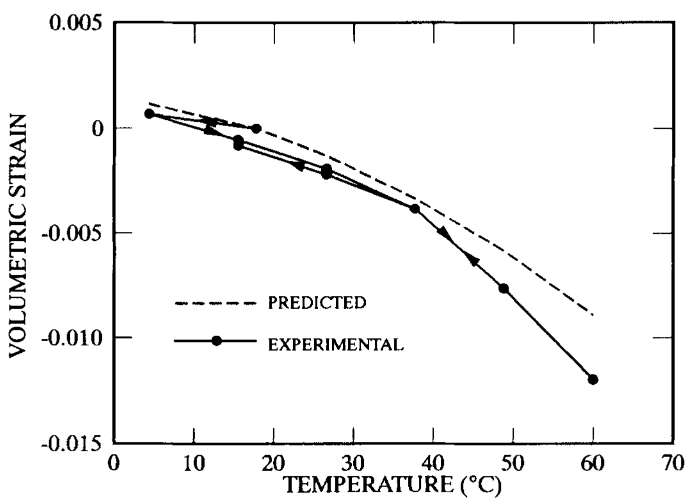

Figure 3. Volumetric strains of NC kaolin due to temperature changes under undrained conditions

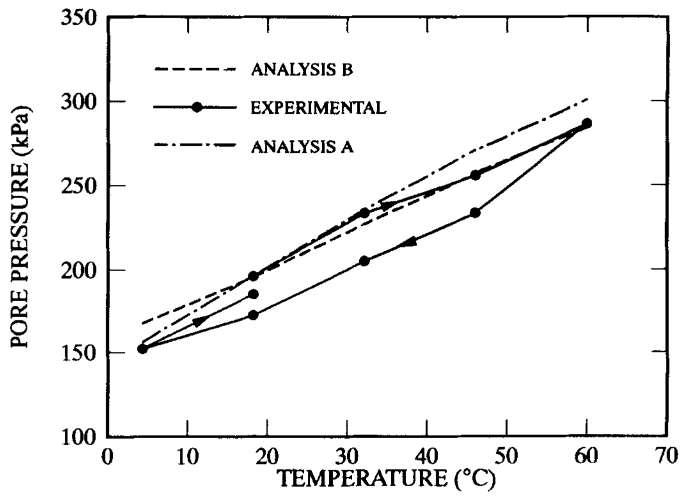

Figure 4. Pore pressures generated due to temperature changes (NC illite)

they are identical for analyses A and B. The experimental results are reasonably well predicted by the analysis. The disagreement is greatest during the initial cooling, where unlike the predicted behaviour the observed volumetric strains are clearly irreversible.

Figure 4 illustrates the observed and predicted excess pore pressures in an illite specimen during an undrained temperature cycle. The sample was initially normally consolidated but subjected to a number of drained temperature cycles before the current undrained cycle; this initial cycling may have overconsolidated the specimen. During the undrained stage, the temperature of the sample was first increased to 60 °C from 19 °C, then decreased in steps to 5 °C before reheating to 19 °C. The effective cell pressure on the sample was 196 kPa at the start of the undrained cycle. Figure 4 also shows the responses predicted by analyses A and B, which also followed the same drained and undrained temperature cycles as the laboratory specimen. It is clear that both analyses predict the average trend in the experimental behaviour well, but the predictions made using a value of β varying with η are slightly better. The main differences between the predictions and observations are the irreversibility and hysteresis of the observed behaviour. This is not surprising, as the modified Cam clay model would not be able to predict such phenomena, even in the usual isothermal consolidation behaviour. Similar agreement is also found between the predictions and the observations on another illite sample reported by Campanella and Mitchell.[5] A complete set of observations of undrained volumetric behaviour (i.e. both volumetric strain and pore pressure measurements) is only available for the kaolinite sample mentioned earlier. However, the authors were unable to predict pore pressures in this case due to lack of data on the material properties and stress history of this sample.

5.2. *Drained volmetric behaviour*

Figure 5 illustrates the volumetric strains observed during the drained heating and cooling cycle of an illite specimen initially normally consolidated at 196 kPa. The temperature of the sample was initially increased to 60 °C from 19 °C, then decreased to 5 °C in steps before reheating to the original value. The predictions from analyses A and B are shown on the same figure. The agreement is very good in both cases except at low temperatures during cooling. An interesting

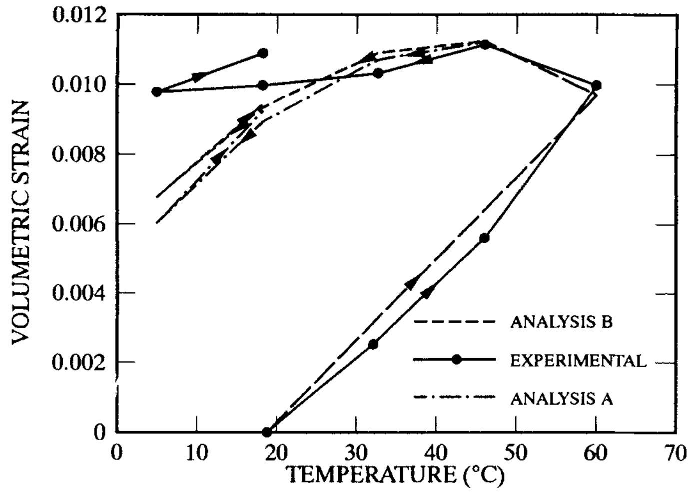

Figure 5. Volumetric strains due to temperature changes under drained conditions (NC illite)

feature in the observed behaviour is the compressive volumetric strains observed during the initial stages of cooling, which is also predicted well by the analyses. In the experiment the soil sample was allowed to attain uniform undrained conditions after a temperature increment before allowing drainage. Since the excess pore pressure generated during cooling are negative, according to the modified Cam clay model, yielding would have occurred during the undrained phase but not during the drained phase. Consequently, there is a substantial difference in the stiffness of the soil during the undrained and drained phases, which is the reason for compressive strains during initial cooling. This effect decreases and becomes insignificant at low temperatures, mainly due to the large reduction in the value of α_w with decreasing temperature (Figure 1).

Figure 6 illustrates the water outflow from the sample observed in the above-mentioned test and the predictions given by analyses A and B. The agreement between the observed behaviour and the predictions is very good in both cases except during the first increment of heating and towards the later stages of cooling. According to Campanella and Mitchell,[5] the contrasting behaviour between the first and subsequent steps of heating may be due to an apparent overconsolidation effect[22] caused by the long period of consolidation prior to the testing (about one week). Similar agreement between the observed and predicted behaviour was found for the data on another sample of illite presented by Campanella and Mitchell.[5]

Figure 7 shows the drained volumetric behaviour of a sample of Boom clay initially normally consolidated at 6 MPa and the predictions made by analyses A and B. The temperature of the specimen was increased from 21 °C to 84·5 °C in steps before cooling back to the original value. In performing these calculations it was assumed that the temperature changes occurred rapidly and were followed by periods of pore pressure equalization, similar to the procedure adopted by Campanella and Mitchell.[5] Based on these assumptions, both analyses predicted the experimental behaviour well. However, it appears that in the laboratory tests conducted on Boom clay, the samples were subjected to very slow, computer-controlled temperature changes, apparently under fully drained conditions (no pore pressure generation). For this type of simulation the model would perform less well, predicting expansive strains for each step during cooling, and not the volumetric compression observed in the first step in the laboratory tests.

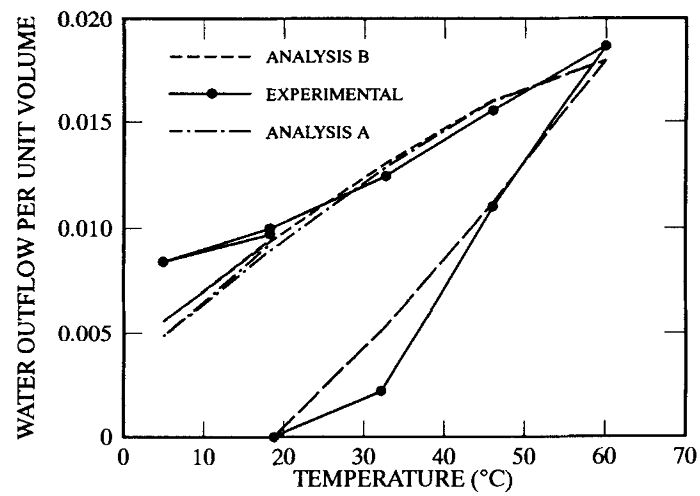

Figure 6. Water outflow due to temperature changes under drained conditions (NC illite)

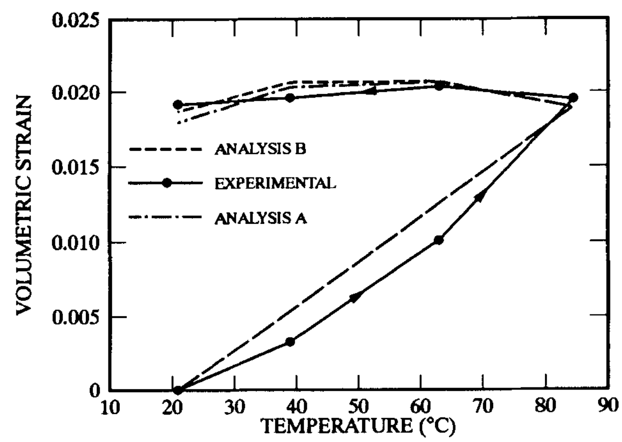

Figure 7. Volumetric strains due to drained heating (NC Boom clay)

5.3. Undrained shear behaviour

Figures 8(a) and 8(b) illustrate the observed behaviour of a triaxial sample of Boom clay subjected to undrained heating under constant total stresses, reported by Hueckel and Pellegrini.[18] The specimen was apparently normally consolidated at an effective pressure of 5·7 MPa initially, and sheared under undrained conditions to a deviator stress of about two-thirds of the undrained shear strength before heating. The heating was performed in steps from the ambient temperature of 21 °C to failure. The failure, associated with a shear band, occurred at a temperature of about 92 °C. The predictions of analyses A and B are also shown in Figures 8(a) and 8(b).

Except close to failure, the observed pore pressure increments during heating are overpredicted by analysis A by about 100 per cent. However, the predictions from analysis B are much better and agree well with the observations. The decrease in pore pressure at higher temperatures

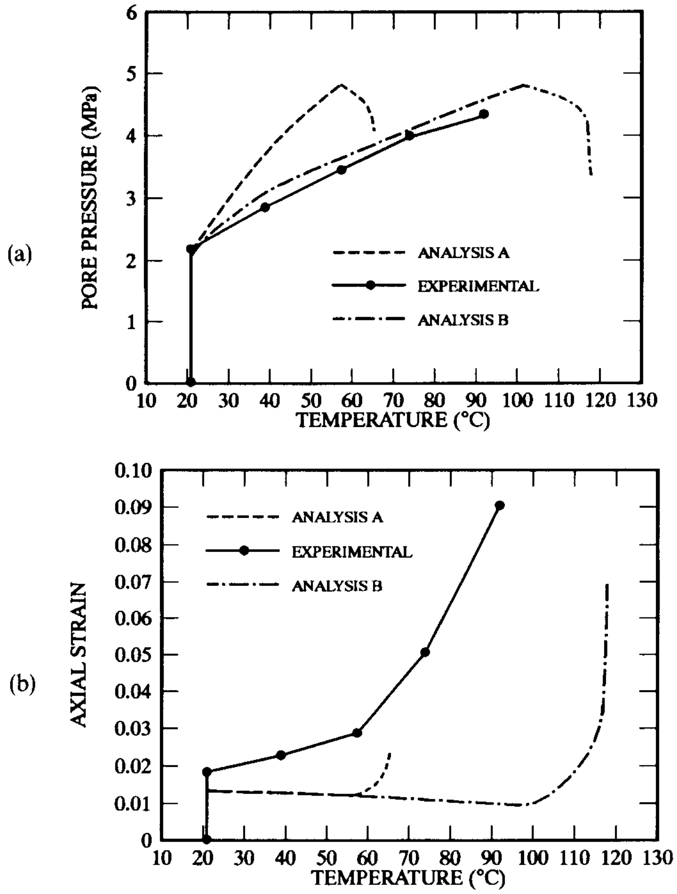

Figure 8. Undrained heating of Boom clay under constant deviatoric stress.

predicted by the analyses is absent in the observed behaviour. This difference between the observed behaviour and modified Cam clay predictions is usually encountered in heavily overconsolidated clays and is associated with the softening behaviour of these materials. Such softening has been observed in tests on a variety of other clays (e.g. References 24 and 9). The temperature at failure is underpredicted by analysis A and overpredicted by analysis B. This may be due to the variation of β with the stress ratio, η assumed in the analyses.

The axial strains (Figure 8(b)) are poorly predicted by the analyses, particularly during the early stages of heating. In fact, at these stages the observed strains are compressive whilst predicted axial strain changes are expansive. Compressive axial strains due to undrained heating in the presence of deviatoric stresses have also been reported by Mitchell and Campanella.[8] They attributed these compressive strains to creep, which is more significant at higher temperatures. However, it is not certain whether creep alone can describe this aspect of shear deformation behaviour.

The predicted stress path during heating is one of constant deviatoric stress, with an initial reduction of mean effective pressure, p', followed by an increase of p' to the critical state value. The predicted behaviour is initially elastic followed by softening on the dry side of the critical state. However, in the experiment p' decreased to failure monotonically.

The analyses seem to be incapable of predicting several aspects of the observed behaviour. The modified Cam clay model is known to be unreliable in the heavily overconsolidated region at which the experimental failure must have occurred. However, this alone cannot explain the differences in the observed and predicted behaviours. Clearly, more research is needed in this area before the shear deformation behaviour during heating can be fully understood. This seems to be the most serious drawback in the use of modified Cam clay in predicting the thermomechanical behaviour of clays. No other analyses of the shear behaviour have been made due to the absence of complete experimental data.

6. COMPARISON WITH THERMOPLASTIC MODIFIED CAM CLAY

The observed deformation and strength behaviour of clays is reviewed here in the light of the modified Cam clay formulation proposed by Britto *et al.*[1] and that presented above, and the thermoplastic model proposed by Hueckel and Borsetto.[2] To avoid any ambiguity, the model proposed by Britto *et al.* is referred to hereafter as the simple modified Cam clay model and that proposed by Hueckel and Borsetto as the thermoplastic modified Cam clay model.

6.1. Modified Cam clay with thermoplasticity

The thermoplastic modified Cam clay yield locus is illustrated in Figure 9. It is identical to the usual modified Cam clay yield surface under isothermal conditions, but varies with temperature regardless of the current state of yielding. Loading and unloading criteria are such that drained heating at a constant state of effective stress is fully plastic, with the thermal softening due to the increase in temperature being compensated for by the mechanical hardening. In contrast, drained cooling at constant effective stress is an elastic process during which the yield surface grows with the decrease in the temperature. The plastic strains are calculated assuming associated flow. The

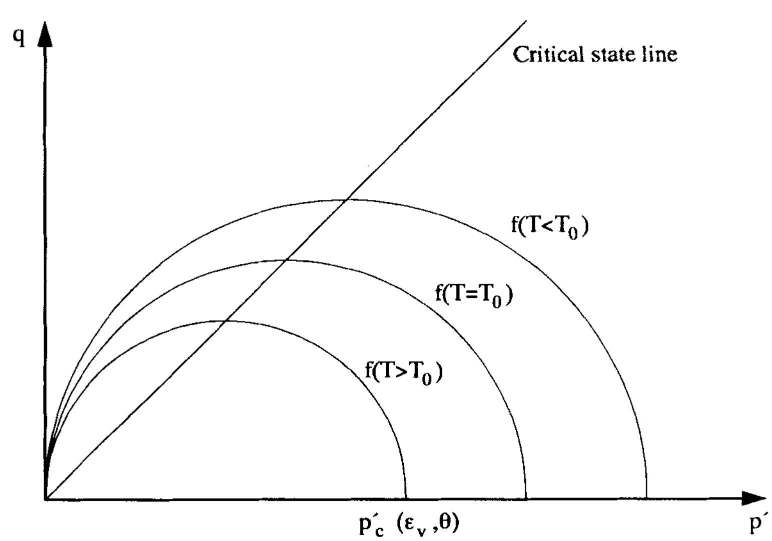

Figure 9. Yield locus for thermoplastic modified Cam clay

formulation given by Hueckel and Borsetto actually incorporates non-associated flow, but that is not considered here.

The isothermal elastic stress–strain behaviour is identical to that used in the usual modified Cam clay model. The elastic thermal behaviour is modelled by the introduction of an isotropic thermal strain determined using an effective thermal volume expansion coefficient. Allowances are made for the coefficient of effective thermal volume expansion and the effective bulk modulus to vary with the temperature and mean effective pressure. In contrast, the simple modified Cam clay model only accounts for the variation of the effective bulk modulus with the mean effective pressure. However, in analysis B presented in this paper, the coefficient of thermal volume expansion was assumed to vary with the stress ratio, η. There is very little evidence at present to justify these additional variations being incorportated into the thermoplastic model; they can be easily added to any existing model if necessary, after the nature and significance of them are fully understood.

6.2. Volumetric behaviour

The authors are unaware of any direct comparisons between experimental observations and predictions made using the thermoplastic model. However, Baldi *et al.*[6] compared the experimental behaviour during heating of normally and overconsolidated samples of Boom clay and Pontida clay using stress–strain relations similar to those given by the thermoplastic model. These comparisons appear to have been made prior to the publication of the thermoplastic model by Hueckel and Borsetto.[2] For the heating of normally consolidated samples, Baldi *et al.*[6] found reasonably good agreement between the predictions and the observed volumetric strains and water flow. However, they found poor agreement between the predicted and observed behaviours of overconsolidated samples. Baldi *et al.*[6] concluded that the difference in the coefficient of thermal volume expansion of free water and adsorbed water was the reason for this disagreement. The authors are of the opinion that the same results may be obtained in normally and lightly overconsolidated clays using the thermoplastic model, ignoring this difference in the coefficient of volume expansion of free water and adsorbed water and allowing for the correct variation of the coefficient of effective thermal volume expansion with overconsolidation ratio. No comparisons of the behaviour during cooling were reported by Baldi *et al.*[6]

Hueckel and Baldi[24] reviewed the observed volumetric behaviour of three clays using the thermoplastic model. They demonstrated that many aspects of the observed behaviour of normally and overconsolidated clays can be derived from the thermoplastic model. Interestingly, except in two cases, the simple modified Cam clay model can also explain the observed behaviour in all the cases considered by them. In the first exception, a normally consolidated soil was observed to behave as if it had been overconsolidated after a heating and cooling cycle (e.g. Reference 20). Hueckel and Baldi explain this effect as due to the thermal growth of the yield surface during cooling. In contrast, if the heating and cooling cycle is fully drained, the simple modified Cam clay model predicts that the soil will remain normally consolidated throughout the cycle. However, the experiments reported in the literature have not, in general, been true drained tests, drainage being allowed only after the thermal equilibrium has been established. In such an experiment negative pore pressures would be generated during the cooling, leading to overconsolidation of the soil. Figure 10 illustrates the stress path predicted by the simple modified Cam clay model in this case. The undrained parts of this stress path (AA′ and BB′) do not lie on the normally consolidated or swelling lines due to the presence of thermal strain, i.e. the simple model can explain the observed behaviour.

In the second case of disagreement, an overconsolidated sample has indicated a lower value of maximum preconsolidation pressure when heated before reloading. This, of course, cannot be

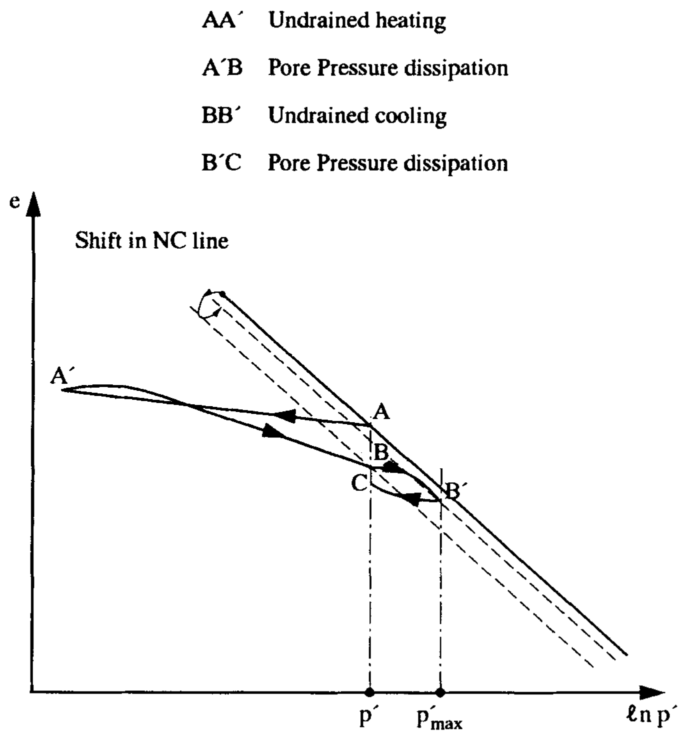

Figure 10. Stress path during heating and cooling cycle

explained by the simple modified Cam clay model without allowing for a temperature dependence of the yield locus. Clearly, further good data on the fundamental thermomechanical behaviour of remoulded clays is needed to verify the validity of either model.

6.3. Soil strength

The observed effects of temperature changes on the shear strength of soils are highly variable. This is partly due to the differences in the methods of sample preparation and testing procedure adopted by various researchers. Sherif and Burrows[25] have reported the effect of heating on the unconfined compressive strength of remoulded kaolin. The initially normally consolidated soil samples were heated undrained before testing. For all specimens, a decrease in strength was observed with an increase in the temperature. This is not surprising, as the positive excess pore pressures generated due to heating should have lowered the effective pressure in the soil specimens. It is not certain whether factors other than the mechanical effect of pore pressure contributed to the observed decrease in the shear strength. Both the thermoplastic model and the simple model would predict a decrease in undrained shear strength under the circumstances given above. However, the reduction in the undrained shear strength predicted by the thermoplastic model should be larger as it incorporates thermal softening.

Considering only the data on undisturbed samples and on remoulded clays prepared from slurry, it is difficult to conclude whether a decrease or an increase of strength is associated with drained heating. On the wet side of the critical state, both the thermoplastic model and the simple model would predict no change in drained shear strength after drained heating. In one particular test, a normally consolidated triaxial specimen was subjected to drained heating under a constant deviator stress, and then sheared to drained failure (reported by Hueckel and Baldi[24]). As can be

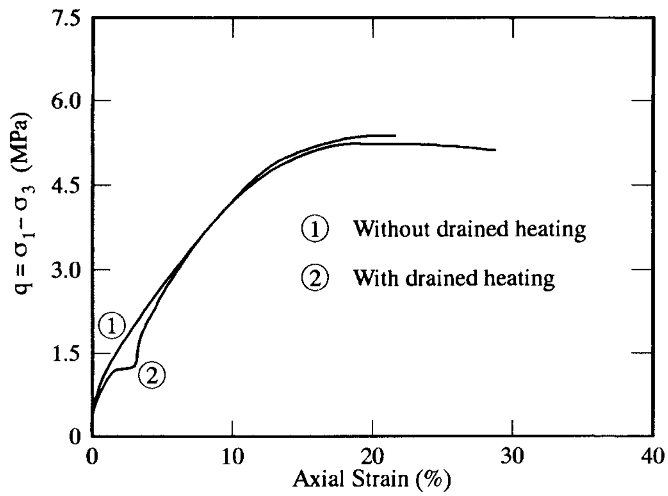

Figure 11. Effect of drained heating on drained triaxial behaviour

seen from Figure 11, the strength of this sample is almost identical to that of an unheated specimen.

On the dry side of the critical state, the thermoplastic model predicts a decrease in the peak shear strength with drained heating. Hueckel and Baldi[24] have presented some experimental evidence to support this prediction. In contrast, the simple modified Cam clay model predicts no change in peak shear strength after heating. The observed ultimate shear strengths in the experiments reported by Hueckel and Baldi appear to be independent of the temperature and hence the effect of temperature on the strength parameter, M, may not be significant.

6.4. *Shear deformation behaviour*

Very little experimental data is available on the shear deformation behaviour of clays in the presence of temperature changes and deviator stress. Hueckel and Baldi[24] have reported the results of a drained triaxial test on a normally consolidated sample. The specimen was first subjected to a drained deviatoric load of about 25% of the value of the deviatoric stress at drained failure. Then it was heated under drained conditions before further drained shearing to failure. Figure 11 illustrates the stress–strain behaviour observed in this test. It is very unlikely that either model would predict the shape of the stress–strain curve after heating. The strains observed in this experiment during the drained heating phase have been almost purely volumetric. This conforms to the simple modified Cam clay model but not to the thermoplastic model.

Hueckel and Baldi also presented some data on the stress–strain behaviour of heavily overconsolidated clay observed at room temperature and at elevated temperatures. The observed volumetric behaviour at elevated temperatures is less dilative, and this is predicted by the thermoplastic model. In contrast, the simple model predicts no difference in dilation behaviour at elevated temperatures.

Hueckel and Pellegrini[18] analysed the failures due to undrained heating of triaxial samples of Boom clay and Pontida clay using the thermoplastic model; the analysis of the same test results on Boom clay using the simple modified Cam clay model was given in Section 5.3. Hueckel and

Pellegrini used a modified coefficient of thermal volume expansion of pore water to account for the difference in the coefficient of volume expansion of free water and adsorbed water. This was carried out using a relation proposed by Baldi *et al.*[6] for low-porosity clays considering microstructural changes. Hueckel and Pellegrini concluded that the predictions given by the thermoplastic model are consistent with the observed behaviour. However, their analysis predicts compressive volumetric strains during undrained heating. Measurement of such strains is complex, but is required to confirm this somewhat unusual prediction. The thermoplastic model is likely to encounter difficulties similar to those experienced by the simple model in predicting the behaviour during undrained heating, particularly in high-porosity clays in which the relation given by Baldi *et al.*[6] is not applicable. Clearly, more experimental work is necessary in this area.

7. CONCLUSIONS

The simple modified Cam clay model proposed by Britto *et al.*[1] can be used with some confidence to predict the volumetric behaviour and pore pressure of clays subjected to changes in temperature under isotropic stress conditions. One advantage of this model is that it can easily be incorporated into an existing finite element package. In contrast, the thermoplastic version of the modified Cam clay model proposed by Hueckel and Borsetto[8] is more complex. Nevertheless, there is some evidence that this model can also be used to predict the isotropic behaviour. Neither of the above models has been tested thoroughly under deviatoric conditions and the initial indications in this regard are not very favourable. The above factors should be considered, particularly in analysing boundary value problems in which deviatoric behaviour is predominant. Clearly, more fundamental experimental data, particularly regarding deviatoric behaviour, is necessary before developing new models or adopting the existing ones to describe completely the thermomechanical behaviour of clays.

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NOTATION

a	vector $(1, 1, 1, 0, 0, 0)^T$
c_s	specific heat of soil grains
c_w	specific heat of pore water
D	matrix of elastoplastic constants
e	voids ratio
e_{cs}	voids ratio at unit pressure on the critical state line
f	thermal conductivity
G	shear modulus
h	vector of heat flux
k	coefficient of permeability
K'	effective bulk modulus
m	heat capacity
n	porosity
OCR	overconsolidation ratio
p	excess pore pressure

p'	effective pressure
p'_c	intercept of yield locus on p' axis
q	deviatoric stress
t	time interval
T	absolute current temperature
T_0	absolute ambient temperature
u	vector of displacement components
v	seepage velocity vector
α	coefficient of undrained thermal volume expansion
α_s	coefficient of volume expansion of soil grains
α_{st}	physicochemical temperature coefficient of soil structural volume change
α_w	coefficient of volume expansion of pore water
β	coefficient of effective thermal volume expansion
γ_w	unit weight of water
ε	vector of strain components
ε_v	volumetric strain
η	stress ratio, p'_c/p'
θ	temperature increase
κ	gradient of swelling line in $e - \ln p'$ space
λ	gradient of normal consolidation line in $e - \ln p'$ space
M	stress ratio q/p' at the critical state
v'	effective Poisson's ratio
ρ_s	density of soil grains
ρ_w	density of pore water
σ	vector of total stress increment
σ'	vector of effective stress increment

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