Three-Dimensional Lower Bound Solutions for Stability of Plate Anchors in Clay

R. S. Merifield[1]; A. V. Lyamin[2]; S. W. Sloan[3]; and H. S. Yu[4]

Abstract: Soil anchors are commonly used as foundation systems for structures that require uplift or lateral resistance. These types of structures include transmission towers, sheet pile walls, and buried pipelines. Although anchors are typically complex in shape (e.g., drag or helical anchors), many previous analyses idealize the anchor as a continuous strip under plane strain conditions. This assumption provides numerical advantages and the problem can be solved in two dimensions. In contrast to recent numerical studies, this paper applies three-dimensional numerical limit analysis to evaluate the effect of anchor shape on the pullout capacity of horizontal anchors in undrained clay. The anchor is idealized as either square, circular, or rectangular in shape. Estimates of the ultimate pullout load are obtained by using a newly developed three-dimensional numerical procedure based on a finite-element formulation of the lower bound theorem of limit analysis. This formulation assumes a perfectly plastic soil model with a Tresca yield criterion. Results are presented in the familiar form of break-out factors based on various anchor shapes and embedment depths, and are also compared with existing numerical and empirical solutions.

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Introduction

Background and Objectives

Soil anchors can be square, circular, or rectangular in shape and are commonly used as foundation systems for structures requiring uplift resistance, such as transmission towers, or for structures requiring lateral resistance, such as sheet pile walls. More recently, anchors have been used to provide a simple and economical mooring system for offshore floating oil and gas facilities. As the range of applications for anchors expands to include the support of more elaborate and substantially larger structures, a greater understanding of their behavior is required.

The theory of soil uplift resistance may also be used to solve a number of geotechnical problems where the primary uplift resistance of a structure is not provided by the addition of soil anchors. For example, structures such as submerged pipelines or buried foundations, although not supported by anchors, may be modeled effectively as soil anchors.

The objective of the present paper is to quantify the effect of anchor shape upon the ultimate pullout capacity. To do this, lower bound solutions for the ultimate capacity of horizontal square, circular, and rectangular anchors in clay are determined. These three-dimensional lower bound solutions are then compared to a previous study of strip anchors in clay (Merifield et al. 2001), along with the available empirical and numerical results presented in the literature. A systematic procedure is proposed that enables the ultimate uplift capacity of various shaped anchors to be determined. The effect of anchor plate roughness upon the ultimate pullout capacity is also considered.

Previous Studies

Over the last thirty years, a number of researchers have proposed approximate techniques to estimate the uplift capacity of various shaped horizontal anchors in clay. The majority of past research has been experimentally based and, as a result, current design practices are largely based on empiricism. Very few rigorous numerical analyses have been performed to determine the capacity of various shaped anchors in clay.

The majority of existing numerical analyses generally assume a condition of plane strain and the anchor is then analyzed as a continuous strip. The authors are unaware of any rigorous three-dimensional numerical analyses to ascertain the effect of anchor shape on the uplift capacity in clay soils. A condition of plane strain is typically assumed for numerical simplicity. However, in reality, anchors come in various shapes and sizes and, therefore, it is unlikely that the assumption of plane strain will be valid for all cases. For example, a state of plane strain will clearly be overconservative when analyzing circular or square anchors.

Most of the results from studies of anchors in clay either consist of simple approximate solutions or are derived empirically from laboratory model tests. These results can be found in the

[1]Research Engineer, Dept. of Civil, Surveying, and Environmental Engineering, Univ. of Newcastle, NSW 2308 Australia. E-mail: Richard.Merifield@newcastle.edu.au

[2]Research Fellow, Dept. of Civil, Surveying, and Environmental Engineering, Univ. of Newcastle, NSW 2308 Australia. E-mail: Andrei.Lyamin@newcastle.edu.au

[3]Professor, Dept. of Civil, Surveying, and Environmental Engineering, Univ. of Newcastle, NSW 2308 Australia. E-mail: Scott.Sloan@newcastle.edu.au

[4]Professor, Dept. of Civil Engineering, Univ. of Nottingham, U.K.

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works of Vesic (1971), Meyerhof and Adams (1968), Ali (1968), Meyerhof (1973), Das (1978, 1980), and Das et al. (1994). The uplift capacity of anchors is typically expressed in terms of a break-out factor which is a function of the anchor shape, embedment depth, overburden pressure, and the soil properties. The break-out factor is back calculated from laboratory results using an expression for the pull-out load which is assumed analogous to the well known bearing capacity equation.

Meyerhof and Adams (1968) and Meyerhof (1973) presented break-out factors for horizontal anchors based on laboratory tests performed by Adams and Hayes (1967). In addition, Meyerhof (1973) performed tests on both circular and strip anchors. A generalized theoretical framework was also presented, however, this is considered approximate as they make several critical assumptions regarding the anchor failure mechanism and the earth pressure distribution along the failure surface. Based on his laboratory findings, Meyerhof (1973) presented an equation for the break-out factor for circular and square anchors.

In an early study, Vesic (1971) proposed an analytical approach for the pullout capacity of horizontal anchors, based on the solutions of Vesic (1965) for the problem of an expanding cavity close to the surface of a semi-infinite rigid plastic solid. These solutions give the ultimate radial pressure needed to break out a cylindrical or spherical cavity embedded at a depth below the surface of a solid. The pullout capacities for strip and circular anchors were then derived by assuming the pullout load was equivalent to the ultimate cylinder and spherical cavity pressure, plus the weight of soil acting directly above the anchor. Vesic also performed a number of laboratory pull-out tests on circular plate anchors in soft and stiff clays and compared the results with his analytical solutions. More recently Yu (2000) derived an expression for the break-out factor for strip and circular anchors based on more accurate analytical solutions for cavity expansion in cohesive frictional soil that account for dilation. In doing so, it was assumed that the collapse of a plate anchor will occur when any portion of the calculated plastic zone reaches the ground surface. In other words, plate anchors break out when the plastic flow is not confined by the outer elastic zone and becomes free.

Das (1978, 1980) and Das et al. (1985a,b) have suggested procedures, based on model laboratory tests, to estimate the ultimate uplift capacity of square, rectangular, and strip anchors embedded vertically or horizontally in clay. These tests were mostly performed in soft clays with a limited number of tests performed in stiff clays. Das et al. (1994) conducted a number of laboratory tests on circular anchors in soft clay to determine the break-out factors and the variation of suction force with embedment ratio. Similarly, Baba et al. (1989) conducted laboratory testing to determine the effect of loading rate and moisture content on the suction force developed below a circular anchor.

A rigorous numerical study of anchors embedded in clay was carried out by Rowe and Davis (1982). In their study, an elastoplastic finite element analysis was used to determine the break-out factors for horizontal and vertical strip anchors, and for horizontal circular anchors. Other displacement finite-element studies of anchors in clay have been made by Ashbee (1969) and Davie and Sutherland (1977), though very few results were reported.

Limit Analysis Method

The upper and lower bound methods constitute what are known as the limit theorems of classical plasticity, and were developed by Drucker et al. (1952). These theorems are applicable to perfectly plastic materials that obey an associated flow rule. Since their proof, the bounding theorems have provided a powerful tool for analyzing stability problems in soil mechanics. Numerical upper and lower bound techniques have recently been used to study numerous problems including the undrained stability of a trapdoor (Sloan et al. 1990), the stability of slopes (Yu et al. 1998), the bearing capacity of foundations (Merifield et al. 1999; Ukritchon et al. 1998; Yu and Sloan 1994), and the stability of strip anchors (Merifield et al. 2001).

The lower bound theorem states that if an equilibrium distribution of stress covering the whole body can be found that balances a set of external loads on the stress boundary and nowhere exceeds the materials yield criterion, the external loads are not higher than the true collapse load. By examining different admissible stress states, the best (highest) lower bound value on the external loads can be found.

Although the limit theorems provide a simple and useful way of analyzing the stability of geotechnical structures, they have not been widely applied to the problem of anchors in clay. The writers have recently performed a rigorous analysis of horizontal and vertical strip anchors embedded in homogeneous and inhomogeneous clay (Merifield et al. 2001). In this study, upper and lower bound solutions for the ultimate pull-out capacity were obtained using the numerical techniques developed by Sloan (1988), and Sloan and Kleeman (1995). Full details of these numerical procedures can be found in Sloan (1988) and Sloan and Kleeman (1995), and will not be repeated here.

The most commonly used numerical implementation of the lower bound theorem is based on a finite-element discretization of the soil mass. This results in a finite-dimensional optimization problem with large sparse constraint matrices. By adopting linear finite elements and a polyhedral approximation of the yield surface, the optimization problem is one which can be solved using classical linear programming techniques. This type of approach has been widely used and is described in detail, for example, in Bottero et al. (1980) and Sloan (1988). Despite its success over the last two decades, the linear programming approach is limited to dealing with two-dimensional problems. Indeed, the optimization problem resulting from any discrete limit analysis formulation in three dimensions cannot be easily reduced to a linear programming problem, and may need to be solved using nonlinear programming methods, such as those developed by Zouain et al. (1993).

Estimates of the ultimate anchor pull-out load presented in this paper have been obtained by using a new three-dimensional lower bound procedure developed by Lyamin (1999). This procedure can be used to obtain a lower bound collapse load for three-dimensional geotechnical stability problems. Full details of the formulation can be found in Lyamin (1999) and Lyamin and Sloan (1997, 2002), and will not be repeated here.

Problem of Anchor Capacity

General Anchor Behavior

Anchors are typically constructed from steel or concrete and may be circular (including helical), square, or rectangular in shape. A general layout of the problem to be analyzed is shown in Fig. 1.

Following Rowe and Davis (1982), the analysis of anchor behavior may be divided into two distinct categories, namely those of ‘‘immediate breakaway’’ and ‘‘no breakaway.’’ In the immediate breakaway case, it is assumed that the soil/anchor interface cannot sustain tension so that, upon loading, the vertical stress

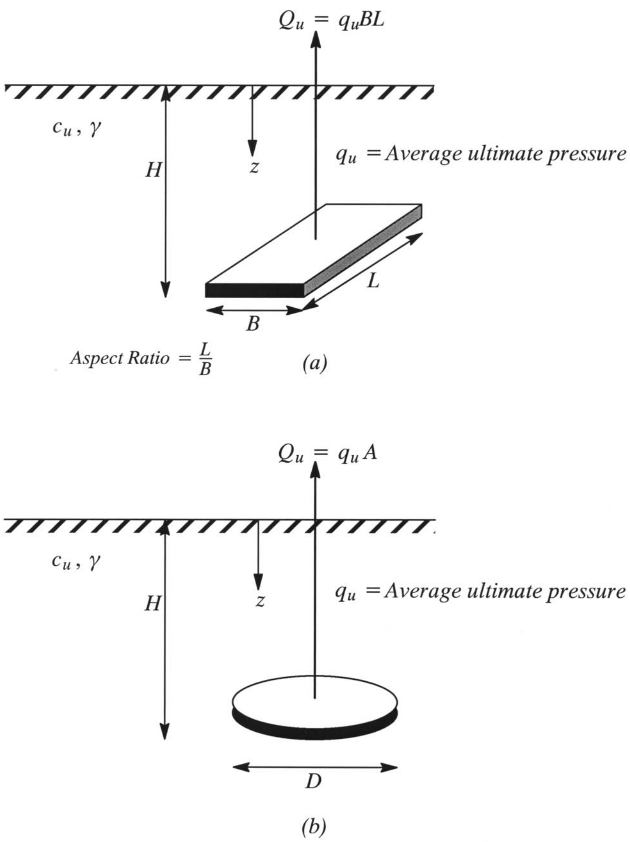


Fig. 1. Problem notation (a) square and rectangular anchors, and (b) circular anchors

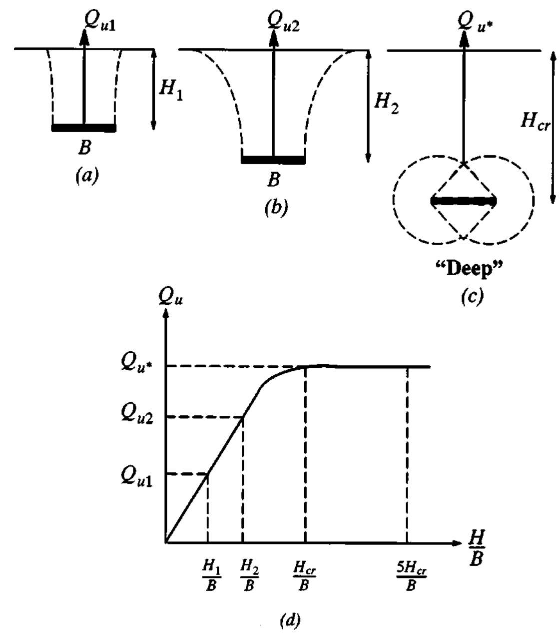


Fig. 2. Shallow and deep anchor behavior

immediately below the anchor reduces to zero and the anchor is no longer in contact with the underlying soil. This represents the case where there is no adhesion or suction between the soil and anchor. In the no breakaway case, the opposite is assumed, with the soil/anchor interface sustaining adequate tension to ensure the anchor remains in contact with the soil at all times. This models the case where an adhesion or suction exists between the anchor and the soil. In reality, it is likely that the true breakaway state of an anchor will fall somewhere between the extremities of the immediate breakaway and no breakaway cases. Due to the uncertainty surrounding the actual magnitude of any suction force, the analyses presented in this paper are for the immediate breakaway case only.

After allowing for immediate and no breakaway behavior, anchors can be further classified as shallow or deep, depending on their mode of failure. This point is illustrated in Fig. 2. An anchor is classified as shallow if, at ultimate collapse, the observed failure mechanism reaches the surface [Figs. 2(a and b)]. In contrast, a deep anchor is one whose failure mode is characterized by localized shear around the anchor and is not affected by the location of the soil surface [Fig. 2(c)].

For a given anchor size, B, and soil properties γ, c_u there exists a critical embedment depth H_{cr} at which the failure mechanism no longer extends to the soil surface. This type of failure mechanism is typically observed for deep anchors, and is localized around the anchor. The significance of such a localized failure mechanism is that the ultimate capacity of the anchor will have reached a maximum limiting value. This arises because the undrained shear strength is assumed to be independent of the mean normal stress.

Pull-Out Capacity of Anchors in Undrained Clay

In the following, it is assumed that the undrained strength of the clay is homogeneous and isotropic. This is in keeping with most conventional undrained analyses, even though real clays are likely to inhomogeneous and anisotropic, and implies that an appropriate undrained shear strength will need to be selected carefully. The lower bound formulation and analyses could be extended to cater to anisotropic strength characteristics using, for example, the model proposed by Ladd (1991). Such a generalization, however, is beyond the scope of this initial study.

The ultimate anchor pull-out capacity (pressure) in undrained clay is usually expressed as a function of the undrained shear strength in the following form

$$q_u = \frac{Q_u}{A} = c_u N_c \tag{1}$$

where A = the anchor area, c_u = the undrained soil strength at the ground surface, and N_c = anchor break-out factor.

For convenience, the anchor break-out factor has been defined for the following two cases:

1. For a homogeneous soil profile with no unit weight ($\gamma = 0$)

$$N_c = N_{co}, \text{ where } N_{co} = \left(\frac{q_u}{c_u}\right)_{\gamma=0} \tag{2}$$

2. For a homogeneous soil profile with unit weight ($\gamma \neq 0$)

$$N_c = N_{c\gamma}, \text{ where } N_{c\gamma} = N_{co} + \frac{\gamma H}{c_u} \tag{3}$$

Eqs. (1)–(3) reflect the complex nature of the break-factor N_c, as observed by Rowe (1978). The break-out factor is a function of the embedment ratio (H/B) and overburden pressure, with the latter being expressed in terms of the dimensionless quantity $\gamma H/c_u$. This indicates that, separate from the overall problem geometry, the soil properties directly influence anchor behavior.

It should be noted that the break-out factor given in Eqs. (1)–(3) does not continue to increase indefinitely, but reaches a limiting value which marks the transition between shallow and deep anchor behavior. The limiting value of the break-out factor is defined as N_{c*}.

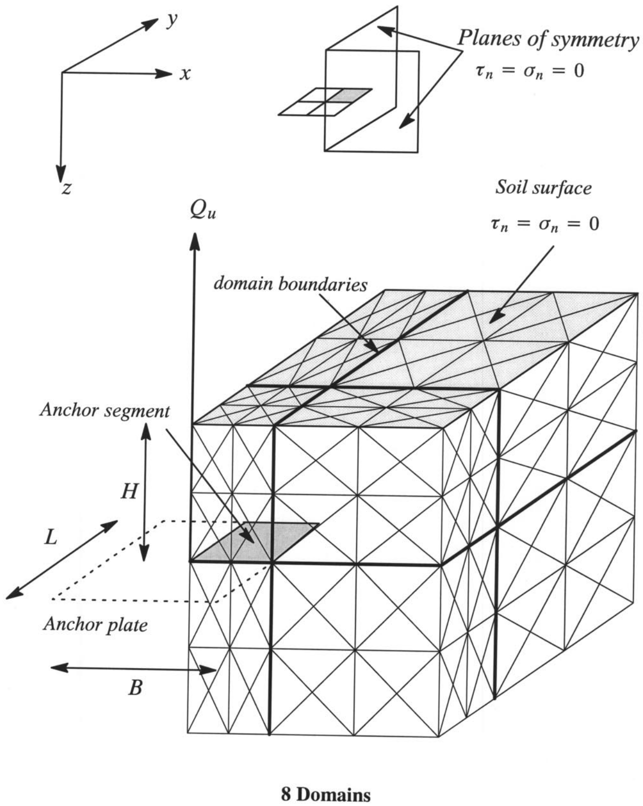


Fig. 3. Typical mesh domains used for analyzing horizontal square or rectangular anchors

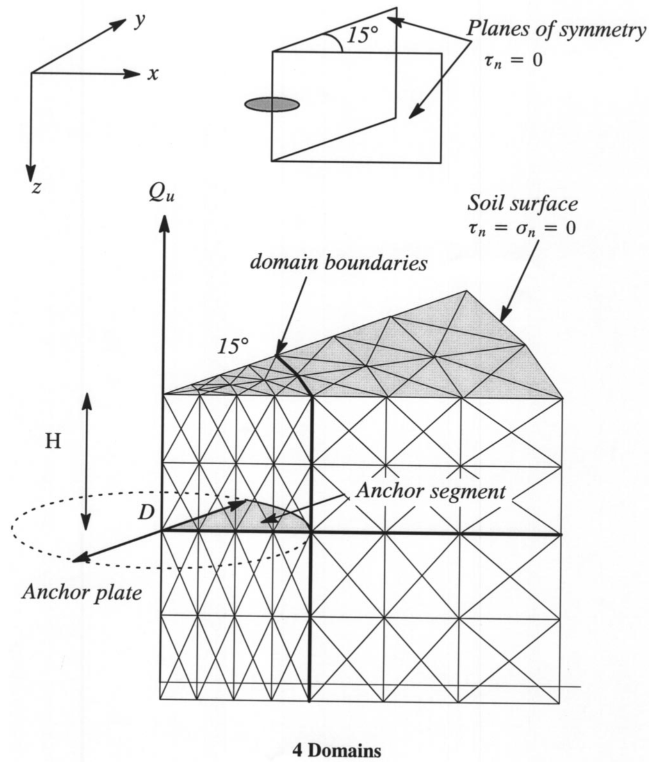


Fig. 4. Typical mesh segments for analysis of circular anchors

Implicit in Eq. (3) is the assumption that the effects of soil unit weight and cohesion are independent of each other and may be superimposed. This assumption was investigated in the previous study of Merifield et al. (2001), where it was found that the principle of superposition can be successfully applied to shallow strip anchors in clay. It is shown, in a later section of this paper, that the superposition principle is equally applicable when estimating the three-dimensional capacity of shallow anchors.

Three-Dimensional Modeling Details

A simplified representation of the lower bound mesh arrangements used to analyze square, rectangular, and circular anchors, is illustrated in Figs. 3 and 4. The soil mass is first discretized into a number of domains where the boundaries between adjacent domains may be specified as a stress discontinuity or rigid joint. Each domain is then subdivided in three-dimensional space to form a number of tetrahedral elements within each domain.

By taking symmetry into account, the overall problem size can be reduced. For square/rectangular anchors, symmetry can be used so that only one quarter of an anchor needs to be analyzed (Fig. 3). Similarly, for circular anchors, only a small 15° slice of the anchor needs to be analyzed (Fig. 4). The boundaries of domains lying on the planes of symmetry are subject to certain stress boundary conditions.

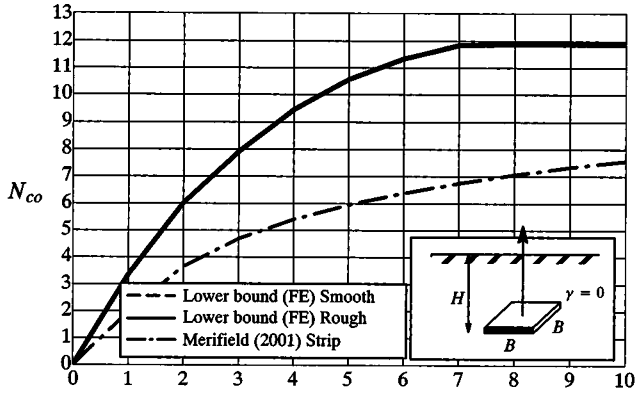
To model a perfectly rough anchor, no constraints are placed on the allowable shear stress at element nodes located directly above the anchor segment (shaded in Fig. 3). The shear stress is therefore unrestricted and may vary up to a value less than or equal to the undrained shear strength of the soil (according to the yield constraint). Alternatively, a smooth anchor may be modeled by insisting that the shear stress is zero at all element nodes along the anchor/soil interface. To allow the under side of the anchor to separate from the soil (immediate breakaway), the stress discontinuity between the domains above and below the anchor segment is removed, and the shear stress and normal stress are forced to be equal to zero. This effectively creates a free surface below the anchor.

Results and Discussion

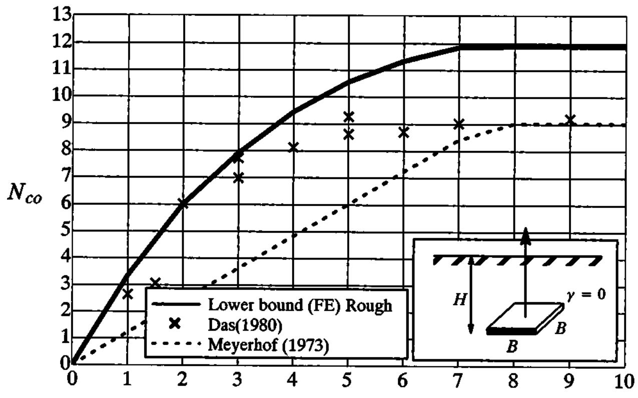
Finite-element limit analyses were performed to obtain a lower bound estimate of the anchor break-out factor N_c for square, circular, and rectangular anchors over a range of embedment depths. These results, along with the effects of plate roughness and overburden pressure, are discussed in the following sections. Where possible, past experimental and numerical results are compared to results obtained from the current study.

Square Anchors

The computed lower bound estimates of the anchor break-out factor N_{co} for square anchors ($L/B=1$) in weightless soil are shown graphically in Fig. 5(a). These break-out factors have been compared with those obtained for strip anchors by Merifield et al. (2001), and are expressed in ratio form in Fig. 6. Fig. 6 indicates that the break-out factors for square anchors in weightless soil are always greater than those obtained for strip anchors at corresponding embedment ratios. For example, at an embedment ratio of $H/B=1$, the break-out factor for a square anchor is approximately 70% greater than that obtained for a strip. This difference reduces to around 55% at $H/B=10$.



(a) Finite Element numerical solutions



(b) Comparison with existing laboratory test results

Fig. 5. Break-out factors for square anchors in clay

The break-out factor was found to increase steeply before reaching a constant value of approximately 11.9 at $H/B \approx 7$. This is in contrast to the results obtained by Merifield et al. (2001) for an infinite strip in a weightless soil, where no limiting value of the break-out factor was observed for $H/B \leqslant 10$. This observation becomes very important when estimating the ultimate capacity of a deep anchor.

The value of the break-out factor N_{co} shown in Fig. 5(a) can be approximated by the following equation:

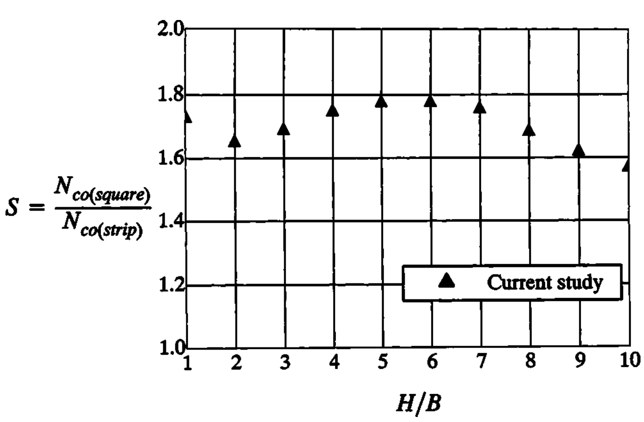


Fig. 6. Ratio of anchor break-out factors for square and strip

$$N_{co} = S\left[2.56 \ln\left(2\frac{H}{B}\right)\right] \quad \text{Square anchors-ROUGH} \tag{4}$$

where S = shape factor illustrated in Fig. 6. The expression within the parentheses is that obtained by Merifield et al. (2001) for an infinite strip.

The authors are unaware of any other numerical studies to determine the ultimate capacity of square anchors. However, a limited number of laboratory testing programs have been undertaken, and are compared to the finite-element limit analysis solutions in Fig. 5(b). For comparison purposes, the break-out factor back figured from laboratory tests has been assumed equivalent to N_{co} as given by Eq. (2). This is based on the assumption that overburden pressures are likely to be very small in these laboratory tests, so the second term in Eq. (3) ($\gamma H/c_u$) becomes insignificant.

A comprehensive testing program was undertaken by Das (1980), who performed pull-out tests on small model square and rectangular anchors in soft to firm clays. Immediate breakaway was ensured by venting the bottom side of the anchor with hollow copper tubing and filter paper. The break-out factors back figured by Das plot below the limit analysis solutions over the full range of embedment ratios [see Fig. 5(b)]. The percentage difference between the Das and limit analysis solutions decreases with increasing embedment ratio up to $H/B = 5$. At embedment ratios of $H/B = 1$ and 5, for example, the variation is 21% and 12% respectively. For embedment ratios of $H/B \geqslant 5$, the difference is around 24%. Das observed that the break-out factor reached a constant value of around 9 at embedment ratios greater than 6. This is below the limit analysis results where a maximum break-out factor of 11.9 was observed for $H/B \geqslant 7$.

The break-out factors obtained using the equation presented by Meyerhof (1973) are clearly overconservative and are as much as 65% below the finite-element bound solutions [see Fig. 5(b)]. This confirms the approximate and conservative nature of this equation as indicated by Meyerhof.

Effect of Anchor Roughness

The effect of anchor roughness on the break-out factor N_{co} was found to be minimal. For an anchor with $H/B \leqslant 5$, for example, changing the roughness from perfectly rough to perfectly smooth reduces N_{co} by around 1%. For embedment ratios of $H/B > 5$, the change in anchor roughness reduces N_{co} by 3–4%. At these embedment ratios, it is likely that lateral shearing of the soil at anchor level takes place and shear stresses are developed along the rough anchor/soil interface. These shear stresses are resisted by the interface and therefore contribute to the capacity of the anchor.

Effect of Overburden Pressure

The numerical results discussed herein are limited to problems with no soil unit weight and, therefore, the effect of soil weight (overburden) needs to be investigated. If our assumption of superposition is valid, then it would be expected that the anchor break-out factor, as given in Eq. (3), would increase linearly with the dimensionless overburden pressure $\gamma H/c_u$. The results from further analyses where both soil weight and cohesion are included, shown in Fig. 7(a), confirm that this is indeed the case.

Fig. 7(a) shows that the ultimate anchor capacity increases linearly with overburden pressure up to a limiting value. This limiting value reflects the transition from shallow to deep behavior, where the mode of failure becomes localized around the an-

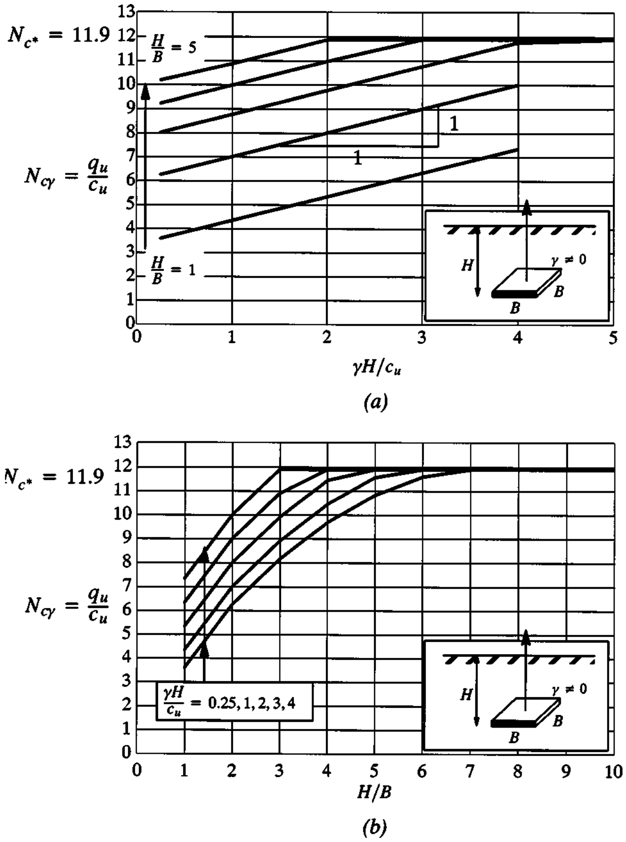


Fig. 7. Effect of overburden pressure for square anchors in clay

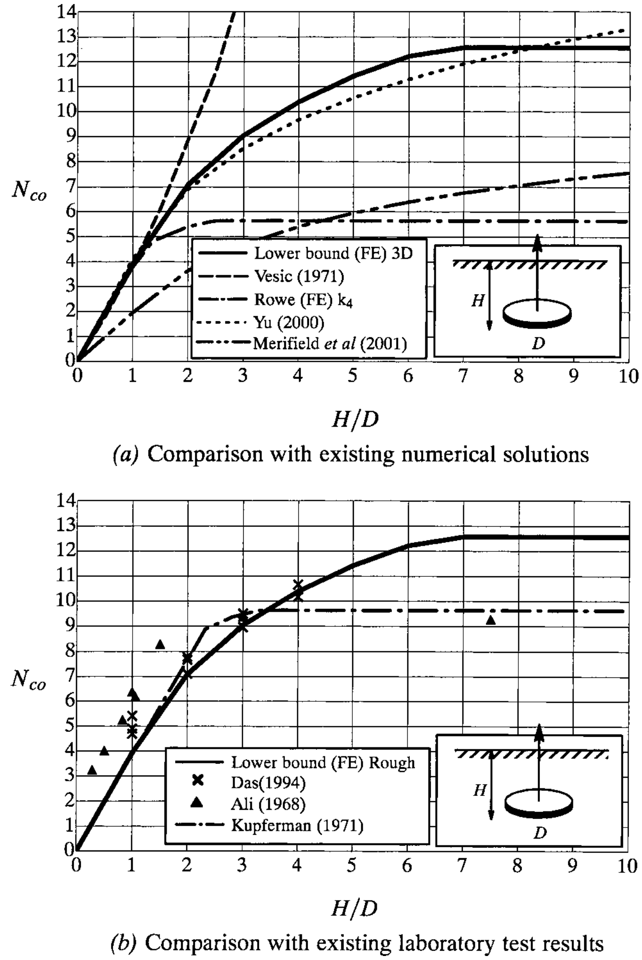


Fig. 8. Break-out factors for circular anchors in clay

chor. At a given embedment depth, an anchor may behave as shallow or deep depending on the dimensionless overburden ratio $\gamma H/c_u$. The critical overburden ratio, which marks the transition from shallow to deep anchor behavior, reduces for increasing embedment ratios. For example, by referring to Fig. 7(a), the critical overburden ratio is approximately 2 for $H/B=5$ and approximately 4 for $H/B=3$.

For deep anchors, the limiting value of the break-out factor N_{c*} was found to be 11.9. This result differs from that obtained by Merifield et al. (2001) for an infinite strip, where the limiting value of the break-out factor N_{c*} was found to be 11.16. Therefore, as expected, the ultimate capacity of a deep square anchor $(11.9c_u)$ is larger than that of a deep strip anchor $(11.16c_u)$.

By combining the lower bounds defined by Eq. (4), together with the limiting value of $N_{c*}=11.9$, an effective anchor design chart can be produced [Fig. 7(b)]. This chart is particularly useful in determining the optimal embedment depth or anchor size in a given soil. The optimal anchor embedment depth or anchor size are those values which correspond to the point of transition from shallow to deep anchor behavior. For example, referring to Fig. 7(b), for an anchor of size $B=0.5$ m embedded in a clay soil with $c_u=20$ kPa and $\gamma=16$ kN/m^3, estimates of H can be made so that the resulting values of H/B and $\gamma H/c_u$ coincide at a value of $N_c=N_c^*=11.9$. For this particular problem, the optimal embedment depth is approximately 2.5 m. Embedding the anchor any deeper than 2.5 m would not lead to an appreciable increase in the pullout capacity. Adopting this iterative process can ensure that the anchor is not buried at an unnecessarily large depth. Similarly, given the embedment depth H, an iterative procedure could be adopted to determine the optimal anchor size B.

Circular Anchors

Lower bound estimates of the anchor break-out factor N_{co} for circular anchors are shown in Fig. 8(a). These break-out factors have been compared with those obtained for strip anchors by Merifield et al. (2001), which have also been included in Fig. 8. The break-out factor N_{co} was found to increase steeply before reaching a constant value of approximately 12.56 at $H/D\approx 7$.

Also shown in Fig. 8(a) are the solutions derived by Vesic (1971), Rowe (1978), and Yu (2000). When compared to the lower bound finite-element predictions, the cavity expansion solutions of Vesic appear to be unconservative for all but the most shallow of anchors where $H/D=1$. In contrast, the solution of Yu compares reasonably well for all embedment ratios. The finite-element solutions of Rowe (1978) plot close to the lower bound solution for anchors at small embedment ratios ($H/D<1.5$), but are conservative for anchors with embedment ratios greater than 1.5. The reason for the latter discrepancy is due to the definition of failure adopted by Rowe and Davis (1982). In their finite-element analyses they found that, although clearly defined collapse loads could be obtained, in many cases the deformations prior to collapse were so great that for practical purposes, failure could be deemed to have occurred at a load below the collapse load. For this type of problem, where the ultimate load is only reached after large deformations, Rowe defined the failure load as the load which would give rise to a displacement four times that predicted by an elastic analysis. This was termed the k_4 failure criterion, and is essentially a serviceability constraint on

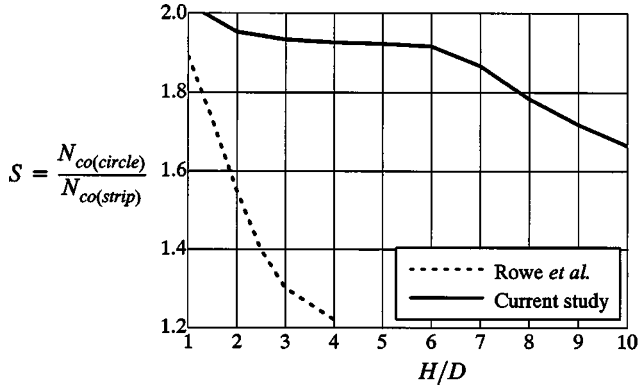


Fig. 9. Ratio of anchor break-out factors for circle and strip

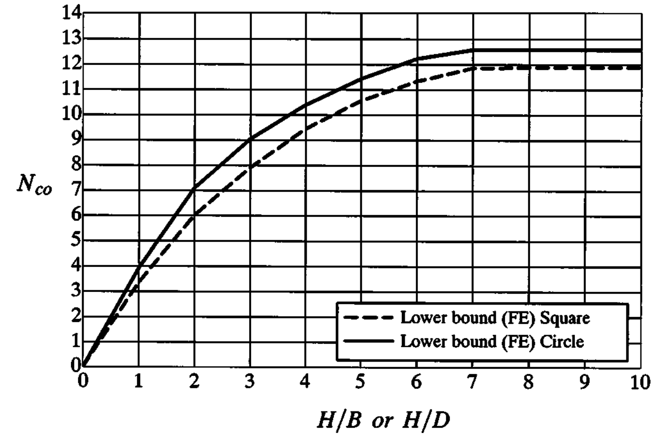


Fig. 10. Comparison of break-out factors

the ultimate load. For embedment ratios greater than about 2, the collapse load was found to be limited by the k_4 condition. This explains the plateau of the curve shown in Fig. 8(a).

The laboratory model tests on circular anchors performed by Das et al. (1994), Ali (1968), and Kupferman (1965) are shown in Fig. 8(b). These tests were typically performed in soft soils. The laboratory results of Das et al. and Kupferman compare reasonably well to the finite-element limit analyses, plotting between 0–16% above the numerical solutions for $H/B \leqslant 4$. However, it should be noted that, in reality, this variation is likely to be smaller due to our assumption that the break-out factor determined from laboratory tests is independent of the overburden pressure. It is not always appropriate to make this assumption, particularly when tests are conducted in soft soils.

The laboratory tests by Ali (1968), which were performed in soft bentonite clay, are difficult to compare to the numerical limit analysis results because suction forces were allowed to develop below the anchor plate while testing. Corrections to the ultimate capacity were then made by estimating the likely suction forces between the anchor and soil. However, suction forces are likely to be highly variable and are a function of several parameters including the embedment depth, soil permeability, undrained shear strength, and loading rate. Therefore, it may be reasonable to conclude that any difference between the results of Ali and the limit analysis solutions may be attributed to the uncertainty in estimating the suction forces developed between the soil and the anchor.

The greatest variation between the limit analysis and laboratory results occurs when the laboratory break-out factors reach a constant limiting value, which clearly indicates deep anchor behavior ($H/B \geqslant 4$). Although the overburden pressure in laboratory model tests is generally small, it appears that the dimensionless overburden ratio $\gamma H/c_u$ in the laboratory results in Fig. 8(b) is sufficiently large so that the anchor collapse mechanism becomes localized around the anchor. Therefore, at embedment ratios past this transition, a comparison between the limit analysis and laboratory results is not strictly appropriate. Nonetheless, results from the laboratory tests reveal a limiting value between 9.5 and 10.5 in comparison to that given by the three-dimensional lower bound of 12.56.

Fig. 9 shows the effect of anchor shape on the collapse load in terms of the ratio of anchor capacity for a circle divided by the anchor capacity of a strip. As was the case for square anchors, the break-out factors for circular anchors are always greater than those obtained for strip anchors at corresponding embedment ratios.

The value of the break-out factor N_{co} determined from the finite-element limit analyses can, with sufficient accuracy, be approximated by the following equation:

$$N_{co} = S\left[2.56\ln\left(2\frac{H}{D}\right)\right] \quad \text{Circular anchors-ROUGH} \qquad (5)$$

where S = shape factor illustrated in Fig. 9.

A comparison between the finite-element limit analysis results for circular and square anchors is shown in Fig. 10. For $H/D \leqslant 4$, the break-out factors for circular anchors were found to be between 9 and 15% greater than those for square anchors. This observation is consistent with the laboratory findings of Das (1980, 1994).

Effect of Overburden Pressure

As was the case for square anchors, the ultimate anchor capacity can be considered to increase linearly with overburden pressure up to a limiting value that reflects the transition from shallow to deep anchor behavior. This is illustrated in Fig. 11(a).

For deep circular anchors, the limiting value of N_{c*} determined from the finite-element lower bound analyses was found to be 12.56. Interestingly this value, which is used to determine the capacity of a deep anchor, is around 20% greater than that found in the laboratory tests of Das and Kupferman [Fig. 8(b)]. Possible explanations for this variation are: (1) The anchor interface conditions in the laboratory tests and the amount of suction that takes place; (2) the boundary conditions below the anchor and whether sufficient material exists to permit a contained collapse mechanism to occur without boundary interference; and (3) uncertainty regarding the precise value of the undrained shear strength.

To aid in the design of circular anchors, Eq. (5) has been combined with the limiting value of $N_{c*} = 12.56$ to produce the design chart shown in Fig. 11(b). This chart indicates that circular anchors at embedment depths (H/D) greater than approximately 5 are likely to behave as deep anchors for most values of the overburden ratio $\gamma H/c_u$.

Effect of Anchor Roughness

Anchor roughness was found to have little influence on the ultimate anchor capacity. For all embedment ratios, only small reductions in the break-out factor N_{co} (<2%) were computed. This suggests that only small shear stresses are developed between the soil and anchor at ultimate failure.

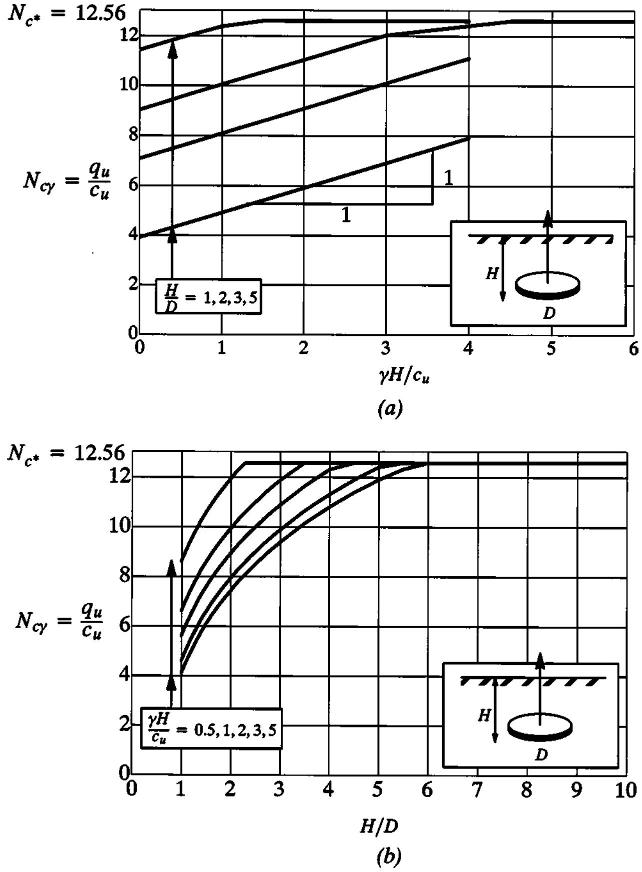


Fig. 11. Effect of overburden pressure for circular anchors in clay

Rectangular Anchors

Lower bound estimates of the anchor break-out factor N_{co} for rectangular anchors in weightless homogeneous soil are shown graphically in Fig. 12(a). Fig. 12 shows that for anchors with aspect ratios (L/B) greater than about 10, the break-out factors may be assumed equal to those obtained for an infinite strip. In other words, above an aspect ratio of 10 a rectangular anchor essentially behaves as a strip anchor.

As previously mentioned, a comprehensive testing program for horizontal anchors in clay was undertaken by Das (1980). Das performed a series of pullout tests on rectangular anchors with aspect ratios of $L/B=2$, 3, and 5 in soft to firm clay. The results of these tests are compared with the current lower bound finite-element solutions in Figs. 12(b) and 13. In general, the results of Das compare reasonably well to the numerical limit analysis results, tending to be slightly conservative for all embedment ratios.

Fig. 13 also plots the results obtained by Rowe (1978) and Das (1980) for horizontal anchors in clay with an aspect ratio of $L/B=5$. These test results of Rowe have been taken from the raw laboratory test data of Rowe (1978) at ultimate load. The results of Rowe also compare reasonably well with the finite-element limit analysis results, and typically plot close to or just below the lower bound solutions up to $H/B=5$. Above this embedment ratio, the results or Rowe plot between 5 and 20% below the lower bound solution.

Effect of Overburden Pressure

The ultimate anchor capacity was found to increase linearly with overburden pressure up to a limiting value that reflects the transition from shallow to deep anchor behavior. This is illustrated in

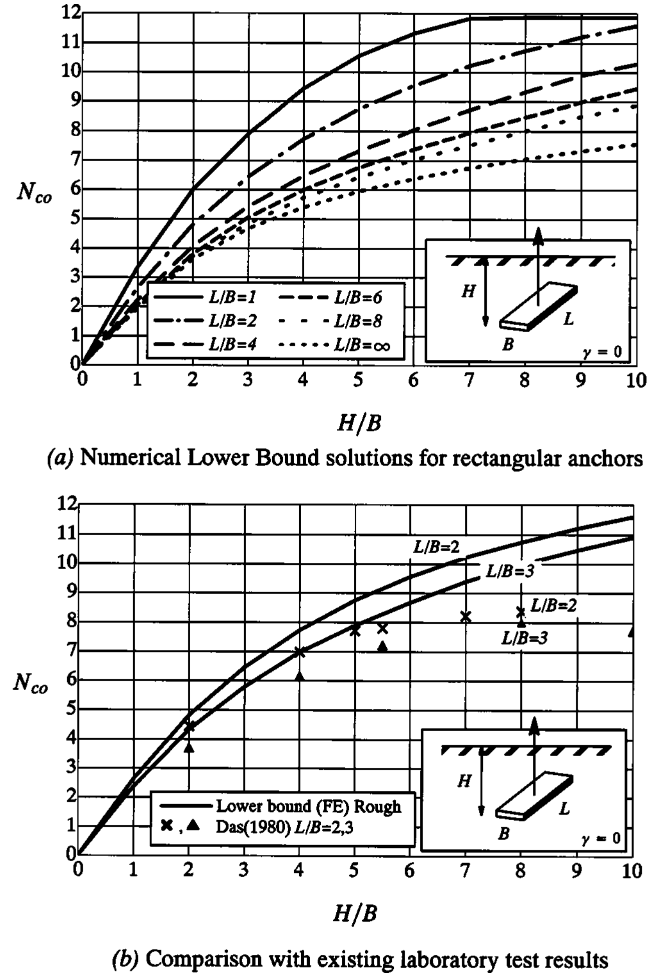


Fig. 12. Break-out factors for rectangular anchors in clay

Fig. 14 for aspect ratios of $L/B=2$ and $L/B=4$. For deep rectangular anchors, the value of the break-out factor N_{c*} will lie somewhere between the limiting values of the break-out factor obtained for a square anchor $(L/B=1)$ and a strip anchor $(L/B=\infty)$. As already mentioned, these limiting values were found to be $N_{c*}=11.9$ and $N_{c*}=11.16$ for square and strip anchors, respectively. It is proposed that the value of N_{c*} for rectangular anchors may be obtained by simple linear interpolation between

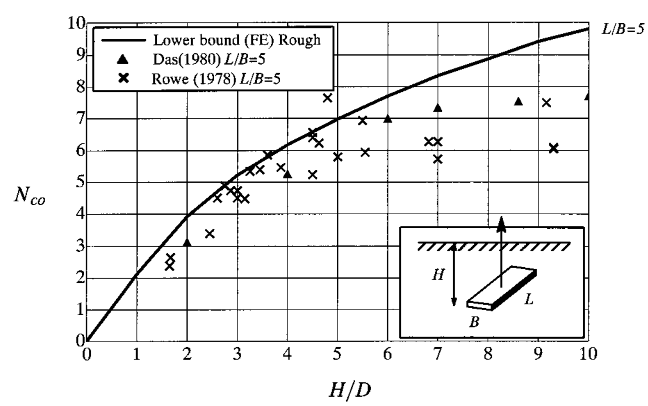


Fig. 13. Comparison of break-out factors—existing laboratory test results

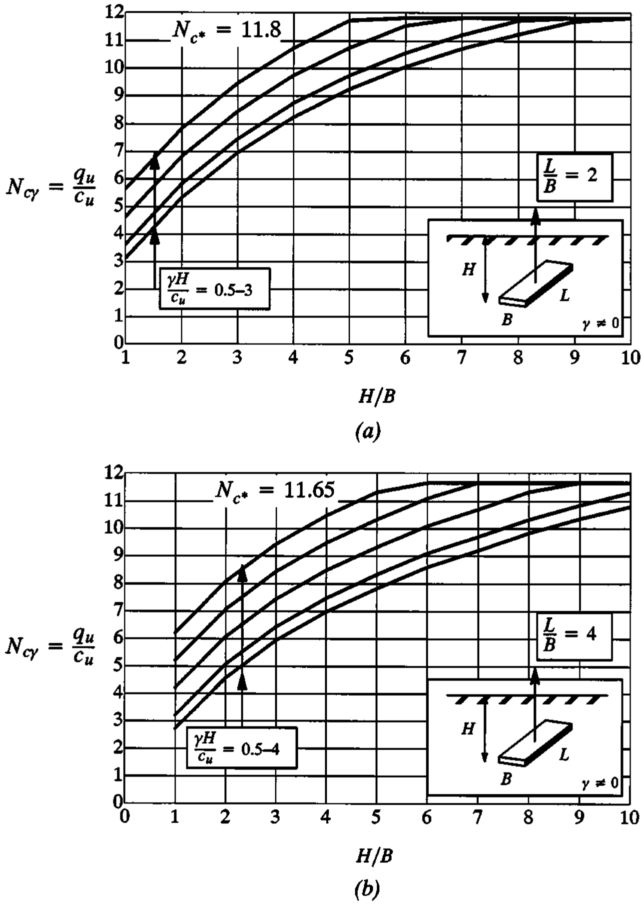


Fig. 14. Effect of overburden on rectangular anchors in clay

these two values assuming that an anchor with an aspect ratio of $L/B = 10$ behaves as a continuos strip anchor.

Fig. 15 has been produced for design purposes, and enables the anchor break-out factor N_{co} to be determined for square or rectangular anchors at various embedment ratios. The effect of overburden pressure can then be added to the break-out factor N_{co} in accordance with Eq. (3) up to the appropriate limiting value of $N_{co} = N_{c*}$.

Suggested Procedure for Estimation of Uplift Capacity

The following list enumerates the suggested procedure for estimating uplift capacity:

1. Determine representative values of the material parameters c_u, and γ.
2. Knowing the anchor size (B, L, D) and embedment depth H, calculate the embedment ratio H/B or H/D (*circular anchor*).
3. Determine the overburden ratio $\gamma H/c_u$.
4. Adopt $N_{c*} = 12.56$ for circular anchors, and $N_{c*} = 11.9$ for square anchors.
5. (i) Calculate the break-out factor N_{co} using either Eqs. (4) and (5) or Fig. 15 depending on the anchor shape.
   (ii) Calculate the break-out factor $N_c = N_{c\gamma}$ using Eq. (3).
   (iii) If $N_c \geqslant N_{c*}$, then the anchor is a deep anchor. The ultimate pull-out load is given by Eq. (1), where $N_c = N_{c*}$.
   (iv) If $N_c \leqslant N_{c*}$, then the anchor is a shallow anchor. The ultimate pull-out load is given by Eq. (1) where N_c is the value obtained in 5(ii).

Example of Application

We now illustrate how to use the results presented to determine the ultimate pullout capacity of an anchor in clay.

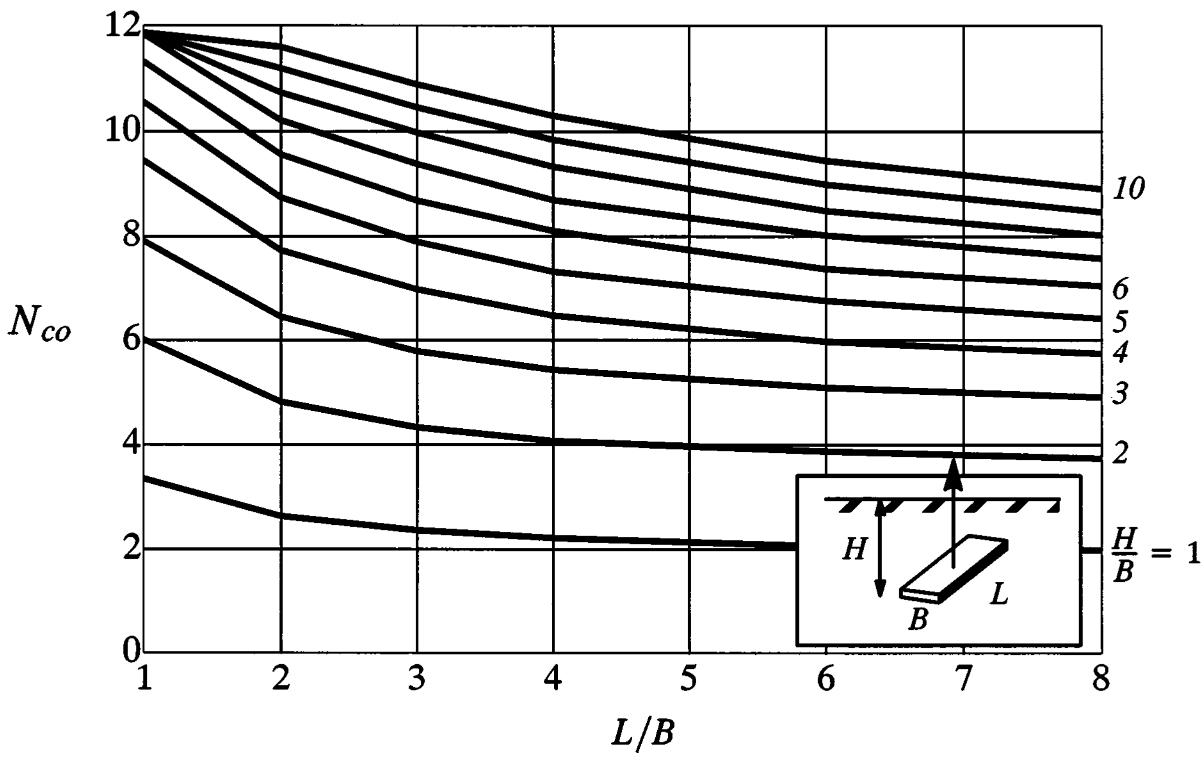


Fig. 15. Design chart for rectangular anchors in clay

Problem. A square horizontal plate anchor of width 0.2 m is to be embedded 1.5 m in a homogeneous clay. Determine the ultimate pull-out capacity given that the clay has a shear strength $c_u = 50$ kPa and unit weight $\gamma = 15$ kN/m^3.

The systematic procedures just given will now be used to determine the ultimate anchor capacity.

1. Given $c_u = 50$ kPa and $\gamma = 15$ kN/m^3.
2. The embedment ratio can be calculated as $H/B = 1.5/0.2 = 7.5$.
3. The dimensionless parameter $\gamma H/c_u = (15 \times 1.5)/50 = 0.45$.
4. Adopt $N_{c*} = 11.9$.
5. (i) From Fig. 6, $S \approx 1.7$. Using Eq. (4)
$$N_{co} = 1.7[2.56 \ln(2H/B)] = 11.79.$$
(ii) From Eq. (3), $N_c = 11.79 + 0.45 = 12.24$.
(iii) $N_c > N_{c*}$ and therefore the anchor is ''deep'' and using Eq. (1)
$$q_u = c_u N_{c*} = 50 \times 11.9 = \mathbf{595.0} \text{ kPa}$$
$$Q_u = 595.0 \times (0.2)^2 = \mathbf{23.8} \text{ kN}.$$

Direct finite-element lower bound calculations using these parameters gives $q_u = 593$ kPa, which is 0.5% less than that obtained herein using the suggested procedure.

Conclusions

The effect of anchor shape on the pull-out capacity of horizontal anchors in undrained clay has been analyzed using a three-dimensional numerical procedure based on a finite-element formulation of the lower bound theorem of limit analysis. Rigorous lower bound solutions for the ultimate capacity of horizontal square, circular, and rectangular anchors have been presented. Consideration has been given to the effect of anchor embedment depth, anchor roughness, and overburden pressure. Results are for the case where no suction forces exist between the anchor and soil, which constitutes what is known as the immediate breakaway condition.

The results obtained have been presented in terms of familiar break-out factors in both graphical and numerical form to facilitate their use in solving practical design problems. A systematic design approach has also been proposed, and the solution to a practical design problem has been included.

The following main conclusions can be drawn from the results presented in this paper:

- As expected, the break-out factors for square, circular, and rectangular anchors in weightless soil are always greater than those obtained for strip anchors at corresponding embedment ratios.
- Rectangular anchors with aspect ratios (L/B) greater than 10 can be considered to behave essentially as a strip anchor.
- A comparison of the three-dimensional numerical limit analysis solutions with those published from small scale laboratory tests show encouraging agreement.
- The ultimate capacity for all anchors was found to increase linearly with overburden pressure up to a limiting value. This confirms that the principle of superposition is valid for shallow square, circular, and rectangular anchors. The limiting value reflects the transition from shallow to deep anchor behavior where the mode of failure becomes localized around the anchor. At a given embedment depth an anchor may behave as shallow or deep, depending on the dimensionless overburden ratio $\gamma H/c_u$.
- The ultimate capacity of horizontal square, circular, or rectangular anchors is not likely to be affected noticeably by anchor roughness. The computed reduction in the ultimate capacity between rough and smooth anchors was typically less than 2%.
- Simple parametric equations have been produced that enable the capacity of square and circular anchors in homogeneous soil profiles to be estimated. These equations can be used to solve practical design problems.

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