Taiebat, H. A. & Carter, J. P. (2002). *Géotechnique* **52**, No. 1, 61–64

TECHNICAL NOTE

Bearing capacity of strip and circular foundations on undrained clay subjected to eccentric loads

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KEYWORDS: bearing capacity; clays; foundations; numerical modelling and analysis.

INTRODUCTION

The bearing capacity of foundations has always been a subject of major interest in soil mechanics and foundation engineering. There is extensive literature dealing with this topic, from both the theoretical and experimental standpoints. A list of principal contributions to this subject can be found in Vesic (1973), Chen & McCarron (1991), and Tani & Craig (1995). Recently, Houlsby & Purzin (1999) investigated possible lower-bound and upper-bound solutions to the problem of the bearing capacity of a strip footing using a scaling procedure. A series of failure loci were presented, which include both lower-bound and upper-bound solutions. It was found that no exact solution exists for the general cases of vertical moment and horizontal loading. Because the normality condition is violated at the interface between the footing and the soil, a unique failure surface may no longer be defined, and the upper-bound or lower-bound theorems may not be valid.

The objective of the current studies is to determine the shape of the failure locus in (V, M) space using the results of a finite element study of this problem. Both strip and circular footings are considered. The (V, M) load case is significant, as it also corresponds to footing problems in which the vertical load is eccentrically applied.

DEFINITION OF THE PROBLEM

Finite element modelling of the problem of the bearing capacity of strip and circular footings under vertical load and moment is described in this section. The footings rest on the surface of a uniform homogeneous soil that deforms under undrained conditions. The soil is assumed to obey the Tresca failure criterion. It has a uniform undrained shear strength s_u and an undrained Young's modulus, $E_u = 300\, s_u$. A Poisson's ratio of $\nu \approx 0{\cdot}5$ (in practice $\nu = 0{\cdot}49$ to avoid numerical difficulties) was assumed for the soil to model the constant-volume elastic response of the soil under undrained conditions. The Young's modulus for the foundations was set as $E_f = 1000 E_u$: that is, the foundations are much stiffer than the soil, and therefore they can be considered as effectively rigid.

The finite element code AFENA (Carter & Balaam, 1995) was employed for the analyses of the foundations. The strip footing was modelled using isoparametric quadrilateral plane strain elements in both load-controlled and displacement-controlled analyses. The circular footing was modelled using three-dimensional wedge elements, which are equivalent to isoparametric brick elements. The analysis using wedge elements was formulated following the 'semi-analytical' approach in finite element modelling described by Zienkiewicz & Taylor (1989). Details of this semi-analytical solution method may be found in Taiebat (1999) and Taiebat & Carter (2001).

Manuscript received 26 April 2001; revised manuscript accepted 1 November 2001.
Discussion on this paper closes 1 August 2002, for further details see inside back cover.
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The contact between the footings and the soil is unable to sustain tension. A thin layer of 'no-tension' elements was used under the foundation to model the interface. The separation of the foundation and the soil is signalled by the occurrence of tensile vertical stress in the interface elements. Immediately after the separation no shear stress can be sustained in the interface elements.

The geometries of the problems considered, as well as the detailed dimensions of the elements in the near-field zone, are presented in Fig. 1. The breadth on the strip footing and the diameter of the circular footing are B and D respectively.

FINITE ELEMENT RESULTS

A series of finite element analyses of circular and strip footings were performed to investigate the shape of the failure envelope for the footings in the (V, M) space. The predicted failure envelopes are compared with the solutions obtained using the lower- and upper-bound theorems of plasticity. The lower-bound solutions used here satisfy equilibrium and do not violate the yield criterion. However, some of the solutions may not adhere strictly to all requirements of the lower-bound theorem. For example, loss of contact at the footing–soil interface implies that the normality principle is not always obeyed. Therefore the term *apparent lower bound* is used for these solutions, as suggested by Houlsby & Purzin (1999). The equation for the apparent lower-bound solutions proposed by Houlsby & Purzin (1999) in V–M loading space is similar to the equation derived from the application of the effective width method, which is commonly used in the analysis of foundations subjected to eccentric loading (e.g. Meyerhof, 1953; Vesic, 1973). In this method, the eccentric bearing capacity of a foundation is assumed to be equivalent to the vertical bearing capacity of another foundation with a fictitious effective area on which the load is centrally applied.

Two-dimensional analyses

The failure envelope predicted by the two-dimensional finite element analyses for a strip footing under both vertical load and moment is presented in Fig. 2. Also shown in this figure are the failure envelopes resulting from the apparent lower-bound and apparent upper-bound solutions suggested by Houlsby & Purzin (1999). The apparent lower-bound envelope suggested by Houlsby & Purzin (1999) can be obtained from

$$\frac{V}{A} = (\pi + 2)\left(1 - \frac{2M}{VB}\right)s_u \tag{1}$$

where V is the vertical load, A is the area of the foundation, and M is the moment applied to the foundation.

The load-controlled finite element method of analysis is unable to predict the failure load when the ratio M/V is high. However, the displacement-controlled method of analysis provides a failure load for all conditions. The predicted failure points resulting from both methods of analysis are entirely consistent to a high degree of accuracy. The deformed shape of the soil and the strip foundation under an eccentric load is shown in Fig. 3.

The predicted failure envelope is very close to the apparent upper-bound solutions and also is in good agreement with the

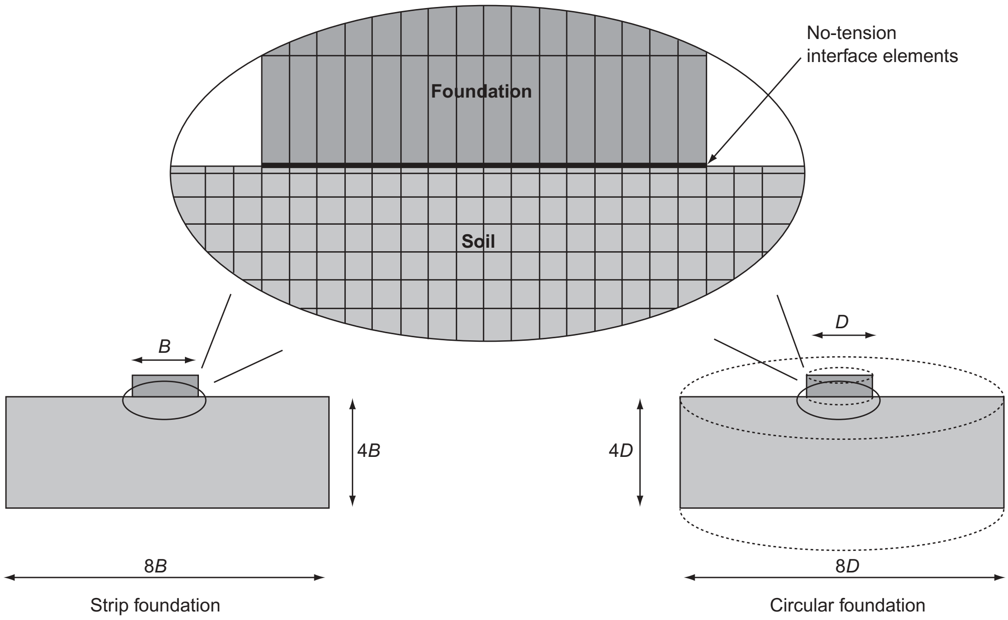

Fig. 1. Geometry of the finite element mesh and details of the mesh in the near field

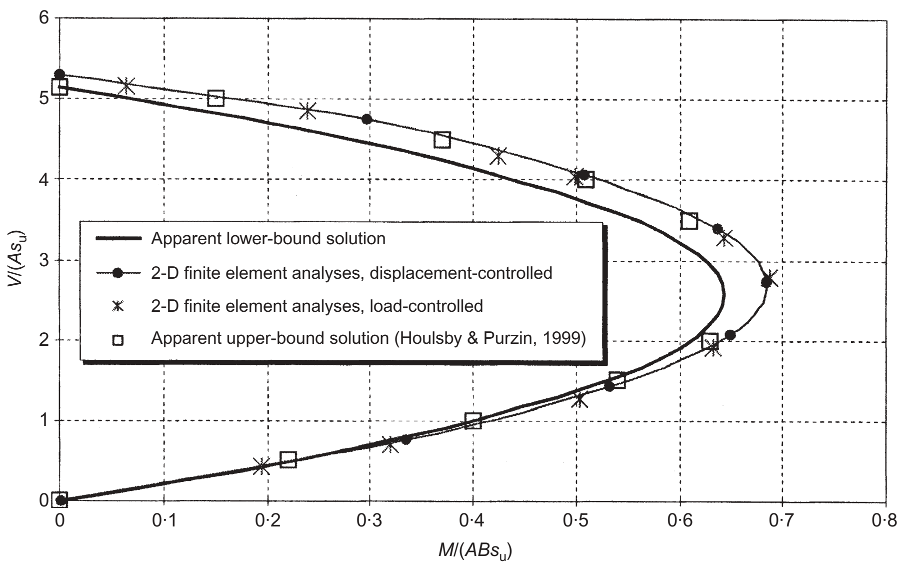

Fig. 2. Failure loci for strip footing under eccentric loading

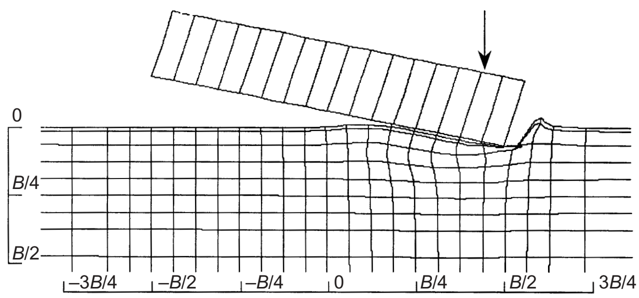

Fig. 3. Deformed shape of the soil and the strip footing under an eccentric load

failure envelope obtained from the apparent lower-bound solutions for most of the loading range.

Three-dimensional analyses

The failure envelope predicted by the three-dimensional finite element analyses for circular footings under both vertical load and moment is presented in Fig. 4. The failure envelope obtained from the apparent lower-bound solutions is also presented in the same figure. For three-dimensional conditions, the apparent lower-bound solutions have been obtained based on the following considerations.

A circular foundation subjected to a vertical load applied with an eccentricity $e = M/V$ can be regarded as an equivalent fictitious foundation with a centrally applied load (Fig. 5), as suggested by Meyerhof (1953) and Vesic (1973). In this case, the area of the fictitious foundation, A', can be calculated as

$$A' = \frac{D^2}{2}\left(\arccos\frac{2e}{D} - \frac{2e}{D}\sqrt{1-\left(\frac{2e}{D}\right)^2}\right) \tag{2}$$

The aspect ratio of the equivalent rectangular area can also be approximated as the ratio of the line lengths b to l, as shown in Fig. 5: that is,

$$\frac{B'}{L'} = \frac{b}{l} = \sqrt{\frac{D-2e}{D+2e}} \tag{3}$$

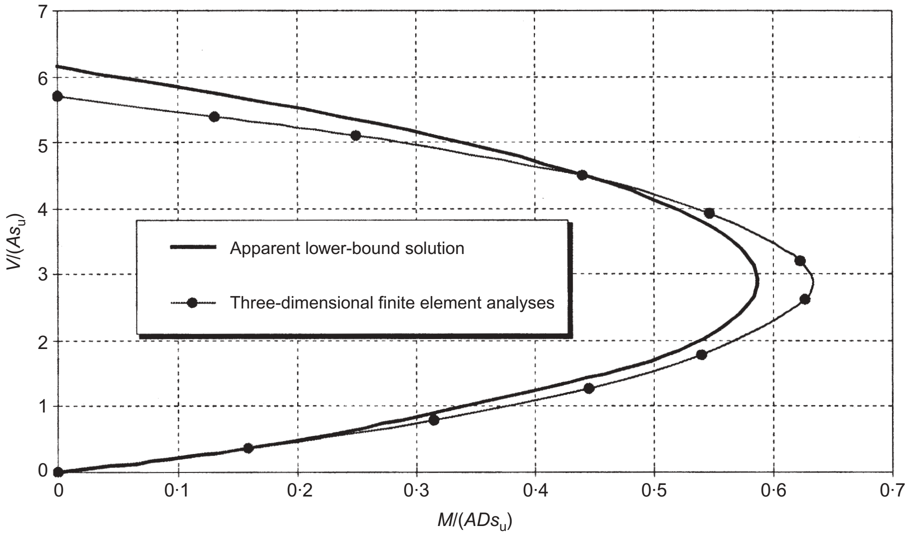

Fig. 4. Failure loci for circular footing under eccentric loading

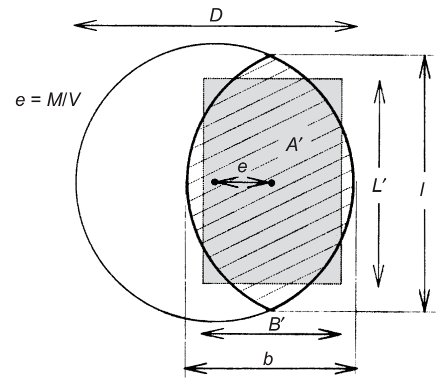

Fig. 5. Effective area of circular footings subjected to eccentric load

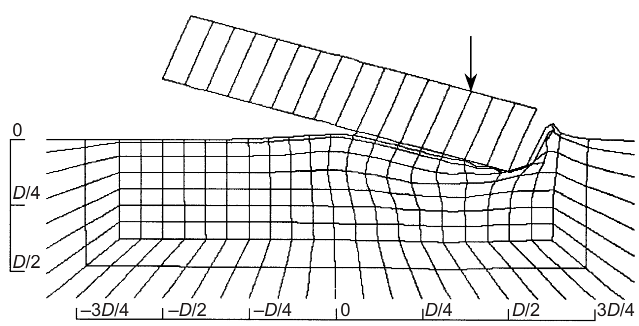

Fig. 6. Deformed shape of the soil and the circular footing under an eccentric load

Therefore, in this case the shape factor for the fictitious rectangular foundation is given by (Vesic, 1973)

$$\xi_s = 1 + 0{\cdot}2\sqrt{\frac{DV - 2M}{DV + 2M}} \quad (4)$$

Hence the bearing capacity of circular foundations subjected to eccentric loading can be obtained from the effective width method as

$$V = \xi_s A'(2 + \pi)s_u \quad (5)$$

Note that based on Vesic's recommendation the shape factor for circular footings under the pure vertical load is predicted by equation (4) as $\xi_s = 1{\cdot}2$. However, exact solutions for the vertical bearing capacity of circular footings on uniform Tresca soil (Shield, 1955; Cox, 1961) suggest the ultimate bearing capacity of $5{\cdot}69\,As_u$ and $6{\cdot}05\,As_u$ for smooth and rough footings respectively. Therefore the appropriate shape factors are actually 1·11 and 1·18 for smooth and rough footings, so that equation (4) will give shape factors and hence bearing capacity values that are in error by about 2–9%. On the other hand, the ultimate load of $V = 5{\cdot}7\,As_u$ predicted by the finite element procedure described here is very close to the exact solution for a smooth footing. The thin layer of soil used under the foundation has effectively resulted in a smooth interface between the foundation and soil.

The failure envelope resulting from the effective width method is compared with the failure envelope predicted by the finite element analyses in Fig. 4. There is reasonable agreement between the two failure envelopes.

A picture of the deformed mesh of the soil and the foundation under the circular footing, in the plane of the applied moment, provided by the finite element analysis, is presented in Fig. 6.

SUMMARY

The failure envelopes for strip and circular footings subjected to vertical load and moment were obtained from finite element analyses and from the simple lower-bound solutions based on the effective width method. Derivation of the upper-bound solution for the problem of bearing capacity of foundations subjected to combined vertical load and moment is complex. However, comparison of the failure envelopes obtained in this study shows that the effective width method, commonly used in the analysis of foundations subjected to eccentric loading, provides good approximations to the collapse loads for these problems.

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