Rock Mech Rock Eng
DOI 10.1007/s00603-011-0183-0

ORIGINAL PAPER

Laboratory Investigation on High Values of Restitution Coefficients

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Received: 26 June 2011 / Accepted: 30 August 2011

Abstract Restitution coefficients are used to quantify the energy dissipation upon impact when predicting rock fall events. These coefficients can be determined in situ or in the laboratory. In any case, the usual values for the normal restitution coefficient k_n are below unity. Values greater than one are quite rare, seen as unusual and barely explained. Previous experimental research conducted in Australia has shown consistent and systematic values of the normal restitution coefficient greater than one. This was tentatively explained by a combination of parameters such as low impacting angle, rotational energy and block angularity. The study presented in this paper aims at (1) identifying the critical parameters conducting to high k_n values and (2) at explaining the associated motion mechanisms. The objective was reached with values of k_n up to almost 2. In addition, the study has confirmed the significance of low impacting angle, rotational energy and block shape in this context.

Keywords Rock fall · Restitution coefficient · Natural hazard · Rotational energy · Impact

1 Introduction

Rock fall is a natural hazard of potentially costly consequences. Design of adequate protection structures involves predicting the possible rock fall events, and in particular, the rocks trajectory. This is usually achieved by means of numerical codes such as RocFall (Stevens 1998) or CRSP (Pfeiffer and Bowen 1989) in which the slope/rocks interaction is defined in terms of restitution coefficients. These latter are crucial for the prediction of trajectory and energy as they quantify the energy dissipation upon impact.

Many in situ rock fall tests have been conducted in order to estimate the restitution coefficients and presumptive values can now be found in the literature (e.g. in Giani et al. 2004; Azzoni et al. 1992; Hungr and Evans 1988; Pfeiffer and Bowen 1989). Interestingly, unity is often seen as the upper bound for restitution coefficients or at least, as the maximum range of usual values [see numerical codes CRSP, RocFall, Pierre (Mathon et al. 2010)]. Yet, the definition of restitution coefficients most commonly used, i.e. in terms of velocity as opposed to energy, does not yield such an upper bound. In fact, normal restitution coefficients in excess of unity have been reported in the literature, even if quite rarely (Azzoni et al. 1992; Paronuzzi 2009; Mathon et al. 2010). After a back analysis of a rock fall event, Paronuzzi (2009) has even obtained values of k_n as high as 2.77. Recently, Spadari et al. (2011) performed in situ tests in an Australian environment and have consistently obtained values of k_n greater than one. Azzoni et al. (1992) explained that high restitution coefficients could be due to the rotational energy of the rocks. Considering the findings from Bourrier et al. (2009), Spadari et al. (2011) suggested that the angularity of the blocks and the low impacting angle were also probably playing a key role. Unfortunately, their results were plagued by a high variability (due to the natural environment) and no definitive conclusions could be drawn. As far as we are aware, high values of normal coefficients were reported only in relation to field testing and mainly for low slope angles (Spadari et al. 2011; Paronuzzi 2009). Laboratory studies tend to produce the usually low values of restitution coefficients (Labiouse and Heidenreich 2009; Chau et al. 1999, 2002; Wu 1985).

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Published online: 15 September 2011

The present study was undertaken to explain the experimental results of Spadari et al. (2011) and provide some understanding on the mechanisms or combination of parameters that could lead to systematically high values of restitution coefficients. In particular, $k_n > 1$ were targeted. The influence of rotational energy, block shape and low impacting angle has been investigated under controlled conditions. The results were found to corroborate the conclusions drawn by Spadari et al. (2011). For the first time, high values of k_n have consistently been achieved in the laboratory when combining non circular blocks, high rotational energy, and low impacting angle. The paper progressively presents the influence of each parameter before discussing the motion mechanisms.

2 Testing Methods

2.1 Testing Apparatus

The apparatus specifically built for this study consists of a ramp, a spinning/releasing mechanism and a landing block (see Figs. 1, 2). The ramp is made of two steel channels (Fig. 2) in which the block shaft is engaged by means of ball bearings (see Fig. 1). These latter were used to limit friction and to avoid parasite rotation due to the blocks down the channels. The spinning device (Fig. 2) is made of an electrical motor with a speed controller and a covered belt. A driving mechanism has been designed to couple/uncouple the block to the engine during the phases of spinning/releasing. The release mechanism, which is operated with a crane, consists of two pins guided through the channels and acting on the block shaft (Fig. 1). With such a device, blocks could be dropped with constant kinetic energy but variable and controlled rotational energy.

2.2 Concrete Blocks

The falling blocks used in this study are all flat with four different forms: circular, elliptic, square and pentagonal (see Fig. 3 and characteristics in Table 1). In the following, these blocks will be referred to as disc, ellipse, square and pentagon. Square and pentagon were used because it was

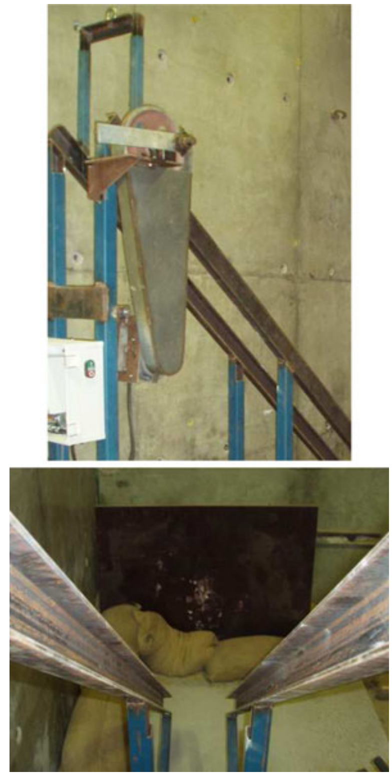


Fig. 2 Photographs of the testing apparatus. *Top* view of the top part of the ramp: spinning device with cover and release mechanism. *Bottom* view of the steel channels making the ramp

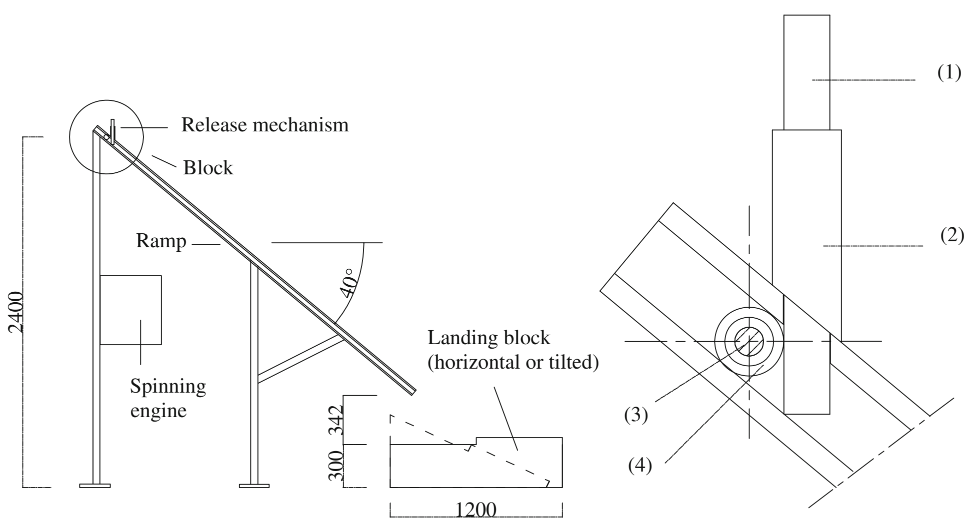


Fig. 1 Schematic representation of the testing apparatus. *Left* general view (dimensions in mm). *Right* detail of the release mechanism and ball bearing. (*1*), release pin; (*2*), guide; (*3*), block shaft; (*4*), ball bearing. The landing block as represented includes a step, the flat block is not shown

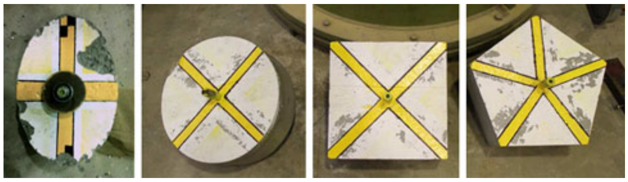


Fig. 3 Ellipse, disc, square and pentagon blocks used for the tests. The *stripes* are markers for the image analysis

Table 1 Mass and dimensions of the falling blocks

Block	Characteristic length	Value (mm)	Mass (kg)
Disc	Radius	196	42.7
Square	Side	347	45.8
Pentagon	Side	265	43.9
Ellipse	Long/short axis	294/196	39.1

thought that the angularity of a falling block might play a role on its motion and could be partly responsible for high values of normal restitution coefficient. The concrete used for the falling blocks has a compressive strength of 60 MPa and was reinforced with steel fibers.

Three geometries were considered for the landing block: (1) horizontal and flat, (2) horizontal with a 85 mm step and (3) inclined by 25° with a step (see Fig. 1). The landing block was designed to be heavy enough (mass ranging from 500 to 1,000 kg) so that it would not move upon impact. It was cast using the same concrete than for the falling blocks. The step would eventually wear upon the successive impacts and dental plaster (compressive strength of 55 MPa) was used to re-shape it when too damaged. The friction angle between the impacted surface and the blocks was found to be about 23°. The height of the step (85 mm) falls in between the range of macro roughness recorded for the testing sites by Spadari et al. (2011).

2.3 Image Analysis and Calculations

The impacts and bounces were recorded by a high speed camera (Model CR600 from Optronis, 500 frames per second). The velocity pre and post impact was estimated from the sequence of photographs and using the image tracking software TEMA 3.3. This software can track the displacement of markers (stripes in Fig. 3) from which it derives speed and acceleration.

The restitution coefficients were calculated using Eq. 1, which is the most commonly adopted definition in the literature (Giani et al. 2004; Labiouse and Heidenreich 2009; Paronuzzi 2009).

$$k_i = \frac{V_i^{\text{post}}}{V_i^{\text{pre}}} \quad (1)$$

where the subscript i is either n for normal or t for tangential, normal and tangential referring to the average impact plan. The superscripts *pre* and *post* refer to the velocity pre and post impact, respectively. In case of a combination of rolling and bouncing: pre-impact refers to the moment just before the first impact and post impact refers to the moment just after the block has left the impacted surface.

2.4 Experimental Program

Three series of tests have been conducted, the details of which are given in Table 2. For each series, two or three block shapes were used at different rotational speeds. Each test was repeated five times to account for the randomness of the impact. In the remainder of the paper, only average results are presented for the sake of clarity.

The in situ rotational speed recorded by Spadari et al. (2011) ranged from 150 to 500 revolutions per minute (RPM), with an average of about 300 RPM. However, for safety reason, a maximum value of 300 RPM was used in the laboratory. In the following and for the sake of comparison, the results will be expressed in terms of rotational speed in lieu of rotational energy as all the blocks do not have exactly the same moment of inertia.

3 Results

3.1 Series 1: Effect of Rotational Speed and Block Shape

The first series of tests was conducted with three block shapes (circle, square and pentagon) on a flat horizontal slab. The rotational speed has been progressively increased

Table 2 Summary of the experimental program: details of the three test series

Series	1	2	3
No. of tests	75	34	74
Blocks used	D, S, P	S, P	S, P, E
Landing surface	Horizontal flat	Horizontal with step	Inclined with step
Impact angle (°)	40	40	15
Rotation (RPM)	0/100/200/250/300	100/200/250/300	100/150/200/250/300

D disc, *S* square, *P* pentagon, *E* ellipse, *RPM* revolutions per minute to 300 RPM. The results, expressed in terms of restitution coefficients, are shown in Fig. 4. First of all, it can be seen that the values of normal restitution coefficient k_n are far from reaching unity. This result suggests that block angularity and rotational speed are not enough to obtain high values of k_n and thus can not explain the experimental results obtained by Spadari et al. (2011).

On the other hand, k_n tends to increase with the rotational speed for pentagonal and square blocks. As discussed by Azzoni et al. (1992), such result is due to the combined rolling and bouncing motion at impact. An angular block can experience multiple contacts and tends to flip over an edge. This multiple contact does not happen for the disc, which tend to bounce directly after impact. This would explain why there is no clear influence of the rotational speed on k_n for the round block.

The response in terms of k_t is quite different. The rotational speed does have an influence on k_t for the three blocks: the higher the rotational speed, the higher k_t. This is due to a transfer of rotational speed into tangential speed when the block comes in contact with the surface. The most consistent trend is for the circle mainly because the contact block/surface is punctual and, hence, more repeatable. As discussed previously, the contact for angular blocks is more complex and variable in nature, leading to a more variable trend. Accordingly, it can be noticed that as the block gets rounder, from square to circle, the trend becomes more consistent.

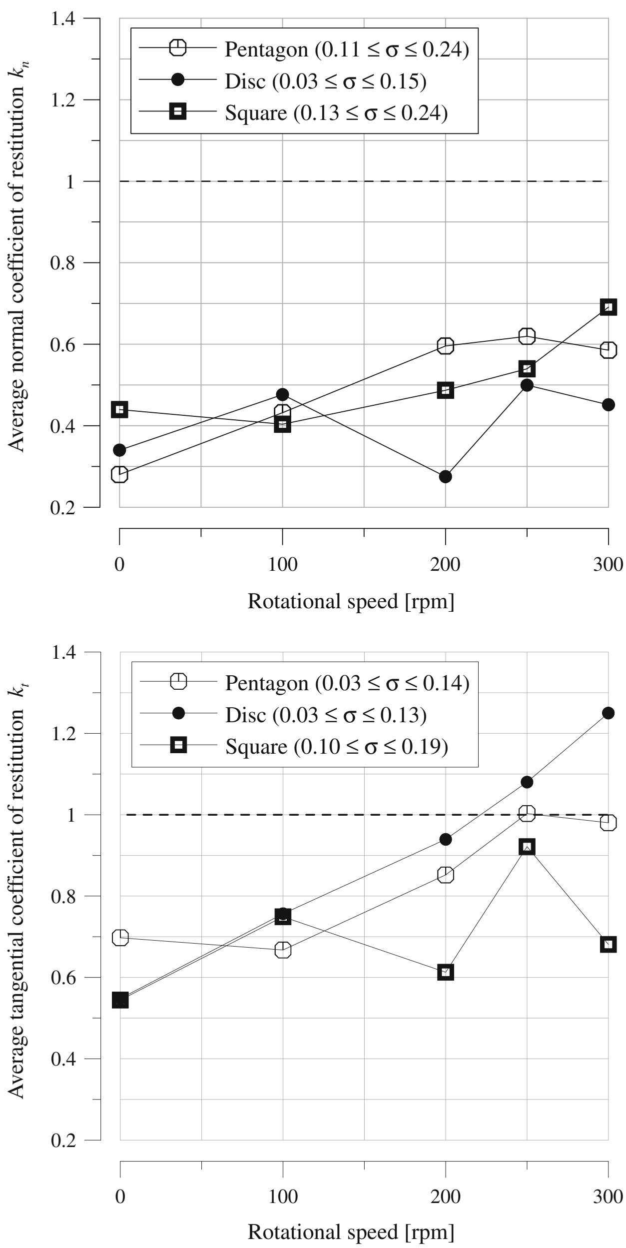


Fig. 4 Test series 1: evolution of the average restitution coefficient k_n (*top*) and k_t (*bottom*) as a function of rotational speed. The range of standard deviation σ is indicated in the legend for five measurements

3.2 Series 2: Effect of the Step

Considering that the combination of block angularity and rotational speed could not lead to values of k_n in excess of unity, some macro roughness has been added to the landing surface under the form of a 85 mm high step (see Fig. 1). According to Azzoni et al. (1995), this type of geometry could create a satisfactory rock/slope interaction provided the moments are not antagonistic. The disc was not used for that second series since only single and punctual contacts were to be expected.

Results for these tests are presented in Fig. 5. Despite an increase of k_n for the pentagon, all values of k_n remained below unity. The type of impact went from a simple bounce for the horizontal flat landing block to a combined rolling/bouncing when adding the step, which is believed to be at the origin of high restitution coefficients. This point will be deepened in Sect. 4. The effect of the rotational speed on k_n is still noticeable but the presence of the step modifies the trend on k_t previously obtained.

3.3 Series 3: Effect of the Impacting Angle

This last series combines the use of an inclined landing block (to reduce the impacting angle to 15°) with a step, three block shapes (ellipse, square, pentagon) and rotational speed up to 300 RPM. This time, values of k_n greater than one were consistently and repeatedly recorded (Fig. 6) with average values up to 1.6. This strongly suggests the importance of the low impacting angle when comparing to the outcomes of series 2. There is also a clear increase of k_n as the block spin faster which confirms the qualitative conclusions by Azzoni et al. (1992) regarding the significance of the rotational moments. The results for the ellipse are more scattered because of its high aspect ratio resulting in more variability at impact.

As previously, the evolution of the tangential restitution coefficient k_t with rotational speed is more variable (Fig. 6) and it is not trivial to define a trend. In the light of the three series, it appears that k_t would increase consistently with rotational speed if the block were rather round and the surface flat and smooth, thus allowing a transfer from rotational speed into tangential speed.

4 Discussion

Since the unusual results reported in the literature mainly deal with with k_n as opposed to k_t (Spadari et al. 2011; Mathon et al. 2010; Paronuzzi 2009), the following discussion focuses only on k_n. As mentioned previously, two types of impact have been observed: a single impact referred to as simple bouncing and a double impact referred to as combined bouncing/rolling. This latter occurred only in presence of the step and constitute the majority of the

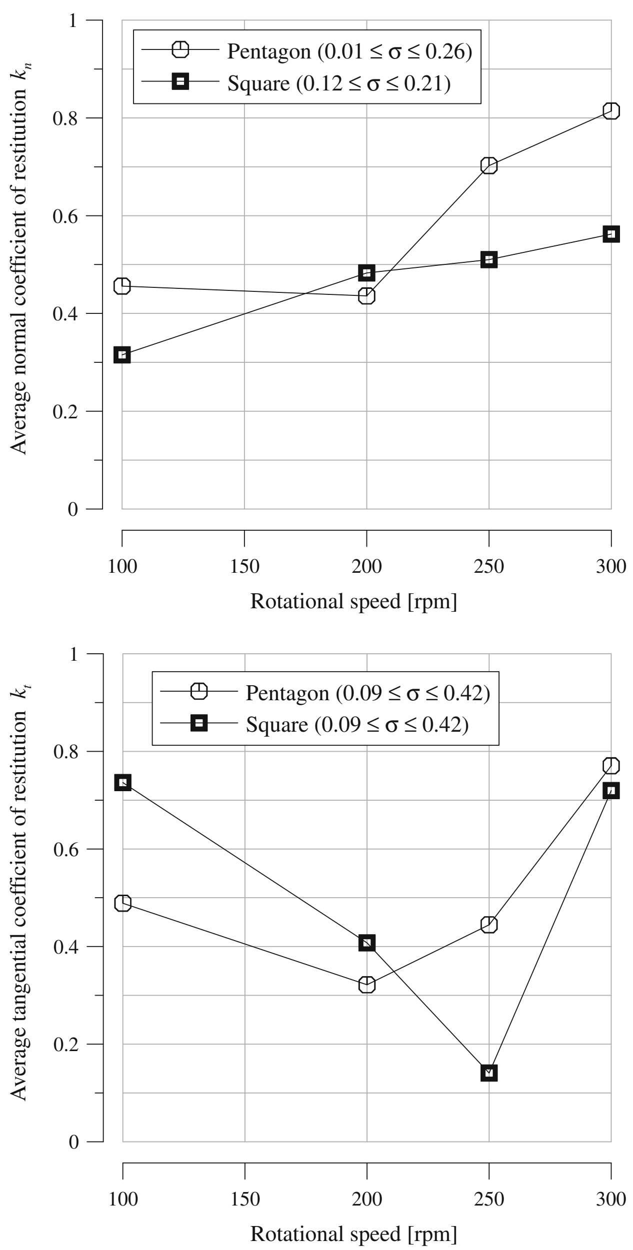


Fig. 5 Test series 2: evolution of the average restitution coefficient k_n (*top*) and k_t (*bottom*) as a function of rotational speed. σ is the standard deviation of three to five measurements

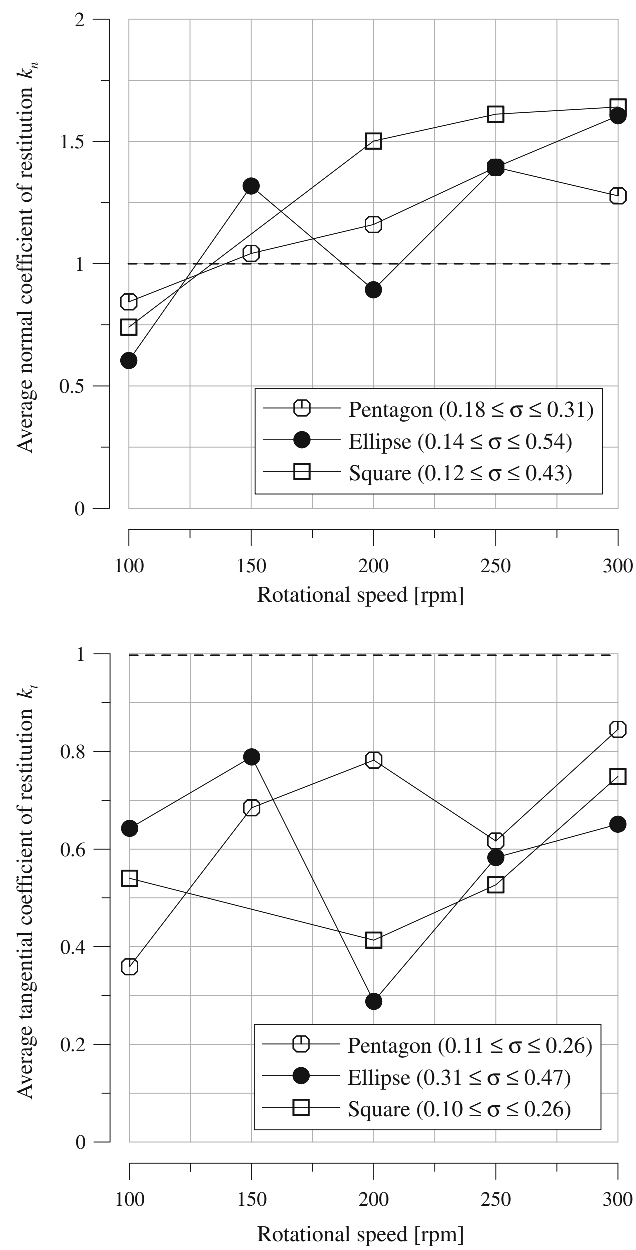


Fig. 6 Test series 3: evolution of the average restitution coefficient k_n (*top*) and k_t (*bottom*) as a function of rotational speed. σ is the standard deviation of three to five measurements

results. The bouncing motion will be analyzed according to a simple mathematical model used by Bozzolo and Pamini (1986) and Azzoni et al. (1995), which was extended to a surface incorporating a step.

4.1 Mathematical Model of Block Motion

For the combined bouncing/rolling impact, the block–surface interaction is a two phase phenomenon, which is reproduced in the mathematical model. In a first phase, the block impacts just before the step and rotation occurs around the contact point. In a second phase, the block hits the edge of the step around which rotation takes place (Fig. 7).

In this model, the velocity of the block, which is actually that of the centre of mass, is inferred from the conservation of the angular momentum (Bozzolo and Pamini 1986; Azzoni et al. 1995). The velocity after the first phase of the interaction becomes the input for the calculation of the velocity after the second phase, which is a simplification. The conservation of angular momentum yields:

$$w_i = \frac{I \cdot w_{(i-1)} + V_{x(i-1)} \cdot d_{yi} - V_{y(i-1)} \cdot d_{xi}}{I + d_{xi}^2 + d_{yi}^2} \quad (2)$$

where $i = 1{,}2$; index 0 refers to the moment before the first impact, index 1 to the moment after the first impact and just before second impact (assumption above mentioned) and index 2 to the moment after the second impact. w is the rotational speed (in rad/s), V_x and V_y are the directions parallel (x) and perpendicular (y) to the landing surface, respectively. I is the moment of inertia per unit mass, d_x and d_y are the distance along x and y between the centre of mass and the point of contact.

The component of velocity after each phase of the interaction can be calculated as:

$$V_{xi} = w_i \cdot d_{yi} \quad \text{and} \quad V_{yi} = -w_i \cdot d_{xi} \quad (3)$$

4.2 Parameters Yielding High k_n Values

Values of normal restitution coefficient k_n greater than unity require two conditions: (1) a bouncing angle higher than the impacting angle and (2) a low level of energy dissipation upon impact.

For the first series of tests on the flat horizontal surface, regardless of the rotational speed, the majority of the blocks experienced a simple bounce, e.g. contact in one corner for angular blocks (see Fig. 8, left) without a second contact. This type of interaction was not found to generate high bouncing angles. Indeed, when examining the motion in the light of the mathematical model, Fig. 8 (left) represents the most favorable case with center of mass being the most on the left of the contact point (highest d_x value, and hence highest V_y value). Using Eq. 3, the ratio of vertical over horizontal velocity is equal to d_x/d_y, which is roughly equal to 0.5 in Fig. 8 (left). This corresponds to an exit of about 25°, which is roughly in accordance with the experimental observations (Fig. 9, left). Consequently, even in the most favorable case, the k_n values tend to be low.

Adding some macro roughness to the landing block (series 2) modifies the motion of the falling block: most of the bounces happen in two phases as illustrated in Fig. 8 (middle and right). The second impact appears to be critical in regard to k_n: the ratio d_{x2}/d_{y2} increases to about one corresponding approximately to $V_{x2} = V_{y2}$ and hence to a bouncing angle of about 45°. This is confirmed by Fig. 9 (left).

Despite the improvement in terms of bouncing angles due to the macro roughness (Fig. 9), values of k_n greater than unity could not be reached during the second series of tests (Fig. 6). This is caused by an adverse effect of the step, which is a higher degree of energy dissipation upon impact as illustrated by Fig. 9. Unlike for the first series of

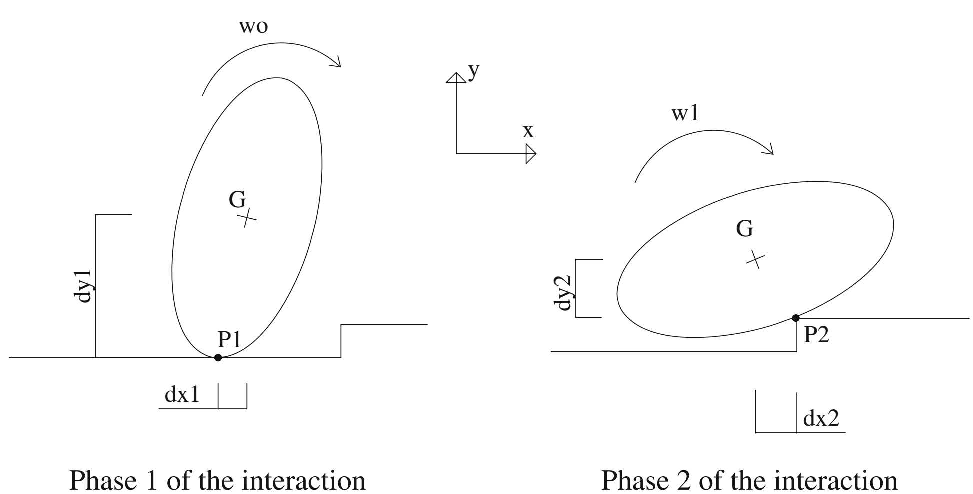


Fig. 7 Two phases of the ellipse/landing block interaction for a mathematical model describing the motion at impact. Initial model (phase 1) after Bozzolo and Pamini (1986)

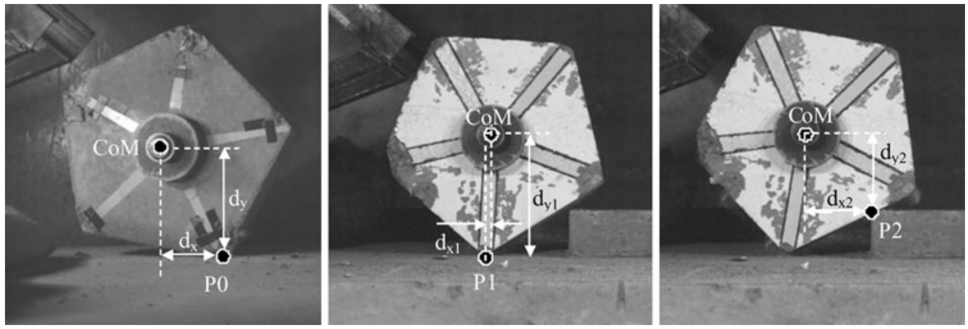


Fig. 8 Photographs of contact. *Left* single contact for the pentagon on the flat landing block during the first series of tests. *Middle* and *right* combined rolling/bouncing impact with two phase interaction for the second series of tests. *Middle* first impact with the landing block, *right* second impact. *CoM* centre of mass, *P* point of contact

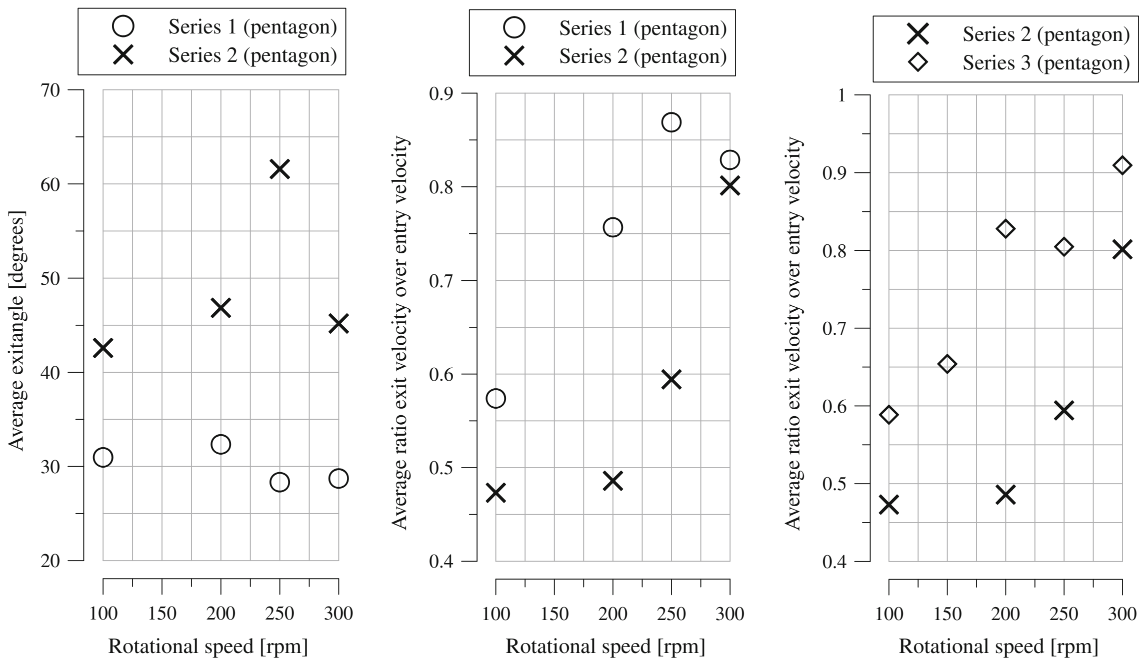


Fig. 9 Influence of the macro roughness and impacting angle on the motion characteristics for the pentagon. *Left* Comparison between series 1 and 2: average exit angle versus rotational speed. Entry angle around 40°. *Middle* Comparison between series 1 and 2: ratio of average exit velocity over average entry velocity versus rotational speed. Entry speed of around 5 m/s. *Right* Comparison between series 2 and 3: ratio of average exit velocity over average entry velocity versus rotational speed

tests, the velocity post impact is lower for the second series. In conclusion, the two conditions required to obtain $k_n > 1$, as mentioned at the beginning of the section, were not met.

The issue of energy dissipation was overcome by lowering the impact angle from 40° to 15°. Figure 9 (right) shows the resulting increase of post impact velocity. The transfer of rotational energy into velocity upon impact was of similar magnitude between the series 2 and 3. Hence, the increase of post impact velocity can solely be imputed to a lower degree of energy dissipation upon impact. Under the conditions of the third series, values of k_n greater than unity have been reached, validating and explaining the findings of Spadari et al. (2011).

As mentioned previously, some cases of simple bouncing occurred during the third series of tests. Analysis of these results shows that a low impacting angle on a flat surface (the block bounces above the step) can also yield high k_n (Fig. 10). This suggests that the presence of the step is not an absolute requirement.

Similarly to the case of an impact on the flat horizontal landing block (see discussion related to Fig. 8 (left) at the beginning of Sect. 4.2), the bouncing angle for a pentagon impacting a flat inclined block will be about 25°, according to the model. This is due to the fact that directions x and y are defined relative to the average impact plane (see Fig. 7).

Consequently, results of Fig. 10 can simply be explained by the fact that the impacting angle has been significantly reduced (from 40° to 15°) and is much lower than the bouncing angle (25°). Unless the energy dissipation is major, such a configuration leads to high k_n values.

In the light of the testing performed, it appears that the results obtained by Spadari et al. (2011) could be explained by a combination of the following parameters:

- low impacting angles (typically lower than 20°)
- some amount of rotational energy
- a block shape allowing obtention of angular moments (not a disc).

The macro roughness helps obtaining high k_n by modifying the impact type but it does not seem to be necessary, as discussed previously.

4.3 Application of the Mathematical Model

It is herein proposed to apply the extended version of the mathematical model for the bouncing motion to the ellipse and the pentagon to determine if it properly reproduces the experimental results. It can be seen in Fig. 11 (top) that the model slightly but consistently underestimates the values of normal restitution coefficient even though values over one were predicted. Comparing the predicted and experimental velocities post impact for one test (Fig. 11 (bottom)) showed that not only the magnitude of velocity is underestimated but so is the bouncing angle. This is reflected by a lower V_y/V_x ratio. The opposite trend would normally be expected as the model does not account for phenomena that would normally reduce the post impact velocity (e.g. block cracking, slippage at impact). However, the mechanisms leading to high values of k_n seem to be correctly captured. See, for example Fig. 11 (bottom), showing one trajectory of the centre of mass of the ellipse. The two contact points can clearly be seen as well as the increase of vertical velocity after the second impact (points are more distant from one another and trajectory is more vertical). It is not the scope of the paper to propose an improvement of the model.

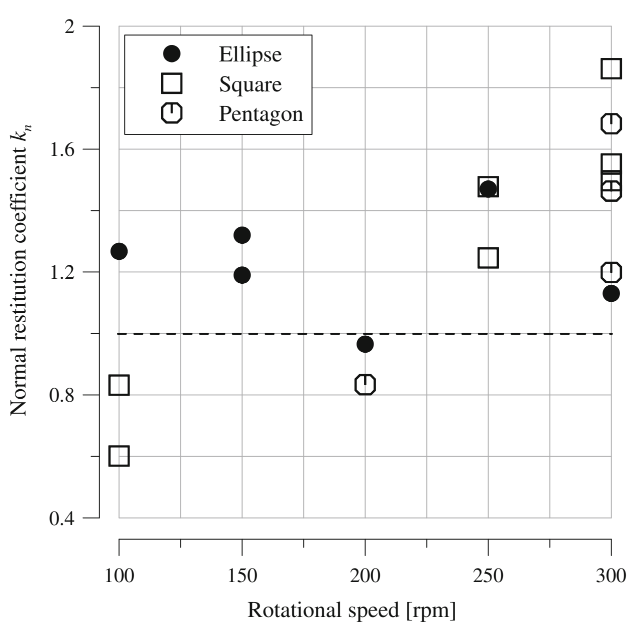
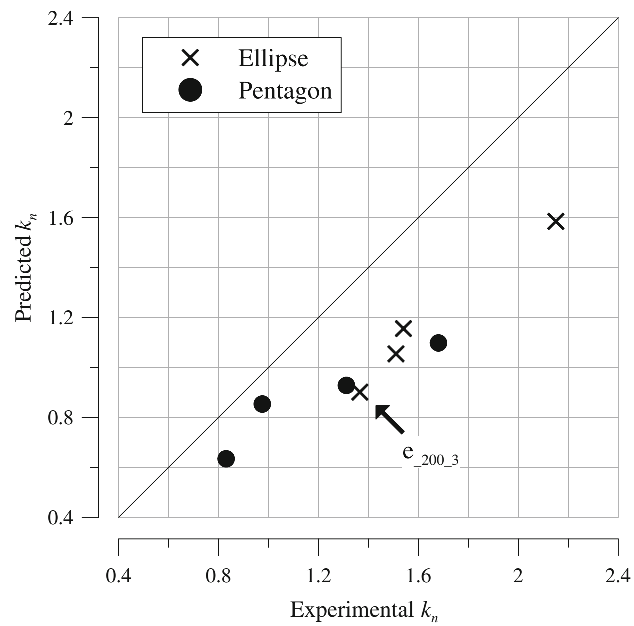


Fig. 10 Values of normal restitution coefficient k_n versus rotational speed for a single impact (no interaction with the step) with low impacting angle

5 Conclusions

Restitution coefficients are critical parameters for most of the models used to predict rock fall. Indeed, they allow quantifying the rock/slope interaction at impact and,



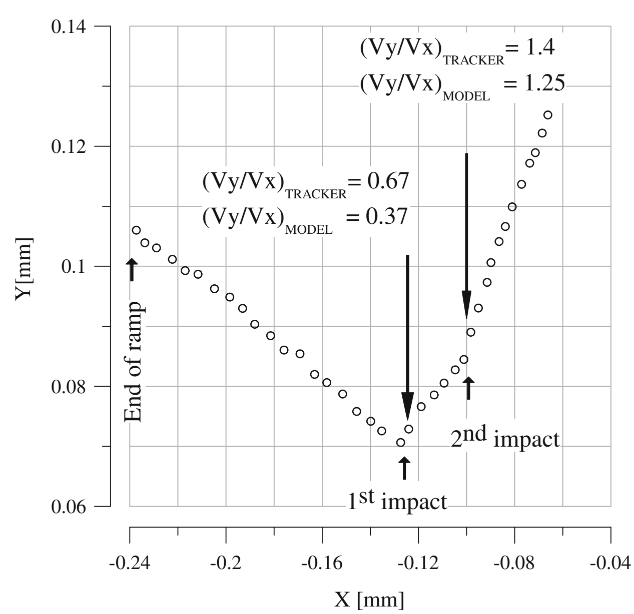


Fig. 11 *Top* comparison between experimental and predicted normal restitution coefficient. *Bottom* trajectory of the centre of mass of the ellipse for test e_{-200-3} with indication of ratio V_y/V_x. x and y are the directions parallel and perpendicular to the landing block, respectively (see Fig. 7)

consequently, the energy dissipation. For a given slope, these coefficients can either be determined by in situ rock fall tests or using presumptive values depending on the material constituting the slope. The presumptive values, considered as usual values, are lower than one for both the normal and tangential restitution coefficients (k_n and k_t). However, following in situ tests, some researchers have observed values for k_n greater than one without clearly providing a comprehensive explanation for it (Azzoni et al. 1992; Paronuzzi 2009; Mathon et al. 2010; Spadari et al. 2011). This study proposes a laboratory study in order to investigate the mechanism leading to high values of k_n and hence, to explain some of the results from the literature. An apparatus was specifically designed to spin blocks of different shapes and to release them on a landing surface. Motion was recorded using high speed cameras. The parameters investigated were the block shape, the rotational energy and the impacting angle. For the first time, high values of k_n have consistently been achieved in the laboratory when combining non circular blocks, some rotational energy and low impacting angle. Values of k_n values up to almost 2 were recorded. The macro roughness tends to turn the single bouncing into combined rolling/bouncing which is also associated with high k_n values. However, this is not a necessary condition, as shown by the results. The study focused on some parameters and motions but other mechanisms not explored herein could also yield high k_n values. In particular, Bourrier et al. (2009) highlighted the influence of incident velocity, which was not accounted for in the present study.

The presumptive values available in the literature for restitution coefficients are material dependent. However, high values of k_n have been observed by Spadari et al. (2011) for vegetated slopes with debris and the results have been explained using a concrete surface. It is believed that the mechanism leading to high k_n prevails regardless of the material constituting the surface provided that the conditions detailed above are met. In that case, high values of k_n should be considered on the basis of possible impact angle and block shape. In any other case, the classification of restitution coefficient by material type and its corresponding value should still be applicable to rockfall studies.

Acknowledgments The authors would like to thank Michael Lawrence and Boe Donaldson for their help with the testing.

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