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DISCUSSION

A failure surface for circular footings on cohesive soils

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C. T. Chatzigogos, *Géodynamique et Structure, Bagneux, France*
A. Pecker, *Géodynamique et Structure, Bagneux, France*
J. Salençon, *École Polytechnique, Palaiseau, France*

The authors are to be commended for their thorough set of numerical calculations for the approximation of the failure locus of a circular footing resting on a purely homogeneous cohesive soil (Taiebat & Carter, 2010), and for revisiting the expression for the inclination factor ζ_i, used in the methodology proposed by Vesic (1973).

The purpose of this discussion is to show how their results can be placed within the framework of yield design theory, and corroborate it.

As correctly pointed out by the authors, the very concept of a limit load in the case of a single loading parameter, or of a yield surface in the case of multi-parameter loading, can be proven only under restrictive theoretical conditions within the framework of the theory of elasto-plasticity. Let these conditions be recalled: under the assumptions of linear elasticity (both physical and geometrical) and of no geometry changes during the elasto-plastic loading path, if the perfectly plastic flow rule obeys the normality conditions, then a yield surface exists such that, whatever the loading path followed, the considered structure will stand any load inside the yield surface, and collapse at the yield surface. This yield surface can then be determined straightforwardly through the two theorems of classical limit analysis.

Usually, assuming all other conditions to be satisfied, the attention is focused on fulfilment of the latter condition, namely that of the associated flow rule. If this condition is not satisfied, counter-examples have been given showing that applying the limit load theorems in what may seem their intuitive wording leads to contradictory results (Salençon, 1973, 1974). Various authors (e.g. Radenkovic, 1961; Palmer, 1966; Salençon, 1977) have endeavoured to produce extended limit load theorems in the case of a non-associated flow rule, with poor results as regards an extended lower-bound theorem. As regards the upper-bound theorem, the results have been clarified through the theory of yield design (Salençon, 1990, 2002), which states that, based upon the only condition of mathematical compatibility between the equilibrium equations and the plastic criteria of the materials, a convex boundary can be determined, which is the locus of the ultimate loads of the system under consideration. Using the associated flow rule as a mathematical tool, this boundary can be approximated from outside. From its very definition this ultimate boundary stands, whatever the actual flow rule and the elastic constitutive law, as an upper bound to any loading path, which cannot be passed through. The main difference from the ideal case of the associated flow rule lies in the assessment of the practical relevance of this upper bound, depending on the loading paths and on the actual flow rule.

In the case of the problem considered in the paper, such upper bounds have been established in the works of Chatzigogos *et al.* (2007) and Salençon *et al.* (2009), within a wider context relevant to earthquake engineering, in which the soil heterogeneity and the effect of seismic inertia forces in the soil are also taken into consideration. Comparing the results of the paper with these upper bounds proves quite relevant, although it was not possible, from the published charts, to perform an exhaustive comparison of the two sets of results. From what could be achieved, it turns out that the results published in the paper as limit loads on the considered loading paths fit inside the convex upper bound derived from yield design theory, and are close to this convex upper bound.

Both statements are significant. On the one hand the numerical method used in the paper is checked as regards the qualitative characterisation of the obtained results; on the other hand the practical relevance of the rigorous upper bounds obtained through yield design theory is corroborated, which may give some confidence in the potential collapse mechanisms used to determine them.

Authors' reply

The authors are grateful to the discussers for drawing attention to their independent and significant contribution to this topic, and for their useful comments on the level of agreement between our two solutions to this important problem of estimating the ultimate capacity of a circular footing under combined loading.

It was gratifying and reassuring to learn from the discussion that the solutions obtained using the upper-bound theorem of plasticity (or the yield design approach) by Chatzigogos *et al.* (2007) and Salençon *et al.* (2009) are close to those obtained using our displacement finite-element approach. We note that the discussers have also included in their work solutions for an inhomogeneous soil medium, in which the strength increases linearly with depth, a case not considered in our paper.

We also note in particular the deformation mechanisms assumed in the convex upper-bound calculations presented by Chatzigogos *et al.* (2007) and Salençon *et al.* (2009). These mechanisms were assumed to produce mainly unilateral and planar velocity fields. Our finite-element analysis shows complete 3D velocity fields, even for cases of footings under large load eccentricities. For example, Fig. 21 shows the velocity fields close to the point of collapse for four points on the V–M failure line, presented on the vertical plane of symmetry and a horizontal section close to the ground. It also shows the contact area of the footing at failure. Fig. 21 reveals that when the eccentricity is zero, the velocity field is totally axisymmetric, as expected. As the eccentricity of load increases, the velocity fields tend to include greater components of lateral movement, but none of the mechanisms considered here becomes completely planar or unilateral.

Despite the differences between the mechanisms considered in the upper-bound solutions of the discussers and those observed in the finite-element analysis, the two techniques provided solutions that are very close, especially for footings under large eccentricities.

We reiterate our thanks to the discussers for bringing these matters to our attention, and we again acknowledge the power and elegance of yield design theory.

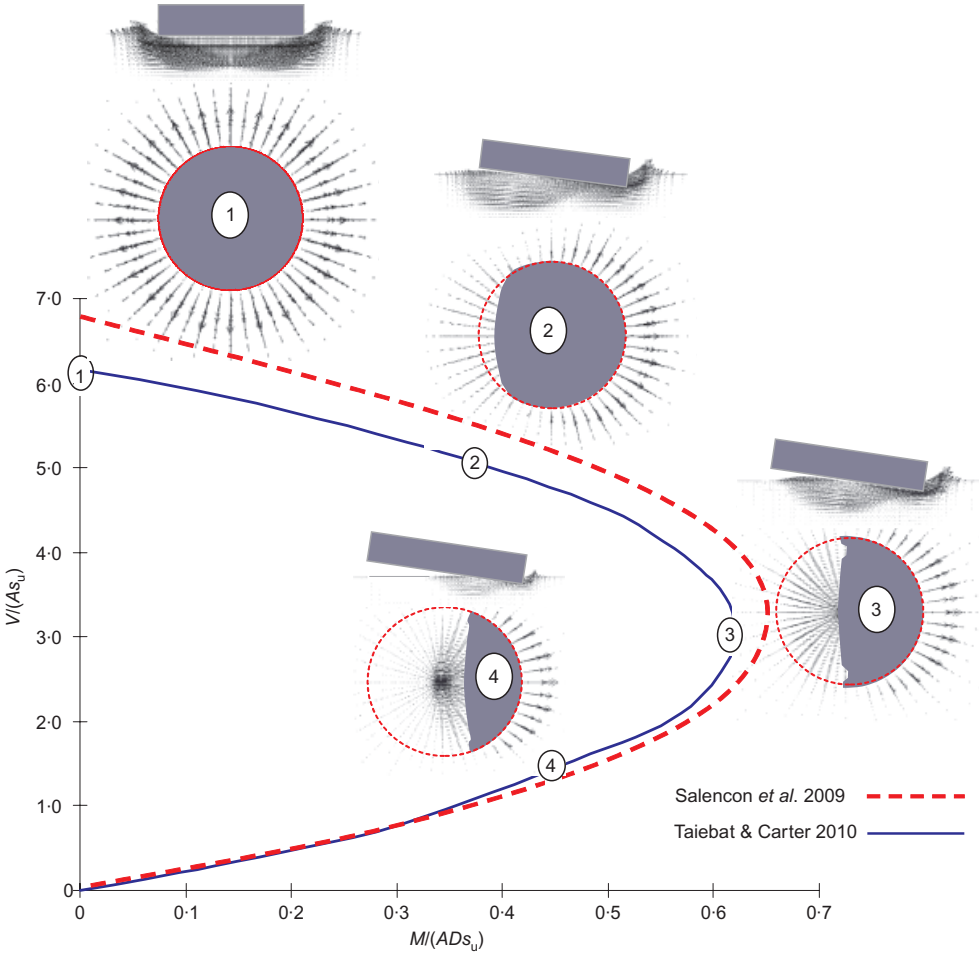

Fig. 21. Velocity fields for four points on the V–M failure line

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