Sydney Soil Model. I: Theoretical Formulation

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Abstract: In this paper a theoretical study of the behavior of structured soils, including both clays and sands, is presented. A new model, which is referred to as the "Sydney soil model," is formulated within the framework of critical state soil mechanics. In the proposed model, the mechanical behavior of soil is divided into two parts, that at a reference state and that attributed to the influence of soil structure. The reference state behavior is formulated according to the soil properties at the critical state of deformation, based on the concept of plastic volumetric hardening. The effects of structure are captured in the model by incorporation of the additional voids ratio that arises owing to the presence of soil structure. The formulation is generalized to include both isotropic compression and general shearing. In part I of this paper, a new theoretical framework for modeling structured soil behavior and the formulation of the proposed Sydney soil model are introduced. In part II of this paper, the Sydney soil model is employed to simulate the behavior of clays and sands, including calcareous clays and sands subjected to both drained and undrained shearing, and the performance of the model is evaluated. **DOI: 10.1061/(ASCE)GM.1943-5622.0000078.**

CE Database subject headings: Clays; Sand, soil type; Calcareous soils; Fabrics; Constitutive relations; Plasticity.

Author keywords: Clays; Sands; Calcareous soils; Fabric/structure of soils; Constitutive relations; Plasticity.

Introduction

In geotechnical engineering soils are usually divided into two broad categories: clays and sands. Traditionally, their mechanical properties have been treated as being fundamentally different, and consequently they have been investigated separately, through both experimental observation and theoretical modeling.

Many constitutive models for clays are based on the well-known Cam clay model, which was developed to describe the behavior of clay in the laboratory reconstituted state (Schofield and Wroth 1968). Modifications to this model have been required for natural clays because they behave differently from the reconstituted material. This has been ascribed to differences in microstructure, and natural soils are commonly referred to as being "structured" (Burland 1990). Recent developments in formulating constitutive models for clays that incorporate the influence of soil structure include those proposed by Whittle (1993), Desai et al. (1986), Wheeler (1997), Kavvadas and Amorosi (2000), Rouainia and Muir Wood (2000), Desai (2001), Liu et al. (2003), Masin (2005), Pedroso et al. (2009), Koliji et al. (2010), and Namikawa and Mihira (2010). A similar model framework has been used to formulate constitutive models for sands (e.g., Nova and Wood 1979; Jefferies 1993; Manzari and Dafalias 1997; Li 2002; Khalili et al. 2005). However, it is frequently the situation that a model developed for clay is assumed to be inappropriate for sand and vice versa.

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Note. This manuscript was submitted on January 10, 2010; approved on July 18, 2010; published online on July 31, 2010. Discussion period open until November 1, 2011; separate discussions must be submitted for individual papers. This paper is part of the ***International Journal of Geomechanics***, Vol. 11, No. 3, June 1, 2011. ©ASCE, ISSN 1532-3641/2011/3-211–224/$25.00.

In this paper, a new constitutive model applicable to both clays and sands is formulated within the framework of critical state soil mechanics. The fundamental hypothesis is that both clays and sands are structured soils with similar forms of intrinsic behavior, but with a variety of different structures. The proposed model, which is referred to as the "Sydney soil model," is suitable for describing the behavior of "structured" soil in general states of stress and strain and under monotonic and cyclic loading for simple stress reversal. In part I of the paper, the theoretical formulation of the new model is presented. In part II the proposed model is validated by comparisons of the model's performance with experimental data. A general discussion on model performance and identification of soil parameters is also included.

Formulation

In the proposed model, structured soil is idealized as an isotropic hardening and destructuring material with the possibility of virgin yielding and subyielding behavior, and with limited memory of the stress history it has experienced. Subyielding is defined as plastic yielding that occurs when the stress state resides within the virgin or "structured" yield surface.

The proposed model is developed from a description of the plastic volumetric deformation of the soil during virgin yielding. The corresponding plastic deviatoric deformation is determined from an assumed flow rule. Plastic deformation of soil during subyielding is related to that during virgin yielding by means of a mapping quantity, obtained using a simplified approximate form of the Mroz-Iwan rule for yield surface translation (Mroz 1967; Iwan 1967).

Soil Structure

The term "soil structure" is generally used to mean the arrangement and bonding of the soil constituents (Mitchell 1976). Thus, strictly speaking, soil in any state possesses structure (Burland 1990). However, soil in its reconstituted state is commonly referred to as unstructured, and differences in mechanical behavior between a soil in its natural and reconstituted states are considered to be the result of structure (e.g., Leroueil and Vaughan 1990; Gens and Nova 1993; Liu and Carter 1999, 2002; Sitharam et al. 2009; Horpibulsuk et al. 2010).

In this study, a new definition of soil structure and the means of measuring its influence are suggested. The idea behind this approach is similar to that of the disturbed state concept proposed by Desai (1974, 2001, 2005). Soil behavior at a hypothesized reference state is used as a base from which to measure the influence of soil structure. It follows that soil in a state other than the reference state is structured, and any difference in mechanical behavior from the soil at its reference state is a result of this structure. The new and the conventional definitions of "structure" will, of course, be the same if the reference state and the reconstituted state are identical. For many sandy soils there is no standard method of reconstitution, and the term "reconstituted behavior" is not very meaningful (e.g., Gajo and Muir Wood 1999). The proposed definition of structure avoids this difficulty.

In order to concentrate on the most important physical concepts of the framework, and to avoid the complexity of mathematical detail, the influences of soil anisotropy and principal stress rotation, which can be affected by soil structure, are not considered. Furthermore, coaxiality between the principal axes of plastic strain increment and those of stress is assumed.

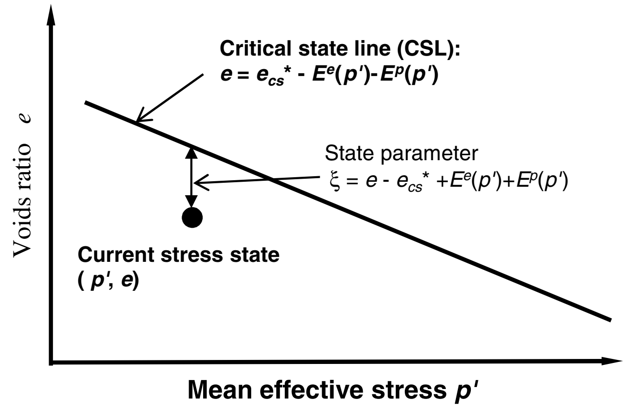

Fig. 1. Critical state line and state parameter

Definitions of Stress and Strain Parameters

The stress and strain quantities that are used to describe the soil behavior are as follows. The mean stress is given by

$$p' = \frac{1}{3}(\sigma'_{11} + \sigma'_{22} + \sigma'_{33}) \tag{1}$$

and the deviatoric stress by

$$q = \frac{1}{\sqrt{2}}\sqrt{[(\sigma'_{11} - \sigma'_{22})^2 + (\sigma'_{22} - \sigma'_{33})^2 + (\sigma'_{33} - \sigma'_{11})^2 + 6(\sigma'^2_{12} + \sigma'^2_{23} + \sigma'^2_{31})]} \tag{2}$$

in which σ'_{ij} = Cartesian components of effective stress.

The stress ratio η is given by

$$\eta = \frac{q}{p'} \tag{3}$$

The corresponding incremental volumetric and deviatoric strains are defined as

$$d\varepsilon_v = d\varepsilon_{11} + d\varepsilon_{22} + d\varepsilon_{33} \tag{4}$$

and

$$d\varepsilon_d = \frac{\sqrt{2}}{3}\sqrt{[(d\varepsilon_{11} - d\varepsilon_{22})^2 + (d\varepsilon_{22} - d\varepsilon_{33})^2 + (d\varepsilon_{33} - d\varepsilon_{11})^2 + 6(d\varepsilon^2_{12} + d\varepsilon^2_{23} + d\varepsilon^2_{31})]} \tag{5}$$

Critical State

The concept of a "critical state" of deformation was first introduced by Cassagrande (1936, 1949) and was adopted in the establishment of Critical State Soil Mechanics (Schofield and Wroth 1968). It is a state at which soil behaves essentially as a frictional fluid, deforming without further change in the stress state and voids ratio. Experimental data overwhelmingly indicate that the mechanical properties of soil at the critical state are independent of the original structure that may have arisen from sample preparation or depositional history and the subsequent modification of the structure resulting from loading (e.g., Been and Jefferies 1985; Bolton 1986; Burland 1990; Muir-Wood et al. 1994; Chu 1995). The critical state strength and the critical state line in $e - p'$ space, where e is the voids ratio, may thus be considered to be intrinsic soil properties, and the mechanical properties of soil at a critical state of deformation can be considered to be independent of soil structure.

It is further assumed that the relationship between the voids ratio and the effective mean stress at critical states is unique for a given soil. This relationship defines the critical state line (CSL) (Fig. 1), which can be described by the following general expression:

$$e = e^*_{cs} - E^e(p') - E^p(p') \tag{6}$$

where e^*_{cs} = soil parameter defining the position of the critical state line in $e - p'$ space [following Burland (1990) intrinsic properties are denoted by the symbol *]; and $E^e(p')$ and $E^p(p')$ = components defining the change in the voids ratio associated with elastic deformation and plastic deformation, respectively. $E^e(p')$ and $E^p(p')$ are assumed to be functions of the current mean effective stress. From the elastic properties of the soil the expression for $E^e(p')$ can be determined. Hence, the additional and irrecoverable volumetric deformation $E^p(p')$, and e^*_{cs}, can be determined from the position of the critical state line in $e - p'$ space.

To allow for general states of stress, and possible noncircularity of the critical state failure loci in the π plane, the stress parameter, f_2, proposed by Liu (1991), is used to define the critical state strength in terms of the three principal stresses where

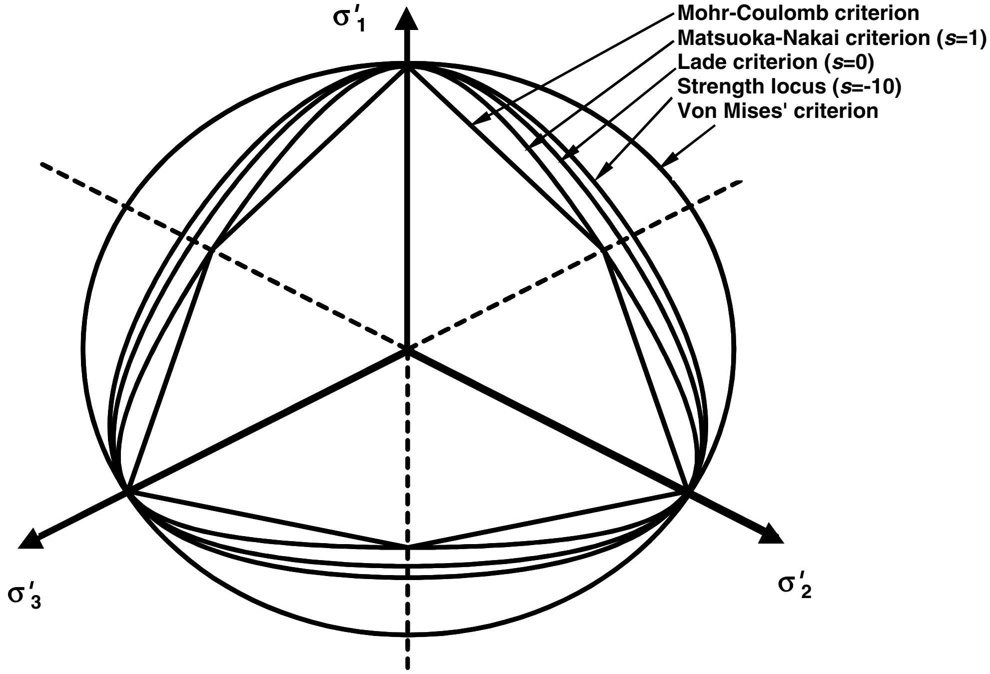

Fig. 2. Critical state strength surfaces in the π plane ($\phi_{cs,cm} = 32°$, $p' = 100$ kPa)

$$f_2 = \frac{(\sigma_1' + \sigma_2' + \sigma_3')[(\sigma_1' + \sigma_2' + \sigma_3')^2 - s^*(\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2)]}{\sigma_1'\sigma_2'\sigma_3'} - 27 + 9s^* \tag{7}$$

where s^* = material constant; and σ_1', σ_2', and σ_3' = principal effective stress components. The parameter s^* can be determined from two stress states (e.g., triaxial compression and extension) at the critical state of deformation, as given in Appendix I.

The variation of the shape of the critical state surface ($f_2 =$ constant) in the π plane with parameter s^* is illustrated in Fig. 2. When $s^* = 1$ the proposed criterion coincides with the Matsuoka-Nakai (1982) criterion. When $s^* = 0$ the new criterion coincides with the Lade (1977) criterion, and it approaches the Von Mises criterion as the value of s^* further decreases (Liu and Carter 2003a). The proposed failure criterion for the critical state is a homogeneous equation. The shape of the critical state surface in principal stress space is an irregular cone, with the vertex being at the origin of stress space. To link the value of f_2 directly to the familiar concept of stress ratio, a generalized stress ratio $\eta^\wedge$ and a generalized shear stress $q^\wedge$ are defined, as follows (the definition of the generalized stress ratio can be seen in Appendix II):

$$\eta^\wedge = \frac{3[f_2 + \sqrt{f_2(f_2 + 27 - 15s^*)}]}{f_2 + \sqrt{f_2(f_2 + 27 - 15s^*)} + 4(27 - 15s^*)} \tag{8}$$

and

$$q^\wedge = p'\eta^\wedge \tag{9}$$

It can be shown that $\eta^\wedge$ is a monotonic function of f_2 and that $\eta^\wedge = 0$ for isotropic stress states. It is assumed that stress states with the same value of $\eta^\wedge$ correspond to the same degree of shear strength mobilization.

The value of the strength parameter M^*, which is equal to the value of $\eta^\wedge$ at the critical state, can be determined according to Eq. (8) from the friction angles in triaxial compression and extension. There are other methods for working out the generalized shear stress and stress ratio (e.g., Sheng et al. 2000; Khalili and Liu 2008).

Elastic Deformation

It is assumed that the deformation of soil can be divided into elastic and plastic parts, $d\varepsilon^e$ and $d\varepsilon^p$, respectively, so that the total strain increment $d\varepsilon$ can be expressed as

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \tag{10}$$

In the Sydney soil model, the familiar elastic constitutive equations, described by Hooke's law with either constant Poisson's ratio ν^* or a constant shear modulus G^*, are adopted. Although they may not be material constants, G^* and ν^* are assumed to be independent of soil structure. The relationship between ν^*, G^*, and the volumetric deformation modulus is given by

$$\nu^* = \frac{3(1+e) - 2G^*[dE^e(p')/dp']}{2G^*[dE^e(p')/dp'] + 6(1+e)} \tag{11}$$

The elastic volumetric and distortional strain increments can be calculated from

$$d\varepsilon_v^e = \left[\frac{dE^e(p')}{dp'}\right]\frac{dp'}{(1+e)} \qquad d\varepsilon_d^e = \frac{\sqrt{2}(1+\nu^*)}{9(1-2\nu^*)}\left[\frac{dE^e(p')}{dp'}\right]\frac{\sqrt{(d\sigma_{11}' - d\sigma_{22}')^2 + (d\sigma_{22}' - d\sigma_{33}')^2 + (d\sigma_{33}' - d\sigma_{11}')^2 + 6(d\sigma_{12}'^2 + d\sigma_{23}'^2 + d\sigma_{31}'^2)}}{(1+e)} \tag{12}$$

The elastic volumetric compression equation $E^e(p')$ and the Poisson's ratio ν^* (or shear modulus G^*) are required to define the elastic behavior, and these may be determined from simple soil tests. The form of the elastic behavior, $E^e(p')$, may be assumed to be linear either in $e-p'$ space or in $e-\ln p'$ space. More sophisticated descriptions of elastic behavior (e.g., Tatsuoka et al. 1997; Atkinson 2000) may also be implemented easily in the proposed framework for modeling soil behavior.

Yield Surface

In elastoplastic soil models such as modified Cam clay (Roscoe and Burland 1968; Muir-Wood 1990), only elastic behavior is predicted whenever the stress state falls within the yield surface and elastoplastic behavior (virgin yielding) occurs whenever the stress state lies on it. The yield surface is dependent on the loading applied to a soil and can be defined directly by its stress history.

A similar approach is adopted in the current study. However, it is assumed that the initial yield surface is influenced by the soil structure, in addition to its stress history (see Fig. 3), and the possibility of plastic deformation inside the yield surface is also considered. A hypothetical "reference yield surface" can be defined, which is the yield surface associated with stress history when the structure of a soil is removed. This is assumed to possess the same shape as the yield locus, but its aspect ratio M may differ from M^*. Loading that involves stress excursions inside the yield surface, is referred to herein as inner loading. On the yield surface, virgin loading results in a variation of the yield surface with the current stress state remaining on it.

The yield surface of a soil in $p'-q^\wedge$ space is assumed to be elliptical, and the value of p' where the ellipse again intersects the mean stress axis, p'_s (Fig. 3), represents the size of the yield surface. The aspect ratio of the yield surface, denoted by M, is assumed to be dependent on soil structure, as has been observed experimentally (e.g., Coop and Lee 1993; Carter et al. 2000) and adopted in some constitutive models (e.g., Yu 1998; Islam et al. 1998). The surface is thus described by the yield function f, where

$$f = q^{\wedge 2} - M^2 p'(p'_s - p') = 0 \tag{13}$$

A comparison of the yield surfaces for the Sydney soil model (SS) for the case where $\varphi_c = \varphi_e = 30°$ and $p'_s = 100$ kPa, and modified Cam clay (MCC) with $\phi_c = 30°$ is shown in Fig. 4. At the critical state in triaxial compression, $q/p' = 1.2$ for MCC, and $q^\wedge/p' = M^* = 0.712$ for SS.

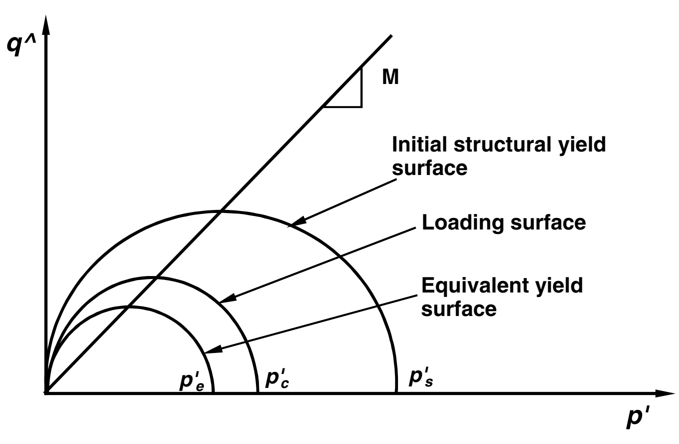

Fig. 3. Yield surface and loading surface for soil

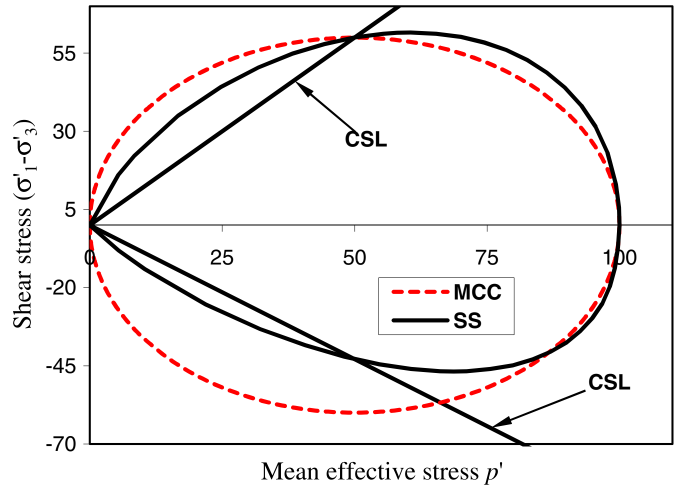

Fig. 4. Comparison of the modified Cam clay and Sydney soil model yield surfaces for conventional triaxial stress conditions

For soil at its unstructured reference state the yield surface is described by Eq. (13), with the aspect ratio, $M = M^*$, the critical state strength, and $p'_s = p'_e$. The yield surface size, defined by the parameter p'_e, can be determined from the current voids ratio and the stress state, as discussed further in the following.

The initial yield surface provides important data on soil structure and is usually determined experimentally. Several tests may have to be performed on intact soil samples to explore the shape and size of the initial structural yield surface in the $p'-q^\wedge$ space to provide a best fit elliptical surface.

Virgin Isotropic Compression Line

Virgin Isotropic Compression at the Reference State

As stated previously, soil behavior at the reference state is used as a base from which the influence of soil structure is measured. The reference state adopted in the Sydney soil model is based on the critical state. It is assumed that a soil of given mineralogy is at a reference state if the size of the yield surface is uniquely dependent on the plastic volumetric deformation, and a critical state of deformation can be reached by continuous deformation through reference states. It follows that the reference yield surface can be determined from the current voids ratio and the current stress state, because the volumetric deformation contributed by elasticity is dependent only on changes in the stress state.

The reference isotropic virgin compression line for soil may thus be derived from the critical state line as follows. Suppose that there are two points A and B on the same yield surface, where point B corresponds to the critical state of deformation. Because the difference in voids ratio between A and B is attributed to elastic deformation only, the following equation can be written:

$$e_A - e_B = [E^e(p')_A - E^e(p')_B] \tag{14}$$

Since point B is on the CSL, its voids ratio can be determined by Eq. (6), and as a result

$$e_A = e^*_{cs} - E^e(p'_A) - E^p(p'_B) \tag{15}$$

$E^p(p')$ is associated with plastic volumetric deformation and is dependent on the size of the reference yield surface. Consequently, the size of the reference yield surface, p'_e, should be substituted for the variable p'_B. Because the yield locus is assumed to be an ellipse, it follows that at the reference state, $p'_{cs} = 0.5p'_e$, and the value of voids ratio for any stress state on the reference yield surface is thus given by the following equation:

$$e^* = e^*_{cs} - E^e(p') - E^p\left(\frac{p'_e}{2}\right) \tag{16}$$

Eq. (16), with $p' = p'_e$, describes the hypothetical virgin isotropic compression line (ICL) for a soil in its unstructured reference state. This line, denoted by ICL* in Fig. 5, may be thought of as an intrinsic soil property.

Virgin Isotropic Compression of a Structured Soil

The virgin isotropic compression behavior of a structured soil may be described in terms of e^*, the voids ratio of the soil on the reference isotropic compression line (ICL*) at the current isotropic yield stress, p'_s, and Δe, the additional "structure-related" voids ratio at the same mean effective stress (Fig. 5), so that

$$e = e^* + \Delta e \tag{17}$$

Values for Δe must be estimated from experiments and are required as input for modeling soil behavior.

For an isotropic stress state $p' = p'_s$, and the additional voids ratio can be rewritten in terms of the size of the yield surface p'_s instead of the current mean effective stress p', i.e.,

$$\Delta e = \Delta E(p'_s) = e - e^* \tag{18}$$

Combining Eqs. (16)–(18), and assuming that the elastic deformation is independent of soil structure, the following expression for isotropic virgin compression is obtained:

$$e = e^*_{cs} - E^e(p') - E^p\left(\frac{p'_s}{2}\right) + \Delta E(p'_s) \tag{19}$$

Volumetric Deformation during Virgin Yielding

Eq. (19), which describes the change of voids ratio of structured soil undergoing virgin isotropic loading, contains three basic terms. The first describes the elastic deformation and in its general form, $E^e(p')$, is valid for any loading. The second, $E^p(0.5p'_s)$, describes the change associated with plastic deformation for soil at the corresponding reference state, and, as discussed earlier, this is also valid for loading along general stress paths. The third term, i.e., $\Delta E(p'_s)$, describes the change associated with the additional voids ratio sustained by soil structure, which is also a plastic deformation. If it is assumed that the hardening and destructuring of structured soil, not at a reference state, are also dependent on the change in size of the yield surface, irrespective of the stress path, then the third term is also valid for loading along general stress paths. The following expression for the voids ratio is obtained for loading along general stress paths:

$$e = e^*_{cs} - E^e(p') - E^p\left(\frac{p'_s}{2}\right) + \Delta E(p'_s) \tag{20}$$

Taking the differential form of Eq. (20) and dividing both sides by $(1+e)$, the following equation for the total volumetric strain increment is obtained:

$$d\varepsilon_v = \frac{dE^e(p')}{dp'}\frac{dp'}{(1+e)} + \frac{dE^p(p'_s/2)}{dp'_s}\frac{dp'_s}{(1+e)} - \frac{d[\Delta E(p'_s)]}{dp'_s}\frac{dp'_s}{(1+e)} \tag{21}$$

Like Eq. (20), Eq. (21) is valid for virgin yielding only.

The first term on the right-hand side of Eq. (21) is the elastic deformation, so that

$$d\varepsilon^e_v = \frac{dE^e(p')}{dp'}\frac{dp'}{(1+e)} \tag{22}$$

Hence, the plastic volumetric strain increment is composed of two parts and can be written as

$$d\varepsilon^p_v = \frac{dE^p(p'_s/2)}{dp'_s}\frac{dp'_s}{(1+e)} - \frac{d[\Delta E(p'_s)]}{dp'_s}\frac{dp'_s}{(1+e)} \tag{23}$$

The first part describes soil behavior at the reference state and is dependent only on the intrinsic soil properties. However, the second part is dependent on soil structure and describes the reduction of the additional voids ratio sustained by soil structure with the expansion of the yield surface; i.e., it describes destructuring.

It has been observed that during destructuring the plastic volumetric deformation is dependent on the magnitude of the current shear stress as well as the change in size of the yield

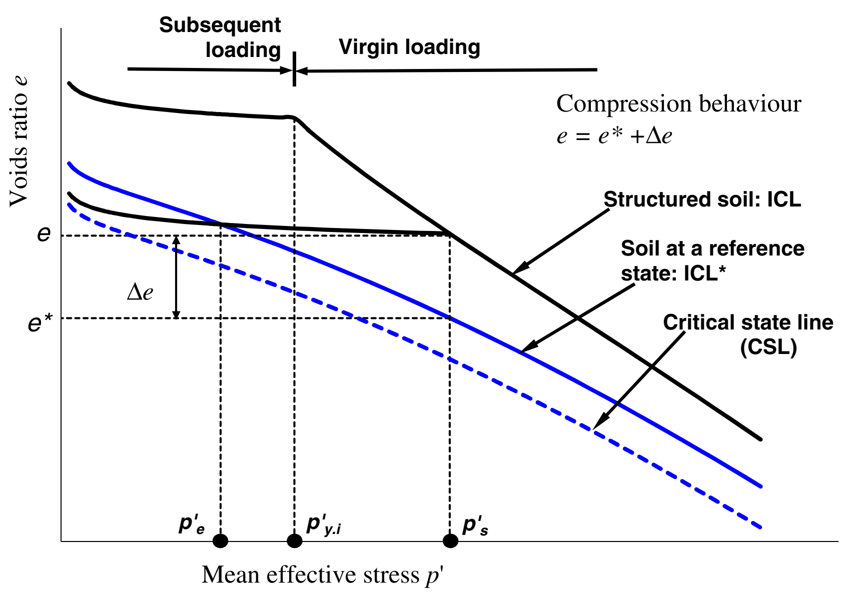

Fig. 5. Compression behavior of soils described in terms of ICL* and the influence of soil structure

surface (Liu and Carter 2003b). Because of the assumption of plastic-volumetric-deformation-dependent hardening, the effect of shearing on destructuring is not included in the derived equation. A generalization of Eq. (23) is proposed by taking into consideration the destructuring due to shearing, via introducing the function $D_s(\Delta e, \eta^\wedge)d\eta^\wedge$. Then Eq. (23) can be generalized as

$$d\varepsilon_v^p = \frac{dE^p(0.5p_s')}{dp_s'}\frac{dp_s'}{(1+e)} - \frac{d[\Delta E(p_s')]}{dp_s'}\frac{dp_s'}{(1+e)} + D_s(\Delta e, \eta^\wedge)d\eta^\wedge \tag{24}$$

A general form for the shear destructuring function, D_s, is presented in Appendix II, from which a modified volumetric deformation equation is obtained as follows:

$$d\varepsilon_v^p = \frac{dE^p(0.5p_s')}{dp_s'}\frac{dp_s'}{(1+e)} - \left(\frac{\Delta e - c}{\Delta E(p_s') - c}\right)\frac{d[\Delta E(p_s')]}{dp_s'}\frac{\langle dp_s'\rangle}{(1+e)} + \frac{\gamma\Delta e}{(1+e)}\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta}{M^* - \eta^\wedge}\right\rangle \tag{25}$$

where

$$\langle a\rangle = \begin{cases} a & \text{if } a > 0 \\ 0 & \text{if } a > 0 \end{cases} \tag{26}$$

and

$$c = \begin{cases} \lim\limits_{p_s'\to\infty} \Delta E(p_s') & \text{if the limit is finite} \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

The second and the third terms in Eq. (25) are associated with destructuring. The second term occurs only when the yield surface expands. The parameter c, which can be derived from $\Delta E(p_s')$ according to Eq. (27), is introduced to allow for the possibility that destructuring may not lead to the virgin isotropic compression line asymptoting to the ICL*. The third term occurs during virgin yielding when the current stress ratio increases or when softening occurs.

It should be noted that in equations such as Eqs. (24) and (25) the symbol Δe represents the value of the current additional voids ratio for general loading, and $\Delta E(p_s')$ represents the value of additional voids ratio for virgin isotropic compression with $p' = p_s'$, which can be calculated from Eq. (18). Because of the enhancement provided by Eqs. (24) and (25), the hardening rule for a structured soil is dependent on both the plastic volumetric deformation and the shear stress ratio, $\eta^\wedge$, and hence, in general, $\Delta e \neq \Delta E(p_s')$.

As may be seen from Eqs. (24) and (25), n and γ are the two parameters defining destructuring due to shearing, $D_s(\Delta e, \eta^\wedge)$. These parameters, n and γ, control different features of the reduction of the additional voids ratio resulting from the increment of shear stress ratio: n controls the shape and γ controls the amplitude of the destructuring function. Their values can be determined directly from the $\Delta e - p'$ or $e - p'$ relationship during a drained virgin shearing test. In situations where sufficient experimental data are not available to define clearly the value of n, experience with the model shows that assuming $n = 1$ will provide a reasonable fit for many soils.

Flow Rule

A new flow rule is proposed based on study of the flow rule of the original Cam clay model (Roscoe et al. 1958) and examination of a large body of experimental data on the shearing behavior of frictional materials (e.g., Iwasaki et al. 1978; Huang and Airey 1998; Hight et al. 1992; Carter and Airey 1994; Cotecchia and Chandler 1997). From this study it is proposed that the plastic volumetric and deviatoric strain increments are related as follows:

$$d\varepsilon_d^p = \frac{\frac{3}{2}\sqrt[3]{\frac{\eta^\wedge}{M^*}}}{\left|1 - \frac{\eta^\wedge}{M^*}\right| + \frac{\omega\eta^\wedge}{M^*}\left|1 - \sqrt{\frac{p_e'}{p_s'}}\right|}\overline{|d\varepsilon_v^p|}$$

$$\text{where } \overline{|d\varepsilon_v^p|} = \left(\frac{1}{1+e}\right)\left\{\left|\frac{dE^p(0.5p_s')}{dp_s'}dp_s' - \left[\frac{\Delta e - c}{\Delta E(p_s') - c}\right]\frac{d[\Delta E(p_s')]}{dp_s'}\langle dp_s'\rangle\right| + \gamma|\Delta e|\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle\right\} \tag{28}$$

where ω = model parameter and $|a|$ = the absolute value of the quantity a. The notation $\overline{|d\varepsilon_v^p|}$ used in Eq. (28) does not necessarily represent the absolute value of $d\varepsilon_v^p$, but rather its meaning is defined as follows:

$$\overline{|d\varepsilon_v^p|} = |d\varepsilon_v^p(\text{due to isotropic stress changes})| + |d\varepsilon_v^p(\text{due to shear stress changes})|$$

It is found that the distortional strain simulated such as for softening may be smaller than that observed when the volumetric strain produced by compression and that by shearing have different signs. Based on trial and error, the scalar quantity $\overline{|d\varepsilon_v^p|}$ is introduced to improve model simulation for this situation. Its magnitude is dependent on plastic volumetric deformation.

The size of the reference yield surface, p_e', may be determined from Eq. (16). However, it should be noted that there may be situations where there is no positive value of p_e' satisfying Eq. (16). In this case it will reasonable to assume that $p_s' \gg p_e'$, and in particular that

$$p_e' = 0.001p_s' \quad \text{if } e_{cs}^* - E^e(p') - E^p(0.001p_s'/2) - e < 0 \tag{29}$$

A comparison of three different flow rules is illustrated in Fig. 6. They are the rule defined by Eq. (28), the one adopted in the original Cam clay model, and the one adopted in the modified Cam clay model. It is seen that the proposed flow rule predicts a greater deviatoric deformation than the MCC model for loading at low stress ratio. This is consistent with experimental observation (Hight et al. 1992; Cotecchia and Chandler 1997).

The plastic deviatoric strain given by Eq. (5) has been defined as a positive scalar quantity. In order to define the increment of the plastic strain, it has been assumed that the deviatoric plastic strain increment, i.e., $(d\varepsilon_{ij}^{p-d}\varepsilon_v^p\delta_{ij}/3)$, is linearly proportional to the

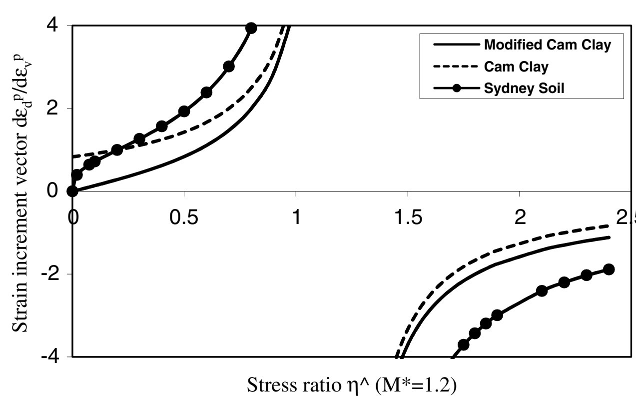

Fig. 6. Comparison of three flow rules

current deviatoric stress, i.e., $(\sigma'_{ij} - p'\delta_{ij}/3)$. δ_{ij} is the Kronecker delta defined by

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{30}$$

In this case the plastic strain increment can be expressed as

$$d\varepsilon^p_{ij} = \frac{1}{3} d\varepsilon^p_v \delta_{ij} + \Lambda\left(\sigma'_{ij} - \frac{1}{3} p' \delta_{ij}\right) \tag{31}$$

where Λ = scalar quantity that specifies the magnitude of the plastic deformation. The value of Λ can be determined from the value of $d\varepsilon^p_d$ calculated from Eq. (28) using the following equation:

$$\Lambda = \frac{\pm 3\sqrt{2} d\varepsilon^p_d}{2\sqrt{[(\sigma'_{11} - \sigma'_{22})^2 + (\sigma'_{22} - \sigma'_{33})^2 + (\sigma'_{33} - \sigma'_{11})^2 + 6(\sigma'^2_{12} + \sigma'^2_{23} + \sigma'^2_{31})]}} \tag{32}$$

Λ is always positive for virgin yielding.

It may be noticed that the earlier assumption also implies the coaxiality between the axes of the principal plastic strain increment with those of the principal stresses.

The parameter ω was introduced in the flow rule. Its value may be determined based on the observed plastic volumetric and deviatoric strain relationship, provided the elastic deformation is known. Where insufficient data are available it is recommended that ω be assigned a value of 1.

Variations of M and p'_s during Virgin Yielding

The aspect ratio of the yield surface is dependent on soil structure (section "Yield Surface") and is specified as follows:

$$M = \frac{M^*}{1 + \mu \ln(p'_s/p'_e)} \tag{33}$$

During destructuring, Δe diminishes and the aspect ratio of the surface approaches M^*, its value at the critical (reference) state. If the influence of soil structure on the shape of the surface is insignificant, it may be assumed that $\mu = 0$ and in this case $M = M^*$.

For situations where the aspect ratio of the yield surface M is not a constant, the change in size of the yield surface during virgin yielding, dM, is derived as follows. Noting that M, p'_s, and p'_e all change with a variation in stress state, differentiating Eq. (33), the change of the aspect ratio can be written as

$$dM = -\frac{\mu M}{1 + \mu \ln(p'_s/p'_e)} \left(\frac{dp'_s}{p'_s} - \frac{dp'_e}{p'_e}\right) \tag{34}$$

Based on Eq. (16), dp'_e can be written as

$$dp'_e = -\frac{\left[\frac{dE^e(p')}{p'} dp' + de\right]}{\frac{dE^p(0.5p'_e)}{dp'_e}} \tag{35}$$

Considering the elastic volumetric strain increment given by Eq. (22) and the plastic volumetric strain increment given by Eq. (25), the value of de can be obtained. Substituting this value of de into Eq. (35), the following expression for dp'_e is obtained:

$$dp'_e = \left\{\frac{dE^p(0.5p'_e)}{dp'_e} dp'_e - \left(\frac{\Delta e - c}{\Delta E(p'_s) - c}\right) \frac{d[\Delta E(p'_s)]}{dp'_s} \langle dp'_s \rangle + \gamma \Delta e \left(\frac{\eta^\wedge}{M^*}\right)^n \left\langle \frac{d\eta^\wedge}{M^* - \eta^\wedge} \right\rangle \right\} \times \left[\frac{dE^p(0.5p'_e)}{P'_e}\right]^{-1} \tag{36}$$

Substituting Eq. (36) into (34), the following expression for dM is obtained:

$$dM = y_1 dp'_s + y_2 \langle dp'_s \rangle + y_3 \left\langle \frac{d\eta^\wedge}{M^* - \eta^\wedge} \right\rangle$$

$$\text{where } y_1 = \frac{-\mu M}{1 + \mu \ln(p'_s/p'_e)} \frac{1}{p'_e\left[\frac{dE^p(0.5p'_e)}{dp'_e}\right]} \times \left[\left(\frac{p'_e}{p'_s}\right) \frac{dE^p(0.5p'_e)}{dp'_e} - \frac{dE^p(0.5p'_s)}{dp'_s}\right]$$

$$y_2 = \frac{-\mu M}{1 + \mu \ln(p'_s/p'_e)} \frac{1}{p'_e\left[\frac{dE^p(0.5p'_e)}{dp'_e}\right]} \times \left\{\frac{\Delta e - c}{\Delta E(p'_s) - c} \frac{d[\Delta E(p'_s)]}{dp'_s}\right\} \quad \text{and}$$

$$y_3 = \frac{\mu \gamma M \Delta e (\eta^\wedge/M^*)^n}{1 + \mu \ln(p'_s/p'_e)} \frac{1}{p'_e\left[\frac{dE^p(0.5p'_e)}{dp'_e}\right]} \tag{37}$$

Differentiating Eq. (13), the following expression can be obtained:

$$2q^\wedge dq^\wedge - 2Mp'(p'_s - p')dM + M^2(2p' - p'_s)dp' - M^2 p' dp'_s = 0 \tag{38}$$

Substituting Eq. (37) and (38), the following equation for dp'_s can be obtained:

$$dp'_s = \begin{cases} \dfrac{(2p' - p'_s)M^2 dp' + 2q^\wedge dq^\wedge - 2y_3 p' M (p'_s - p') \langle \frac{d\eta^\wedge}{M^* - \eta^\wedge} \rangle}{[p'M^2 + 2(y_1 + y_2)p'M(p'_s - p')]} & \text{for } dp'_s \geq 0 \\ \dfrac{(2p' - p'_s)M^2 dp' + 2q^\wedge dq^\wedge - 2y_3 p' M (p'_s - p') \langle \frac{d\eta^\wedge}{M^* - \eta^\wedge} \rangle}{[p'M^2 + 2y_1 p'M(p'_s - p')]} & \text{for } dp'_s < 0 \end{cases} \tag{39}$$

The parameter μ defines the variation of the aspect ratio of the yield surface with soil structure. When experimental data are available, μ can be determined from values of the peak shear stress $q^\wedge$ observed in conventional undrained triaxial tests on soils with the same structure but with different values of Δe_i. If the soil is experiencing virgin yielding, the peak strength is strongly influenced by μ. In the absence of data it may be assumed that the aspect ratio is unaffected by structure, i.e., $\mu = 0$.

Stress-Strain Relationships for Virgin Yielding

Virgin loading corresponds to a variation of the current yield surface. Softening behavior also falls into the category of virgin yielding and is described by the same set of equations. Based on the plastic volumetric deformation given by Eq. (25), the plastic deviatoric deformation given by Eq. (28), and considering the elastic deformations given by Eq. (12), the following stress and strain relationships for virgin yielding are obtained:

$$d\varepsilon_v = d\varepsilon_v^e + \frac{dE^p(0.5p_s')}{dp_s'}\frac{dp_s'}{(1+e)} - \left[\frac{\Delta e - c}{\Delta E(p_s') - c}\right]\frac{d[\Delta E(p_s')]}{dp_s'}\frac{\langle dp_s'\rangle}{(1+e)} + \Delta e\frac{\gamma}{(1+e)}\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle \tag{40}$$

$$d\varepsilon_d = d\varepsilon_d^e + \left(\frac{1}{1+e}\right)\frac{\left(\frac{3}{2}\right)\sqrt[3]{\eta^\wedge/M^*}}{|1-\eta^\wedge/M^*| + \omega(\eta^\wedge/M^*)|1-\sqrt{p_e'/p_s'}|} \times \left\{\left|\frac{dE^p(0.5p_s')}{dp_s'}dp_s' - \left[\frac{\Delta e - c}{\Delta E(p_s') - c}\right]\frac{d[\Delta E(p_s')]}{dp_s'}\langle dp_s'\rangle\right| + \gamma|\Delta e|\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle\right\} \tag{41}$$

It should be noted that dp_s' is calculated from Eq. (39) and $d\eta^\wedge$ can be evaluated from Eq. (8).

Loading within the Yield Surface

The early elastoplastic models for soil assumed elastic behavior within the yield locus, implying a dramatic change in stiffness as the yield locus is reached. However, it is now well established that all soils have a very limited truly elastic response, and that some plastic strain occurs within the yield locus. As a consequence, the yield locus can sometimes be difficult to detect precisely. For some soils a rapid change of soil stiffness can be observed, and this is taken as the commencement of virgin yielding (e.g., Burland 1990; Leroueil and Vaughan 1990; Hight et al. 1992; Liu and Carter 2000a). However, for other types of soil, such as many sands, there may be no clear difference in soil stiffness (e.g., Liu et al. 2000; Martins et al. 2001).

To capture the different responses within the yield locus, two types of behavior are considered in the Sydney soil model. The different responses can be related to the prior stress history of the soil. If the soil has experienced prior stresses such that the previous maximum stress state lies on or beyond the current reference yield surface (defined by p_e'), then Type 1 behavior will result. Clayey soils would be expected to be in this category. Alternatively, if the soil has experienced little prior loading so that $p_{\max}' < p_e'$, then Type 2 behavior will result. Most sandy and gravelly soils would be expected to be in this category. This distinction is necessary to capture the additional volumetric strain that occurs if the soil has not been previously stressed.

To model these responses, a loading surface, following Hashiguchi (1980), is defined on which the current stress state lies. The loading surface is assumed to be elliptical (Fig. 3), the same shape as the yield surface, and its size is denoted by p_c'. The loading surface serves as a mapping quantity to indicate the closeness of the current stress state to the yield surface, allowing the strains to be calculated making use of an approximation to the multiyield surface theory (Mroz 1967; Iwan 1967; Krieg 1975; Dafalias and Popov 1976). To simplify the model, and enable the formulation of explicit stress-strain relations, it is also assumed that the yield surface is unaffected by any plastic straining that occurs within it, i.e., $dp_s' = 0$, and $dM = 0$.

Following previous work of the writers (Liu and Carter 2000b, 2003b) and considering that the plastic deformation during subyielding (i.e., within the yield locus) will reduce the quantity Δe for both $\eta^\wedge < M^*$ and $\eta^\wedge > M^*$, the following expression for plastic volumetric deformation is proposed for clayey (Type 1) soils, based on trial and error evaluation:

$$d\varepsilon_v^p = \frac{\alpha}{(1+e)}\left(\frac{p_c'}{p_s'}\right)\left\{\left(\frac{M^{*2} - \eta^{\wedge 2}}{M^{*2} + 0.3\eta^{\wedge 2}}\right)\left[\frac{dE^p(0.5p_c')}{dp_c'}\right]dp_c' + \gamma\Delta e\left(\frac{M^* + \eta^\wedge}{M^{*2} + 0.3\eta^{\wedge 2}}\right)\left(\frac{\eta^\wedge}{M^*}\right)^n\langle d\eta^\wedge\rangle\right\} \tag{42}$$

The effects of stress history on soil behavior during inner loading are approximated by the following simple scalar expression for α:

$$\alpha = \begin{cases}\left(\frac{\sqrt{p_c'} - \sqrt{p_u'}}{\sqrt{p_s'} - \sqrt{p_u'}}\right)^m & \text{if } dp_c' \geq 0\\ \left(1 - \sqrt{\frac{p_c'}{p_s'}}\right)^m & \text{if } dp_c' < 0\end{cases} \tag{43}$$

where m = material constant and p_u' = minimum size of the loading surface previously attained, i.e., $p_u' = \min\{p_c'\}$, since the soil last experienced virgin yielding. When the stress state returns to virgin yielding, the memory of the previous loading inside the structural yield surface is lost, and p_u' will be equal to p_s', until the stress state falls within the yield locus when again $p_u' = \min\{p_c'\}$. Generally α takes a value between 0 and 1, with $\alpha = 0$ corresponding to purely elastic deformation and $\alpha = 1$ corresponding to virgin yielding.

The deviatoric plastic strain increment during loading within the structural yield locus is proposed as follows:

$$d\varepsilon_d^p = \begin{cases}\dfrac{\frac{3}{2}|d\varepsilon_v^p|\sqrt[3]{\frac{\eta^\wedge}{M^*}}}{\left|1-\frac{\eta^\wedge}{M^*}\right| + \frac{\omega\eta^\wedge}{M^*}\left|1-\sqrt{\frac{p_e'}{p_s'}}\right|} & \text{for loading, i.e., } dp_c' \geq 0\\ \dfrac{\frac{3}{2}|d\varepsilon_v^p|\sqrt[3]{\frac{|\eta^\wedge - \eta_u^\wedge|}{M^*}}}{\left|1-\frac{\eta^\wedge}{M^*}\right| + \frac{\omega\eta^\wedge}{M^*}\left|1-\sqrt{\frac{p_e'}{p_s'}}\right|} & \text{for unloading, i.e., } dp_c' < 0\end{cases} \tag{44}$$

where $\eta_u^\wedge$ = stress ratio at which unloading commences.

The increment of the plastic strain can be computed according to Eqs. (31) and (32), in which case Λ is positive for $dp_c' > 0$ and negative for $dp_c' < 0$.

It has been found that the behavior of sands and gravels that have not previously experienced high values of stress are not well described by Eq. (42), and additional volumetric strain occurs within the yield locus. For these (Type 2) soils it has been found necessary, based on data from many sandy soils, to include a term involving the additional voids ratio, $\Delta E(p')$, similar to that in virgin yielding, but dependent on p_c'. Because p_s' is assumed constant inside the yield locus, the definition of Δe for Type 2 soils, within the yield locus, has been modified as follows:

$$\Delta e = e - e^*(p_{c,\max}') \tag{45}$$

Fig. 7 shows the compression behavior of a sandy soil to illustrate the modified definition of Δe. This figure shows a situation where Δe is always negative because the structured ICL lies below the hypothetical ICL*. This is consistent with data from some sands that show virgin yielding behavior even when they have large negative values of the state parameter (Ishihara 1993; Golightly and Hyde 1988).

When the size of the current loading surface is expanding and the state is within the yield locus, referred to here as first loading, the plastic volumetric strain can be calculated from

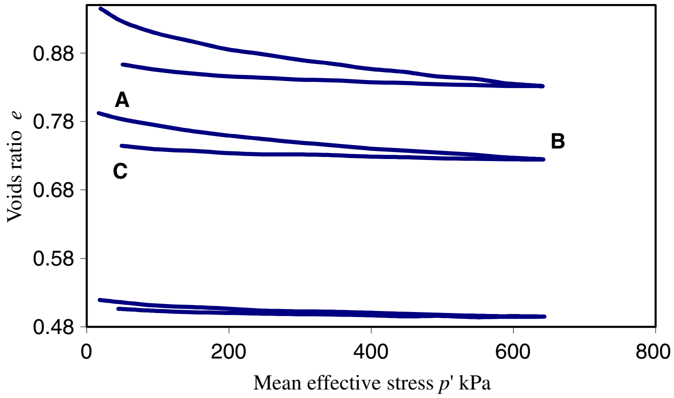

Fig. 7. Isotropic compression test on Fuji sand (test data after Tatsuoka 1973)

$$d\varepsilon_v^p = \frac{1}{(1+e)}\left\{\left(\frac{M^{*2}-\eta^{\wedge 2}}{M^{*2}+0.3\eta^{\wedge 2}}\right)\left[\frac{dE^p(0.5p'_c)}{dp'_c} - \frac{\Delta e - c}{\Delta E(p'_s)-c}\frac{d\Delta E(p'_c)}{dp'_c}\right]dp'_c + \gamma\Delta e\left(\frac{p'_c}{p'_s}\right)\left(\frac{M^*+\eta^\wedge}{M^{*2}+0.3\eta^{\wedge 2}}\right)\left(\frac{\eta^\wedge}{M^*}\right)^n\langle d\eta^\wedge\rangle\right\} \tag{46}$$

Eq. (46) differs from Eq. (42) by the addition of the term in the additional voids ratio, $\Delta E(p')$, and also by setting $\alpha = 1$.

To illustrate how the model works, we will consider the isotropic loading from A to D in Fig. 8 for a clay-type soil and from A to C in Fig. 7 for a sand-type soil. For a clay-type soil the plastic volumetric strain during loading from A to B, and during unloading CD, is given by Eq. (42). Point B is where virgin yielding of the soil takes place. Thus the plastic volumetric strain during loading from B to C is given by Eq. (40).

For a sand-type soil Eq. (42) also applies for loading and unloading, except when first loading. In this case the sand has been artificially prepared with the given voids ratio at A, and this soil will be undergoing first loading from A. Thus Eq. (42) only applies for unloading BC. From A to B the size of the current loading surface (p'_c) is less than the current yield surface (p'_s), the soil is within the yield locus undergoing first loading (i.e., $p'_c = p'_{c\max}$ and $dp'_c > 0$),

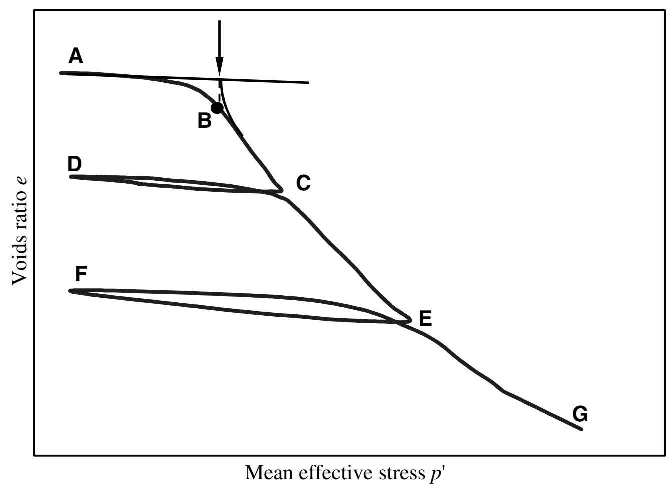

Fig. 8. Behavior of clay-type soil during cyclic isotropic compression (test data after Nash et al. 1992)

and the plastic volumetric strain is given by Eq. (46). For all soil types, once virgin yielding occurs, the plastic volumetric strain is given by Eq. (40). Also, once a sand-type soil reaches virgin yielding, Eq. (45) is no longer relevant and Δe is given by Eq. (18) for all subsequent loading and unloading; i.e., a difference in the values of Δe for clay-type soils and sand-type soils exists only if $p'_s > p'_{c,\max}$.

The aspect ratios for both the yield surface and the loading surface are assumed to be identical and constant during subyielding. From Eq. (33) it can be seen that M is related to the ratio of (p'_s/p'_e) which will vary because $dp'_s = 0$ and $\Delta e \neq 0$ ($dp'_e \neq 0$). To prevent a change in aspect ratio, the constant μ has to change, and this can be evaluated from Eq. (33). For example, if at the start of subyielding the ratio of (p'_s/p'_e) is r_1 and $\mu = \mu_1$, and when virgin yielding commences the ratio of (p'_s/p'_e) is r_2, then μ_2 can be evaluated as follows:

$$\frac{M^*}{1+\mu_1\ln(r_1)} = \frac{M^*}{1+\mu_2\ln(r_2)} \tag{47}$$

which implies

$$\mu_2 = \mu_1\frac{\ln(r_1)}{\ln(r_2)} \tag{48}$$

Values for the parameters α and m, which influence the inner loading behavior can be identified from experimental data. The parameter m can be most simply determined from isotropic loading. For example, consider the isotropic unloading behavior of soil from p'_c to p'_D (see Fig. 8). For this case $\eta^\wedge = 0$ and $\alpha = [(p'_C - p')/p'_C]^m$, and integrating Eq. (46) provides

$$\varepsilon_v = \int_{p'_C}^{p'_D} d\varepsilon_v = \int_{p'_C}^{p'_D}\frac{d[E^e(p')]}{dp'}\frac{dp'}{(1+e)} + \int_{p'_C}^{p'_D}\left(\frac{p'_C-p'}{p'_C}\right)^m\frac{d[E^p(0.5p'_s)]}{dp'_s}\frac{dp'_s}{(1+e)} \tag{49}$$

The value of m can be determined from Eq. (49) since it is the only unknown variable in the equation. For soil that exhibits purely elastic behavior within the yield surface, a large value, such as $m = 10$, may be adopted. Where sufficient experimental data are not available to define the value of m, it may be assumed that $m = 1$, as this has been found to give a reasonable fit for many soils (Airey et al. 2011).

Instability

Mechanical instability in soil has been widely studied (e.g., Muir-Wood 1990; Lade and Pradel 1990; Chu and Leong 2001). It is noted that instability may be predicted by the proposed model, and the particular case of instability during undrained shearing is discussed here.

As illustrated in Fig. 9, for circumstances where $\mu < 0$ and $\Delta e > 0$ (or alternatively $\mu > 0$ and $\Delta e < 0$), the stress state of a given soil may remain on the elliptical yield surface with $M < \eta^\wedge < M^*$ (point A). According to the proposed flow rule, i.e., Eq. (28), the stiffness of the soil during shear deformation is finite at this point, and thus the soil should not fail at this stage. Suppose there is a disturbance with $dq' < 0$ and $d\eta^\wedge > 0$ (as shown by the shaded region in Fig. 9) which results in $dp'_s > 0$. For undrained tests there can be no volume change, i.e.,

$$d\varepsilon_v^e + d\varepsilon_v^p = 0 \tag{50}$$

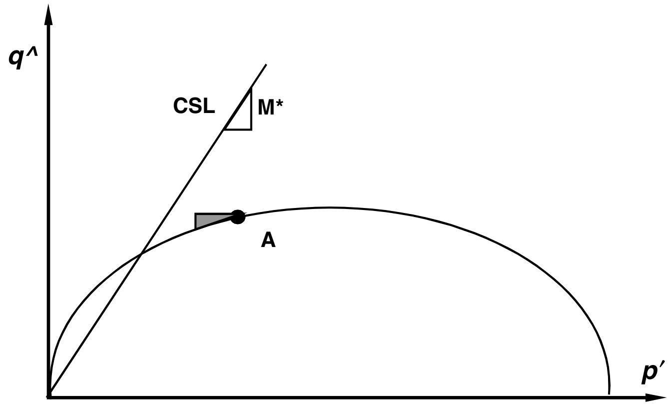

Fig. 9. Instability of soil during undrained shearing

Generally, for most soils

$$\frac{dE^e(p')}{dp'} > 0 \qquad \frac{dE^p(0.5p'_s)}{dp'_s} - \left(\frac{\Delta e - c}{\Delta E(p'_s) - c}\right)\frac{d[\Delta E(p'_s)]}{dp'_s} > 0$$
$$\frac{\gamma\Delta e}{(1+e)}\left(\frac{\eta^\wedge}{M^*}\right)^n > 0 \quad \text{for } \Delta e > 0 \tag{51}$$

The expression on the left-hand side of the second inequality represents the plastic volumetric deformation of a soil associated with the assumption of purely plastic deformation-dependent hardening [see Eq. (23)], and normally it should not be negative. This is because compressive plastic volumetric deformation is normally produced when the current yield surface expands. To satisfy Eq. (51), it is therefore necessary that $dp' < 0$. This reduction in the mean effective stress will result in another increment in p'_s, which has to be balanced by further reduction in dp'. Thus for a given small disturbance, the stress state the soil can sustain has to be reduced in terms of both mean effective stress and shear stress. The original stress state cannot be sustained by the soil, and so the soil is not in a stable condition. This circumstance may continue as long as the current stress state remains on the yield surface with $\eta^\wedge < M^*$. Thus, instability occurs and the soil will finally fail at a critical state of deformation. It should also be observed that this kind of instability is often associated with compressive pore pressure increment during undrained loading and can take place during both virgin yielding and within the yield locus. However, as soon as instability occurs during subyielding, the soil behavior will be governed by the equations for virgin yielding.

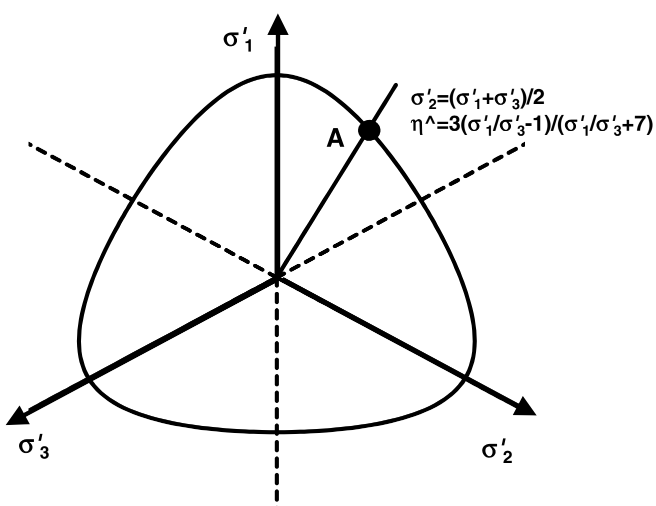

Fig. 10. Generalized stress ratio $\eta^\wedge$

Typical Soil Characteristics

The equations describing the soil model have been presented earlier. In addition to the model parameters, the model requires the specification of three expressions that control the volumetric response. These three equations are required to define the elastic response, $E^e(p')$, the location of the critical state line, $E^p(p'_s)$, and the structure-related component, $\Delta E(p'_s)$. In this section general mathematical forms for these equations are given, representing the behavior of many different types of soil, and they are used to develop explicit stress-strain relations. Obviously, the exact form appropriate for a particular soil is frequently much simpler than the general form specified here, in which case some of the terms in these equations will vanish.

Effects of Structure

Let the critical state line be represented by the following equation:

$$e = e^*_{cs} - l^*_1 p' - \lambda^* \ln p' + c^*_1 \exp(-c^*_2 p') \tag{52}$$

The contribution from elasticity can be described as

$$E^e(p') = l^*_2 p' + \kappa^* \ln p' \tag{53}$$

so that

$$\frac{dE^e(p')}{dp'} = l^*_2 + \frac{\kappa^*}{p'} \tag{54}$$

Consequently, the critical state line given by Eq. (52) can be rewritten in terms of elastic and plastic parts as follows:

$$e = e^*_{cs} - (l^*_2 p' + k^* \ln p') - [(l^*_1 - l^*_2)p' + (\lambda^* - k^*)\ln p' - c^*_1 \exp(-c^*_2 p')] \tag{55}$$

The plastic part of the CSL can be written as

$$E^p(p') = (l^*_1 - l^*_2)p' + (\lambda^* - \kappa^*)\ln(p') - c^*_1 \exp(-c^*_2 p') \tag{56}$$

so that

$$E^p(0.5p'_s) = (l^*_1 - l^*_2)\frac{p'_s}{2} + (\lambda^* - \kappa^*)\ln(0.5p'_s) - c^*_1 \exp(-0.5c^*_2 p'_s) \tag{57}$$

and

$$\frac{dE^p(0.5p'_s)}{dp'_s} = \frac{l^*_1 - l^*_2}{2} + \frac{\lambda^* - \kappa^*}{p'_s} + 0.5c^*_1 c^*_2 \exp(-0.5c^*_2 p'_s) \tag{58}$$

Based on Eq. (16), the ICL* at the reference state can be described by the following equation:

$$e^* = e^*_{cs} + (\lambda^* - \kappa^*)\ln 2 - (l^*_2 p' + \kappa^* \ln p') - \frac{(l^*_1 - l^*_2)}{2}p'_s - (\lambda^* - \kappa^*)\ln p'_s + c^*_1 \exp(-0.5c^*_2 p'_s) \tag{59}$$

Let the additional voids be represented by the following equation:

$$\Delta E(p'_s) = a_o - a_1(p'_s)^{b_1} - \lambda_1 \ln(p'_s) + a\left(\frac{p'_{s,i}}{p'_s}\right)^b + c_3 \exp(-c_4 p'_s) + c_5 \exp(-c_6 p'_s) \tag{60}$$

where $p'_{s,i}$ = specifies the size of the initial yield locus. Two terms of exponential form are included in Eq. (60) because $\Delta E(p'_s) = e - e^*$ [Eq. (18)], and both e and e^* may vary independently exponentially with p'_s. It then follows that

$$\frac{d\Delta E(p'_s)}{dp'_s} = -a_1 b_1 (p'_s)^{b_1 - 1} - \frac{\lambda_1}{p'_s} - \frac{ab}{p'_s}\left(\frac{p'_{s,i}}{p'_s}\right)^b - c_3 c_4 \exp(-c_4 p'_s) - c_5 c_6 \exp(-c_6 p'_s) \tag{61}$$

Explicit Constitutive Equations

Based on the proposed general forms for $E^e(p')$, $E^p(p'_s)$, and $\Delta E(p'_s)$ defined earlier, a set of explicit constitutive equations for the Sydney soil model can be obtained.

Purely Elastic Deformation

The elastic deformation can be expressed as

$$\begin{aligned} d\varepsilon_v^e &= \left(l_2^* + \frac{\kappa^*}{p'}\right)\frac{dp'}{(1+e)} \\ d\varepsilon_d^e &= \frac{\sqrt{2}(1+v^*)}{9(1-2v^*)}\left(l_2^* + \frac{\kappa^*}{p'}\right) \times \frac{\sqrt{(d\sigma'_{11} - d\sigma'_{22})^2 + (d\sigma'_{22} - d\sigma'_{33})^2 + (d\sigma'_{33} - d\sigma'_{11})^2 + 6(d\sigma'^2_{12} + d\sigma'^2_{23} + d\sigma'^2_{31})}}{(1+e)} \end{aligned} \tag{62}$$

Virgin Yielding

The following stress and strain relations describe the virgin yielding behavior:

$$\begin{aligned} d\varepsilon_v = d\varepsilon_v^e &+ \left[\frac{(l_1^* - l_2^*)}{2} + \frac{(\lambda^* - \kappa^*)}{p'_s} + 0.5 c_1^* c_2^* \exp(-0.5 c_2^* p'_s)\right]\frac{dp'_s}{(1+e)} + \frac{\langle dp'_s\rangle}{(1+e)} \\ &\times \frac{\Delta e - c}{\Delta E(p'_s) - c}\left[a_1 b_1 (p'_s)^{b_1-1} + \frac{\lambda_1}{p'_s} + \frac{ab}{p'_s}\left(\frac{p'_{s,i}}{p'_s}\right)^b + c_3 c_4 \exp(-c_4 p'_s) + c_5 c_6 \exp(-c_6 p'_s)\right] \\ &+ \frac{\gamma\Delta e}{(1+e)}\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle \end{aligned} \tag{63}$$

$$\begin{aligned} d\varepsilon_d = d\varepsilon_d^e &+ \Bigg\{\Bigg|\left[\frac{(l_1^* - l_2^*)}{2} + \frac{(\lambda^* - \kappa^*)}{p'_s} + \frac{c_1^* c_2^* \exp(-0.5 c_2^* p'_s)}{2}\right]dp'_s \\ &+ \left[a_1 b_1 (p'_s)^{b_1-1} + \frac{\lambda_1}{p'_s} + \frac{ab}{p'_s}\left(\frac{p'_{s,i}}{p'_s}\right)^b + c_3 c_4 \exp(-c_4 p'_s) + c_5 c_6 \exp(-c_6 p'_s)\right]\frac{\Delta e - c}{\Delta E(p'_s) - c}\langle dp'_s\rangle\Bigg| \\ &+ \gamma|\Delta e|\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle\Bigg\}\left(\frac{1}{1+e}\right) \\ &\times \frac{\left(\frac{3}{2}\right)\sqrt[3]{\eta^\wedge/M^*}}{|1 - \eta^\wedge/M^*| + \omega(\eta/M^*)|1 - \sqrt{p'_e/p'_s}|} \end{aligned} \tag{64}$$

The incremental form of the size of the yield surface dp'_s can be calculated from Eq. (39) where the equations for y_1, y_2, and y_3 can be deduced from the forms for $dE^p(p'_s)/dp'_s$ and $d\Delta E(p'_s)/dp'_s$ given by Eqs. (58) and (61), respectively.

Loading inside the Yield Locus

The volumetric and deviatoric stain increments can be obtained from the relationships presented in section "Loading within the Yielding Surface."

For loading and unloading, within the yield locus, except the first loading of sand-type soil

$$\begin{aligned} d\varepsilon_v = \left(\frac{p'_c}{p'_s}\right)\Bigg\{&\left(\frac{M^{*2} - \eta^{\wedge 2}}{M^{*2} + 0.3\eta^{\wedge 2}}\right)\left[\frac{(l_1^* - l_2^*)}{2} + \frac{(\lambda^* - \kappa^*)}{p'_c} + \frac{c_1^* c_2^* \exp(-0.5 c_c^* p'_s)}{2}\right]dp'_c + \gamma\Delta e\left(\frac{M^* + \eta^\wedge}{M^{*2} + 0.3\eta^{\wedge 2}}\right) \\ &\times \left(\frac{\eta^\wedge}{M^*}\right)^n\langle d\eta^\wedge\rangle\Bigg\}\left(\frac{\alpha}{1+e}\right) + d\varepsilon_v^e \end{aligned} \tag{65}$$

For sand-type soil under first loading

$$\begin{aligned} d\varepsilon_v = d\varepsilon_v^e + \Bigg\{&\left(\frac{M^{*2} - \eta^{\wedge 2}}{M^{*2} + 0.3\eta^{\wedge 2}}\right) \times \left[\frac{(l_1^* - l_2^*)}{2} + \frac{(\lambda^* - \kappa^*)}{p'_c} + \frac{c_1^* c_2^* \exp(-0.5 c_c^* p'_c)}{2}\right]dp'_c \\ &+ \left(\frac{M^{*2} - \eta^{\wedge 2}}{M^{*2} + 0.3\eta^{\wedge 2}}\right)\left[a_1 b_1 (p'_s)^{b_1-1} + \frac{\lambda_1}{p'_s} + \frac{ab}{p'_s}\left(\frac{p'_{s,i}}{p'_s}\right)^b + c_3 c_4 \exp(-c_4 p'_s) + c_5 c_6 \exp(-c_6 p'_s)\right] \times \frac{(\Delta e - c)\langle dp'_c\rangle}{\Delta E(p'_s) - c} \\ &+ \gamma\Delta e\left(\frac{p'_c}{p'_s}\right)\left(\frac{M^* + \eta^\wedge}{M^{*2} + 0.3\eta^{\wedge 2}}\right)\left(\frac{\eta^\wedge}{M^*}\right)^n\langle d\eta^\wedge\rangle\Bigg\}\left(\frac{1}{1+e}\right) \end{aligned} \tag{66}$$

The deviatoric strain increment for all inner loading ($dp'_c > 0$) is

$$d\varepsilon_d = d\varepsilon_d^e + \frac{\left(\frac{3}{2}\right)\sqrt[3]{\eta^\wedge/M^*}}{|1 - \eta^\wedge/M^*| + \omega(\eta^\wedge/M^*)|1 - \sqrt{p'_e/p'_s}|}\overline{|d\varepsilon_v^p|} \tag{67}$$

The definition of the term $\overline{|d\varepsilon_v^p|}$ is given in section "Flow Rule." The deviatoric strain increment for unloading ($dp'_c < 0$) is

$$d\varepsilon_d = d\varepsilon_d^e + \frac{\left(\frac{3}{2}\right)\sqrt[3]{|(\eta_u^\wedge - \eta^\wedge)/M^*|}}{|1 - \eta^\wedge/M^*| + \omega(\eta^\wedge/M^*)|1 - \sqrt{p'_e/p'_s}|}\overline{|d\varepsilon_v^p|} \tag{68}$$

Summary

A new approach to modeling soil behavior within the critical state soil mechanics framework is introduced. Starting from the assumptions that there exist critical states of deformation for soils and the soil properties at such states are reference properties, a hypothetical reference state that is independent of soil structure is proposed.

The behavior of real soil is divided into that exhibited at the reference state and that owing to the influence of soil structure. In the theoretical formulation, soil characteristics such as the elastic volumetric deformation and the plastic volumetric deformation during isotropic virgin yielding are allowed to assume any mathematical form, as determined by the specific features of a given soil.

A new constitutive model, which is referred to as the Sydney Soil model, is proposed within the new framework. The model is based on adequate representation of the plastic volumetric deformation of soil during virgin yielding. The corresponding plastic deviatoric deformation is determined from an assumed flow rule. Destructuring of soil is described in terms of both the effect of isotropic loading and shearing. The observed differences in behavior between some clay-type soils and sand-type soils are also represented by the introduction of the concept of first loading in addition to conventional virgin yielding and subyielding.

Finally, mathematical forms describing characteristics such as elastic volumetric deformation and plastic volumetric deformation under isotropic virgin yielding are specified, and explicit constitutive equations for soil are obtained.

The model has several parameters, which can be divided into two categories.

Parameters in the first category are independent of soil structure. They are parameters defining the final failure strength φ_c, s^*, parameters defining the elastic deformation $E^e(p')$ and ν^* (or G^*), a parameter defining subyielding behavior m, and parameters defining $E^p(p')$ and e^*_{cs}. All these parameters describe intrinsic (reference) soil properties. Therefore they can be determined from laboratory tests on soil samples at the reference state. As will be demonstrated in part II of the paper, for most clayey soils the reference states defined in this model are identical to reconstituted states. For other soils, these properties may be obtained from soil behavior at critical states of deformation.

Parameters in the second category are dependent on soil structure. They are $p'_{s,i}$, n, γ, and c parameters defining the additional voids ratio sustained by soil structure, $\Delta E(p'_s)$, μ, a parameter defining the variation of the aspect ratio of the yield surface, and the parameter ω defining the flow rule. These parameters can be determined only from tests on intact (structured) soil samples.

Parameters m, n, γ, μ, s^*, and ω are parameters that are particular to the proposed model. Based on the present level of information, it is proposed that parameters s^*, μ, and ω can be treated effectively as known constants, i.e., $s^* = 1$, $\mu = 0$, and $\omega = 1$, unless there is strong experimental data to define clearly alternate values. Further, in cases where there are insufficient data, it is also suggested that the values $m = 1$ and $n = 1$ be adopted, as these values are appropriate for many soils as demonstrated in part II.

The performance of the Sydney soil model, and the methods of parameter determination are examined in part II of the paper.

Appendix I: Generalized Shear Stress and Shear Stress Ratio

Eq. (7) can be expressed in terms of the six components of a general stress state as follows:

$$f_2 = \frac{(\sigma'_{11}+\sigma'_{22}+\sigma'_{33})[(\sigma'_{11}+\sigma'_{22}+\sigma'_{33})^2 - s^*(\sigma'^2_{11}+\sigma'^2_{22}+\sigma'^2_{33})]}{\sigma'_1\sigma'_2\sigma'_3} - 27 + 9s^* \tag{69}$$

The value of s^* can be determined from two stress states at the critical state of deformation, because these two stress states will yield the same value of f_2 for a given material. The identification of s^* from conventional triaxial tests is given here in terms of the critical frictional angles for both compression, φ_c, and extension, φ_e. The condition $f_2(\varphi_c) = f_2(\varphi_e)$ allows determination of s^* as follows:

$$s^* = \frac{a}{b} \tag{70}$$

where

$$a = (1+\sin\varphi_{cm})(1-\sin\varphi_{cm})^2(3+\sin\varphi_{ex})^3 - (1-\sin\varphi_{ex})(1+\sin\varphi_{ex})^2(3-\sin\varphi_{cm})^3 \tag{71}$$

$$b = (1+\sin\varphi_{cm})(3+\sin\varphi_{ex})(1-\sin\varphi_{cm})^2(3+2\sin\varphi_{ex} + 3\sin\sin^2\varphi_{ex}) - (1-\sin\varphi_{ex})(3-\sin\varphi_{cm})(1+\sin\varphi_{ex})^2 \times (3-2\sin\varphi_{cm}+3\sin\sin^2\varphi_{cm}) \tag{72}$$

For a stress state where the intermediate principal stress is equal to the mean stress (see point A in Fig. 10), the following relationships apply:

$$\sigma'_1/\sigma'_3 = \frac{2f_2}{27-15s^*} + 2\sqrt{\left(\frac{f_2}{27-15s^*}\right)^2 + \left(\frac{f_2}{27-15s^*}\right)} + 1 \tag{73}$$

and

$$\eta^\wedge = \frac{3(\sigma'_1/\sigma'_3 - 1)}{\sigma'_1/\sigma'_3 + 7} = \frac{3[f_2 + \sqrt{f_2(f_2+27-15s^*)}]}{f_2 + \sqrt{f_2(f_2+27-15s^*)} + 4(27-15s^*)} \tag{74}$$

Appendix II: Destructuring Associated with Searing

Been and Jefferies (1985) demonstrated that the effect of voids ratio and stress level on the peak strength can be described by a single parameter, the state parameter, ξ, defined as (Fig. 1)

$$\xi = e + \lambda \ln p' - \Gamma \tag{75}$$

Based on study of a large amount of experimental data for soils, Yu (1998) proposed the following general equation for the yield surface:

$$\left(\frac{\eta}{\bar{M}}\right)^{\bar{n}} = 1 - \frac{\xi}{\xi_r} \tag{76}$$

where $\bar{n}$ = material constant and ξ_r = value of state parameter for a soil state on the yield surface and with $\eta = 0$. η = shear stress ratio conventionally defined by Eq. (3), and $\bar{M}$ = value of η at the critical state.

Differentiating Eq. (76), an expression for the change in the value of the current state parameter with the current stress ratio is obtained as follows:

$$d\xi = -\frac{\bar{n}(\eta)^{\bar{n}-1}\xi_r}{\bar{M}^{\bar{n}}}d\eta \tag{77}$$

It is assumed that the effect of the current shear stress on the additional voids ratio can be represented by an equation with mathematical format similar to Eq. (77). Suppose the current state is denoted as (η, Δe); it is then necessary to discover the corresponding value of additional voids ratio at a stress state with $\eta = 0$. Linear interpolation is used here, so that

$$\Delta e_r = \left(\frac{M^*}{M^* - \eta}\right)\Delta e \tag{78}$$

Based on the preceding analysis, the following equation is proposed for describing the effects of shearing on destructuring:

$$de = -\gamma\Delta e\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle \tag{79}$$

In this case the decrement in the additional voids ratio increases with the current stress ratio. The operator $\langle\,\rangle$ implies the following: (1) for virgin loading with $\eta^\wedge < M^*$ there is no destructuring associated with shearing if the shear stress ratio does not increase, and (2) for softening (virgin yielding with $\eta^\wedge > M^*$) destructuring occurs. It may be noted that $\eta^\wedge$ generally decreases for softening.

The total plastic strain increment for virgin yielding is proposed as

$$\begin{aligned} d\varepsilon_v^p = &\left[\frac{dE^p(p_s'/N)}{dp_s'}\right]\frac{dp_s'}{(1+e)} \\ &- \left[\frac{\Delta e - c}{\Delta E(p_s') - c}\right]\left\{\frac{d[\Delta E(p_s')]}{dp_s'}\right\}\frac{\langle dp_s'\rangle}{(1+e)} \\ &+ \frac{\gamma\Delta e}{(1+e)}\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle \end{aligned} \tag{80}$$

where parameter c is given by

$$c = \begin{cases} \lim\limits_{p_s'\to\infty} \Delta E(p_s') & \text{if the limit is definite} \\ 0 & \text{otherwise} \end{cases} \tag{81}$$

Two modifications are made to the plastic volumetric deformation associated with the additional voids ratio. The operator $\langle\,\rangle$ has been applied so that the reduction in additional voids ratio associated with the change in size of the yield surface is not recoverable in terms of p_s'. The term $(\Delta e - c)/(\Delta E - c)$ is also added. Δe is the value of the current additional voids ratio, calculated using Eq. (17), the difference in voids ratio between the current soil state and the soil at the corresponding reference state.

Acknowledgments

Some of the work described here formed part of the research program of the Special Research Centre for Offshore Foundation Systems, established and supported under the Australian Research Council's Research Centres Program. In addition, Discovery Grants from the Australian Research Council for research into structured soil behavior are also gratefully acknowledged.

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