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NUMERICAL AND ANALYTICAL CALCULATION OF THE ORTHOTROPIC HEAT TRANSFER PROPERTIES OF FIBRE REINFORCED MATERIALS

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Abstract

This paper is on the investigation of the orthotropic heat transfer properties of unidirectional fibre reinforced materials. The orthotropic effective thermal conductivity of such composite materials is investigated based on two different approaches: the finite element method as a representative for numerical approximation methods and an analytical method for homogenised models based on the solution of the respective boundary value problem. It is found that fibre reinforced composites possess strong orthotropic heat transfer properties, which are getting more distinctive with increasing deviation of the thermal conductivities of matrix and reinforcements. Furthermore, the effect of small perturbations of the periodic configuration of fibres in the matrix on the thermal conductivity is investigated.

Keywords: Heat transfer; Thermal conductivity; Composites; Reinforced material; Orthotropic behaviour

1 Introduction

Fibre reinforced composite materials (cf. Fig. 1) combine many interesting properties, such as low thermal expansion, high specific stiffness, very high specific strength and an excellent fatigue resistance [1]. In conjunction with the long history of fibre reinforced materials, a wide field of industrial application has been created.

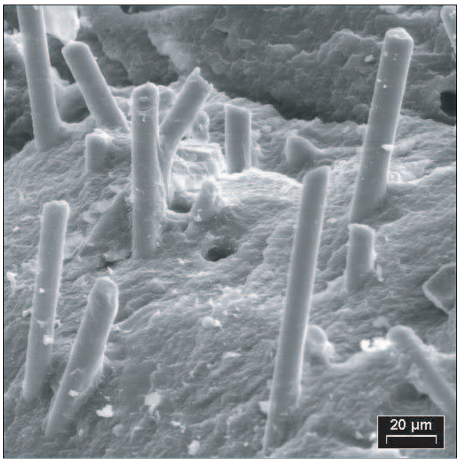

Figure 1: REM photograph of a glass fibre reinforced material, [2].

For the optimum utilisation of these materials, specific material parameters are required. In addition to the well researched mechanical behaviour, many applications include the functionality of heat insulation or of heat conduction in order to avoid the overheating and thermal damage of the matrix or the reinforcements. An interesting area of research is as well the application of carbon fibre reinforced materials as energy storage units [4]. Fibre reinforced materials possess a strong orthotropic behaviour which has to be taken into consideration while determining the heat transfer properties. The most important structural parameters which characterise a composite material are the topology and the volume fraction of the reinforcements $f_{\mathrm{fib}} = V_{\mathrm{fib}}/V_{\mathrm{t}}$ (V_{fib} is equal to the volume of the fibres, V_{t} represents the total volume of the composite). Since a unidirectional reinforced composite is considered and accordingly all fibres are parallel to the x_3-axis (cf. Fig. 2), the topology is determined by the position of the fibres in the x_1-x_2-plane (cf. Fig. 2). In this paper, the geometrical effective thermal conductivity is determined based on conduction in the matrix with a conductivity λ_{m} and the fibres with a conductivity λ_{fib}. Two different methods, namely a numerical and a mathematical, are applied.

The finite element method allows for consideration of arbitrary nonlinearities (e.g. material or boundary conditions). However, the numerical approach is in many cases restricted to the consideration of a single unit cell or periodic structures in order to reduce the computation time. The mathematical approach allows for an easy consideration of perturbations in the fibre arrangement. Results obtained from both approaches are presented and compared. The numerical model is shown in Fig. 3. In order to account for the orthotropic behaviour of a fibre reinforced composite, the thermal conductivity will be determined in three different directions. First within the analytical and mathematical approach, the thermal conductivity λ_1 parallel to the x_1-axis (according to the symmetry equivalent to λ_2) will be investigated. Furthermore, the FE approach allows for the determination of the conductivity λ_3 in the direction of the fibres (parallel to the x_3-axis) and in special cases ($\lambda_{\text{fib}} = 0$) the conductivity λ_{12} in the direction of the bisecting line of the x_1- and x_2-axis.

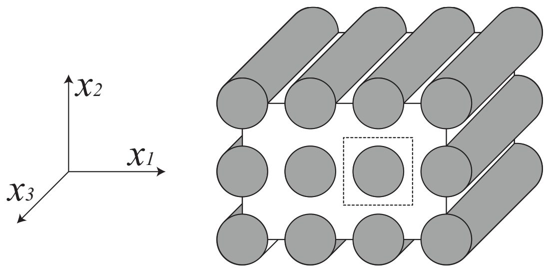

Figure 2: Idealised model of a fibre reinforced composite.

2 FE analysis

The numerical investigation is performed with the commercial finite element (FE) code MSC.Marc. The basic idea of the finite element method is the decomposition of a domain with a complicated geometry into geometrically simple elements, such that the governing differential equation can be solved (approximately) for these finite elements. The single element solutions are then assembled to obtain the complete system solution using given boundary conditions. The assembly process uses appropriate balance equations at the nodes which are used to define the elements and serve also as connection points between the elements. In the scope of this study, 2D structures equivalent to the cross sectional area of the illustrated composite material (cf. Figures 2 and 3a) and 3D models, generated by extruding the 2D model along the x_3-axis, (cf. Fig. 3b) are investigated.

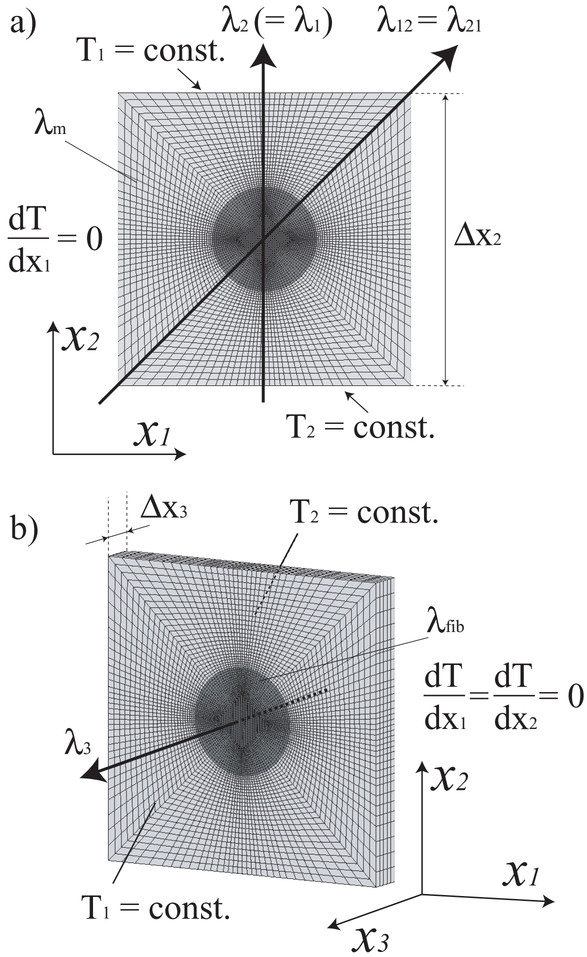

Figure 3: Finite element models with boundary conditions of the fibre reinforced material.

The volume fraction of the reinforcements f_{fib} is varied by modifying of the pitch of the inclusions inside the x_1-x_2-plane, whereas their size (= diameter) remains constant. Since the available computer hardware limits the number of degrees of freedom, not the whole 2D respectively 3D structure can be meshed. Alternatively, only one unit cell is modelled and specific symmetry boundary conditions (2D: $\mathrm{d}T/\mathrm{d}x_1 = 0$, 3D: $\mathrm{d}T/\mathrm{d}x_1 = \mathrm{d}T/\mathrm{d}x_2 = 0$) are introduced (cf. Fig. 3) in order to simulate the influence of the adjacent cells. Corresponding to these symmetric boundary conditions, all FE models describe the thermal behaviour of an infinite structure, where the influence of a free boundary is disregarded. This homogenisation is accurate for structures that consist of more than 10 to 15 units cells in every direction. The number of nodes for the different meshes are summarised in Tab. 1.

Table 1: Numbers of nodes of the applied finite element models.

Volume fraction f_{fib}	2D model	3D model
0.3	13953	83718
0.5	4161	24966
0.7	4673	23365
0.9	6209	37254

The heat conductivities of the components of the investigated composites are listed in Tab. 2:

Table 2: Thermal conductivities of the composite componentes in W / (m · K); [3, 4].

Matrix material λ_{m}	Fibre material λ_{fib}		
Polyphenylene sulfide	Carbon	Glass	Aramid
0.2	190	1.04	0.04

According to Fourier's law, the area-related conductivity is defined by

$$\lambda = \frac{\dot{Q}}{A} \cdot \frac{\triangle x_i}{\triangle T} \ . \qquad i = 2, 3 \tag{1}$$

Since the cross sectional area $A = \triangle x_2^2$ and the spatial distance $\triangle x_i$ $(i = 2, 3)$ are defined by the geometry, respectively the gradient $\triangle T = T_2 - T_1$ by the boundary conditions (cf. Fig. 3), only the heat flux $\dot{Q}$ remains to be determined. This is done by summing up all nodal values k of the 'reaction heat flux' at the top or bottom line (2D) or face (3D) where a temperature boundary condition is prescribed:

$$\dot{Q} = \sum_k \dot{Q}_k \ . \tag{2}$$

3 Analytical solution

A 2D composite material with periodically situated cells including periodic and perturbed arrays of non-overlapping circular reinforcements is considered within the mathematical approach. To study the thermal conductivity λ_1 of such composite materials, it is sufficient to investigate one of its periodic cells in order to represent the whole material [5, 6].

In the framework of this approach, conductive properties of composite materials can be described in terms of conjugation conditions $u^+ = u^-$, $\lambda_{\text{m}} \frac{\partial u^+}{\partial n} = \lambda_{\text{fib}} \frac{\partial u^-}{\partial n}$ on the boundary of the reinforcements ∂D_k with respect to the function $u(x, y)$ (e.g. the temperature) sectionally harmonic in D^+ and D^- [5, 6]. These conjugation conditions correspond to a perfect contact between different materials. Here, D_k are simply connected domains modelling reinforcements of conductivity λ_{fib} in the matrix material D^+ of conductivity λ_{m}, $D^- := \bigcup_{k=1}^{N} D_k$.
If the field is potential, i.e. the function $u(x, y)$ satisfies the Laplace equation $\nabla^2 u = 0$ in a domain D, then one can introduce the function $\varphi(z) = u(z) + \imath v(z)$, $z = x + \imath y$, $\imath^2 = -1$, analytic in D which is called the complex potential and reformulate the problem in terms of this potential.

The effective conductivity of macroscopically isotropic composite materials with small concentration ν of nonrandom reinforcements (which is equal to the volume fraction f_{fib}) is described by classical Clausius-Mossotti formula (for history and application see [7])

$$\lambda_1 = \frac{1 + \mu f_{\text{fib}}}{1 - \mu f_{\text{fib}}} \cdot \lambda_{\text{m}} \ , \tag{3}$$

where $0 < \lambda_{\text{m}} < +\infty$ is the conductivity of the matrix and $\mu = \frac{\lambda_{\text{fib}} - \lambda_{\text{m}}}{\lambda_{\text{fib}} + \lambda_{\text{m}}} \in [-1, 1]$ is the contrast parameter which expresses the difference between the conductivity of both materials.
The generalisation of this formula for quasi periodic composites with random reinforcements of conductivity $\lambda_{\text{fib}} > 0$ is obtained in [5] as:

$$\lambda_1 = \left(1 + \frac{2\mu f_{\text{fib}}}{N} \sum_{i=1}^{N} \psi_i(a_i) \right) \cdot \lambda_{\text{m}} \ . \tag{4}$$

Variables r, N, a_i $(i = 1, \ldots, N)$ are the radius, the number and the centres of the reinforcements, respectively, $\psi_i(z)$ are the derivatives of complex potentials in the cell. The sum on the right hand side of Eq. (4) is reduced to a power series approximation with respect to f_{fib}:

$$\lambda_1 = \left\{ 1 + 2\mu f_{\text{fib}} [A_0 + A_1 f_{\text{fib}} + A_2 f_{\text{fib}}^2 + ...] \right\} \cdot \lambda_{\text{m}} \ . \tag{5}$$

The coefficients A_p can be found for square and pseudofractal arrays of reinforcements in [5, 8]. A square array of reinforcements refers to their distribution for which the centres form a square. The effect of an increasing conductivity due to small perturbations of reinforcements having higher conductivity than the matrix (e.g. carbon or glass fibre reinforcements) is verified [5, 8]. A small perturbation means that each disk lies in the prescribed part of the original cell and does not cross or touch the boundary of corresponding parts. Thus, a periodic array guarantees the minimum of the effective conductivity.

4 Results

In the following, the results of the finite element and analytical approach are discussed. Figure 4 visualises the thermal conductivity λ_1 in the 1-direction (equal to λ_2) in dependence on the volume fraction f_{fib}. The results have been independently obtained within the analytical and mathematical approach and show good correlation.

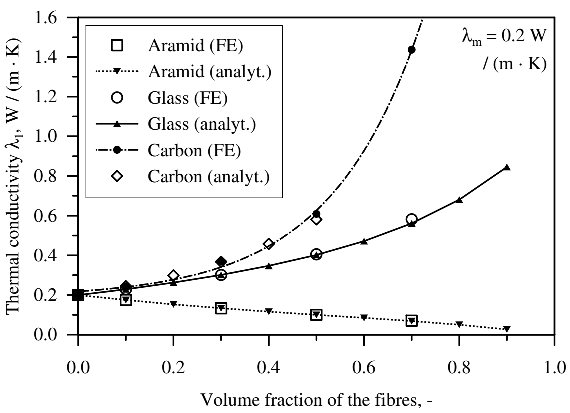

Figure 4: Thermal conductivity λ_1 in dependence on the volume fraction f_{fib}.

For each fibre material, different slopes can be observed. In the case of aramid fibres (Kevlar), $\lambda_{\text{fib}} < \lambda_{\text{m}}$ is valid and consequently the effective thermal conductivity of the composite decreases with rising volume fraction f_{fib}. The thermal conductivity of the glass and carbon fibres exceeds the thermal conductivity of the matrix ($\lambda_{\text{fib}} > \lambda_{\text{m}}$). Thus, a distinct increase of λ_1 with increasing f_{fib} is visible. However, compared to the high thermal conductivity of the reinforcements (e.g. carbon: 190 W / (m · K)), the increase in the relevant area of volume fractions f_{fib} is very low. According to earlier investigations on porous materials [9], where the thermal conductivity of the circular inclusions λ_{fib} was set equal to zero, the orientation of the inclusions in the x_1-x_2-plane shows no significant effect on the thermal conductivity of the structure and therefore similar results for λ_{12} as for λ_1 can be expected.

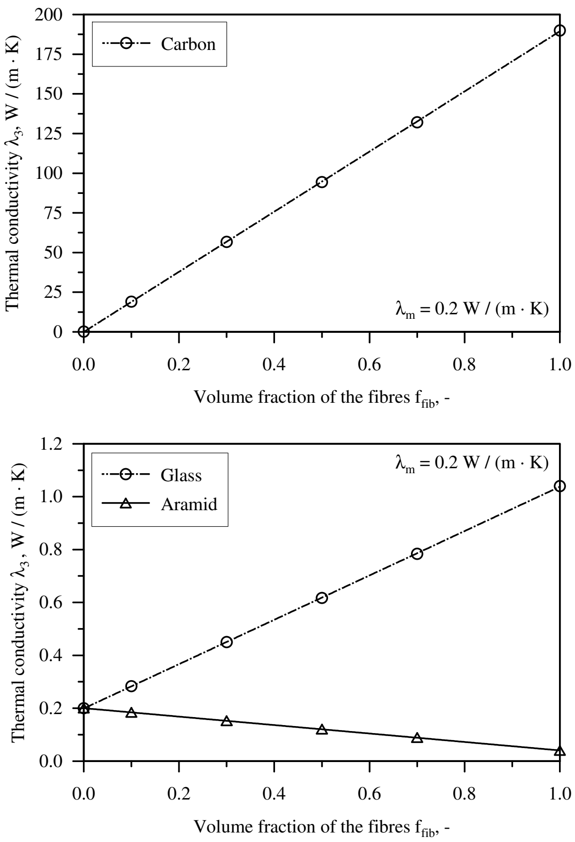

Figure 5: Finite element results of the thermal conductivity λ_3 in dependence on the volume fraction f_{fib}.

The thermal conductivities λ_3 in the 3-direction have been exclusively obtained in the scope of the FE approach. The material parameter shows a linear behaviour in dependence on the volume fraction f_{fib} (cf. Fig. 5). The slope can be analytically described in dependence on the volume fraction of the fibres and thermal conductivities of the components based on a classical Voigt mixing rule:

$$\lambda_3 = (1 - f_{\text{fib}}) \cdot \lambda_{\text{m}} + f_{\text{fib}} \cdot \lambda_{\text{fib}} \, . \tag{6}$$

As an indicator for the orthotropic heat transfer properties of the fibre reinforced material, the quotient $\chi = \lambda_3/\lambda_1 = \lambda_3/\lambda_2$ is defined (cf. Fig. 6) for the results of the finite element analysis. If $f_{\text{fib}} = 0$ the structure is isotropic and the quotient consequently equal to 1. With increasing volume fraction f_{fib}, the orthotropic behaviour and therefore χ increases. In general, it can be observed that the conductivity parallel to the fibres λ_3 exceeds the conductivities perpendicular to this direction $\lambda_1 = \lambda_2$, since $\lambda_3/\lambda_1 > 1$. According to the fibre material, different slopes can be observed. For carbon or glass fibre reinforcements ($\lambda_{\text{fib}} > \lambda_{\text{m}}$), the ratio χ decreases after reaching a maximum in the area of $f_{\text{fib}} \approx 0.4$. In the case of the low thermal conductive aramid reinforcements, a continuous rise of the orthotropic indicator χ with increasing f_{fib} is visible. However, for a theoretical value of $f_{\text{fib}} = 1$, the material is isotropic and therefore $\lambda_3/\lambda_1 \to 1$ is expected for all types of reinforcements. Consequently, an increasing deviation of the thermal conductivities of the components λ_{fib} and λ_{m}, expressed in the contrast parameter μ, seems to shift the maximum of the orthotropic parameter χ to lower values of f_{fib}.

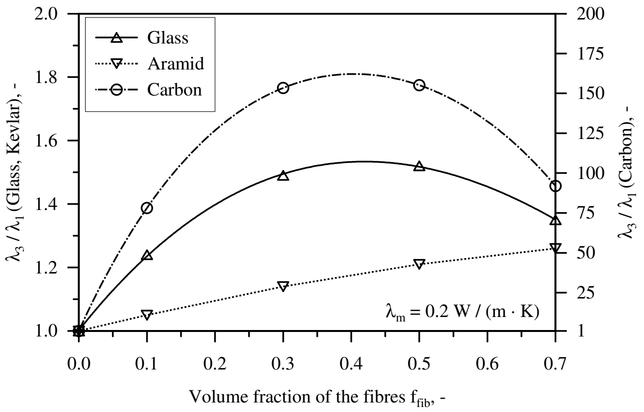

Figure 6: Finite elements results for the orthotropic behaviour of a fibre reinforced composite.

So far, all results refer to a periodic arrangement of the fibres in the matrix as visualised in Fig. 2. The mathematical approach also allows for the consideration of the effect of small perturbations of the position of the reinforcements on the thermal conductivity λ_1. If the contrast parameter μ is positive ($\lambda_{\text{fib}} > \lambda_{\text{m}}$), an increase of the thermal conductivity λ_1 can be observed. Tables 3 and 4 summarise the results for the case of a glass fibre, respectively carbon fibre reinforced composite.

Table 3: Effect of small perturbations on the thermal conductivity λ_1 of a glass fibre reinforced composite.

Vol. fract. f_{fib} -	Periodic $\frac{\text{W}}{\text{m}\cdot\text{K}}$	Perturbed $\frac{\text{W}}{\text{m}\cdot\text{K}}$	Deviation %
0	0.2	-	-
0.1	0.2290	0.2291	0.04
0.2	0.262	0.263	0.16
0.3	0.301	0.303	0.40
0.4	0.347	0.350	0.84
0.5	0.402	0.409	1.65

Table 4: Effect of small perturbations on the thermal conductivity λ_1 of a carbon fibre reinforced composite.

Vol. fract. f_{fib} -	Periodic $\frac{\text{W}}{\text{m}\cdot\text{K}}$	Perturbed $\frac{\text{W}}{\text{m}\cdot\text{K}}$	Deviation %
0	0.2	-	-
0.1	0.2442	0.2444	0.08
0.2	0.299	0.300	0.36
0.3	0.368	0.371	0.95
0.4	0.458	0.468	2.10
0.5	0.583	0.607	4.15

A maximum increase of 1.65% (glass fibres), respectively 4.15% (carbon fibres), is observed for the volume fraction $f_{\text{fib}} = 0.5$. A further increase of the deviation can be expected for higher values of f_{fib}. However, for high contrast parameters μ and volume fractions f_{fib}, more coefficients A_p (cf. Eq. (5)) need to be determined in order to obtain accurate results. This necessity leads to a significant increase of the required computing time and for volume fractions $f_{\text{fib}} > 0.5$, the hardware requirements exceed the available resources.

5 Conclusions

The thermal conductivity λ_1 which is according to the symmetry equal to λ_2 has been determined within the numerical and analytical approach and an excellent correlation of the results was found. Furthermore, the thermal conductivity λ_3 in fibre direction was obtained in the scope of a 3D finite element analysis. A classical analytic equation describes this material parameter in dependence on the volume fraction f_{fib}. In order to quantify the orthotropic behaviour, the quotient χ was introduced. Depending on the thermal conductivities of the reinforcements, an increase of χ was observed with increasing contrast parameter μ. Finally, an increase of the thermal conductivities λ_1, λ_2 could be shown for small perturbations of the periodic arrangement of the reinforcements in the case of a positive contrast parameter μ.

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