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Analysis of the remediation of a contaminated aquifer by a multi-well system

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Abstract

Various numerical techniques have been developed and used to design waste repositories such as landfills, in order to reduce the impact of contamination. However, even with good design, potential contamination such as groundwater contamination may still arise in the future due to unforeseen circumstances or negligence. Thus there is a need to seek efficient, cost-effective and carefully designed remediation strategies for the cleanup of contaminated groundwater. This paper presents a study of the remediation of a contaminated aquifer of uniform thickness by multi-well systems, which include both discharge wells and recharge wells. These investigations show that an appropriately designed pump and treat system (PAT) can have a significant effect on the decontamination of a polluted aquifer and can preclude the further spreading of a contaminant plume. However, if the system is not designed appropriately, it may cause a further serious spreading of the contamination. This possibility is illustrated by the examples presented in the paper, which highlight the need for care in the design of remediation strategies. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Remediation strategy; Multi-well system; Groundwater contamination

1. Introduction

Many numerical techniques (e.g. [1–9]), have been developed and can be used to design waste repositories such as landfills, in order to reduce the impact of contamination. However, even with the availability of good design tools, potential contamination such as groundwater contamination may still arise in the future due to unforeseen circumstances or negligence. There is a need to seek efficient, cost-effective and appropriately designed remediation strategies for the cleanup of contaminated

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groundwater. The need for care in the selection of the remediation strategy is illustrated by the examples presented in this paper.

Aquifer remediation by the pump and treat (PAT) method is the prescribed strategy at approximately two thirds of the existing 1200 Superfund sites [10] as well as numerous other sites of groundwater contamination, and it appears to be the only available method for the remediation of deep aquifers. Generally, a common PAT method of aquifer cleanup is to extract the polluted groundwater and after reducing the concentration of contaminants in the water to an acceptable level, the treated water is either reinjected into the aquifer or, if it is permitted and feasible, it is released to a surface water body [11]. The treatment aspect in the PAT system is important, because without treatment the cleanup action would be just a way of transferring contaminants to some other location.

It is found that in-situ treatments commonly provide the opportunity to greatly reduce cleanup costs [12]. A detailed discussion of the treatment aspect in a PAT system is beyond the scope of this paper. However, it will be shown how the effect of a PAT system can be modelled by a Laplace Transform Finite Element Method (LTFEM) [9]. Investigations of different remediation strategies, using a multiple-well system to remove contaminants from a polluted confined aquifer of uniform thickness, will be discussed.

2. Numerical modelling

2.1. Governing equation

For the case of steady, incompressible and saturated groundwater flow, the governing equation of contaminant transport problems in soil and rock in which advection, dispersion-diffusion, adsorption and decay processes take place is given by:

$$\nabla \bullet (n\mathbf{D} \bullet \nabla c) - n\mathbf{V} \bullet \nabla c = nR_d \left(\frac{\partial c}{\partial t} + \lambda^* c \right) \quad (1)$$

in which t is time, c is the concentration of contaminant, \mathbf{D} is the tensor of hydrodynamic dispersion coefficients, \mathbf{V} is the vector of pore water velocity, n is the porosity of the soil, R_d is the retardation factor ($R_d = 1 + \rho K_d/n$) where ρ is the dry density of the soil and K_d is the partitioning or distribution coefficient, and $\lambda^* = \lambda/R_d$, where λ is the decay constant.

2.2. Laplace transformation

In the LTFEM, a Laplace transform with respect to time is applied to the governing equation and its associated boundary conditions before the implementation of the finite element scheme. After applying the Laplace transform to the governing equation and making use of the basic operational property to eliminate the time derivative, the transformed governing equation in Laplace transform space is given by:

$$\nabla \bullet (n\mathbf{D} \bullet \nabla \bar{c}) - n\mathbf{V} \bullet \nabla \bar{c} = nR_d[(s + \lambda^*)\bar{c} - c_0] \tag{2}$$

in which \bar{c} is the Laplace transform of concentration; $\bar{c} = \int_0^\infty ce^{-st} dt$, where s is the Laplace transform parameter, and c_0 is the initial concentration.

2.3. The finite element scheme in Laplace transform space

The spatial variation of the Laplace transform of the contaminant concentration may be approximated using a finite element discretisation in the spatial domain. The Galerkin finite element method may be used to obtain the finite element approximation of the transformed transient mass transport equation (2) and its associated boundary and initial conditions [13] This leads to a system of linear equations in which the unknowns are the concentrations at a finite number of nodal points and which may be cast in matrix form as

$$[\mathbf{M}]\{\bar{\mathbf{c}}\} = \{\mathbf{F}_i\} + \{\mathbf{F}_f\} \tag{3}$$

in which $[\mathbf{M}]$ is the global matrix including advection–dispersion, sorption and decay matrices which may be obtained by combining the individual element matrices, $\{\bar{\mathbf{c}}\}$ is a vector containing the unknown nodal concentrations in Laplace transform space, $\{\mathbf{F}_i\}$ is a vector which depends on the known initial conditions, $\{\mathbf{F}_f\}$ is a vector due to the prescribed flux (f_n), in which f_n is the specified outward normal flux along the boundaries.

2.4. Numerical inversion of the Laplace transformation

Eq. (3) may be solved to obtain the values of the nodal transformed concentration for any particular value of the Laplace transform variable s . This may be written symbolically as

$$\{\bar{\mathbf{c}}(s)\} = [\mathbf{M}(s)]^{-1} \{\mathbf{F}(s)\} \tag{4}$$

The values of the nodal concentrations in the time domain are then obtained using Stehfest’s algorithm [14,15]. Specifically,

$$\{\mathbf{c}(t)\} = \sum_{i=1}^{N_s} W_i \{\bar{\mathbf{c}}(s_i)\} \tag{5}$$

in which $\{\mathbf{c}(t)\}$ is the vector of the concentration value at time t , $\{\bar{\mathbf{c}}(s_i)\}$ is the vector of the concentration in Laplace transform space, and s_i are sample points given by $s_i = (\ln 2/t) \times i$ in the Stehfest method. N_s must be an even number in Stehfest’s algorithm and W_i are the coefficients given by

$$W_i = \frac{\ln 2}{t} \bullet (-1)^{(N_s/2)+i} \bullet \sum_{k=j}^{\min(i, N_s/2)} \left[\frac{k^{N_s/2} (2k)!}{((N_s/2) - k)! k! (k - 1)! (2k - i)!} \right] \tag{6}$$

in which $j = (i + 1)/2$ if i is an odd number and $j = i/2$ if i is an even number.

For each value of time, Eq. (3) is solved for values of $s_i (i = 1, N_s)$ thus giving the values of $\{\tilde{c}(s_i)\}$. These values are then combined as described by Eq. (5) to obtain the solution in the time domain.

3. Test case of a radial dispersion

A case of dispersion (shown in Fig. 1) due to a steady diverging flow from an injection well is used to demonstrate how the LTFEM can be used to model transportation from a well. In this case a constant radial flow rate (Q) is considered and it is assumed that the water injected at the well has a constant concentration (c_0) of contaminant. This situation has been considered using either approximate or numerical analyses by various investigators [16–19].

The governing equation of radial transport in the presence of radial flow [20] may be written as

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r D_r \frac{\partial c}{\partial r} \right] - V_r \frac{\partial c}{\partial r} = \frac{\partial c}{\partial t} \quad (7)$$

where

- r is the radial distance from the centre of well,
- D_r is the dispersion coefficient,
- V_r is the seepage velocity ($V_r = Q/2\pi r h n$ in which Q is the steady radial flow rate, h is the thickness of the aquifer and n is porosity.).

For steady plane radial flow, rV_r remains constant and when it is assumed that molecular diffusion is negligible, it is found that

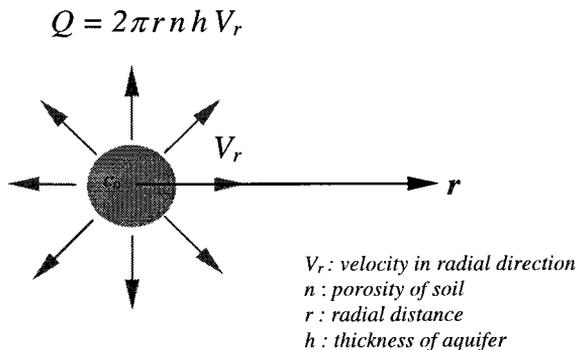


Fig. 1. A radial dispersion problem due to diverging flow from an injection well.

$$\alpha_L V_r \frac{\partial^2 c}{\partial r^2} - V_r \frac{\partial c}{\partial r} = \frac{\partial c}{\partial t} \tag{8}$$

in which α_L is the longitudinal dispersivity in radial flow.

If r_w represents the radius of the well, the initial and boundary conditions considered in this test case are

$$\begin{aligned} c &= 0 & \text{if } r > r_w & \text{ and } t = 0 \\ c &= c_0 & \text{if } r = r_w & \text{ and } t > 0 \\ c &\rightarrow 0 & \text{if } r \rightarrow \infty & \text{ and } t > 0 \end{aligned}$$

Introducing dimensionless parameters (r_D , t_D and c_D), the dimensionless governing equation becomes

$$\frac{1}{r_D} \frac{\partial^2 c_D}{\partial r_D^2} - \frac{1}{r_D} \frac{\partial c_D}{\partial r_D} = \frac{\partial c_D}{\partial t_D} \tag{9}$$

where,

$$\begin{aligned} c_D &= c/c_0 \text{ in which } c_0 \text{ is the concentration at the well,} \\ r_D &= r/\alpha_L \text{ in which } \alpha_L \text{ is the dispersivity,} \\ t_D &= Qt/(2\pi h n \alpha_L^2) \text{ in which } Q \text{ is the steady radial flow rate and } h \text{ is the uniform aquifer thickness.} \end{aligned}$$

The dimensionless initial and boundary conditions are therefore

$$\begin{aligned} c_D &= 0 & \text{if } r_D > r_{Dw} & \text{ and } t_D = 0 \\ c_D &= 1 & \text{if } r_D = r_{Dw} & \text{ and } t_D > 0 \\ c_D &\rightarrow 0 & \text{if } r_D \rightarrow \infty & \text{ and } t_D > 0 \end{aligned}$$

where,

$$\begin{aligned} r_D & \text{ is the dimensionless radial distance,} \\ r_{Dw} & \text{ is the dimensionless well radius.} \end{aligned}$$

The solution of these equations in Laplace transform space was given by Moench and Ogata [19], as

$$\bar{c}_D = \frac{1}{s} \exp\left[\frac{r_D - r_{Dw}}{2}\right] \frac{Ai(Y)}{Ai(Y_0)} \tag{10}$$

where,

$$\begin{aligned} s & \text{ is the Laplace transform variable,} \\ Ai & \text{ is the Airy function,} \\ Y & = s^{-2/3}(sr_D + 1/4), \\ Y_0 & = s^{-2/3}(sr_{Dw} + 1/4). \end{aligned}$$

Moench and Ogata [19] have used Stehfest's algorithm [14,15] to invert Eq. (10). Their results were in close agreement with the numerical solutions to this problem obtained independently by Hoopes and Harleman [18].

For the numerical simulation in this paper, it is only necessary to model a typical sector in the LTFEM since the problem being considered is radially symmetric. Thus a finite element mesh of 160 linear quadrilateral elements was used, as shown in Fig. 2. The constant concentration (c_o) was specified at the boundary nodes of the well and a no flux condition was assumed at the outer boundary which was placed at the position $r = 9$ m. Fig. 3 shows that the solutions obtained from the LTFEM are in close agreement with the solutions of Moench and Ogata [19]. Although the comparison is presented in terms of the dimensionless parameters (t_D and r_D), an examination was also made to confirm that no noticeable boundary effect on the solutions obtained by the LTFEM occurred due to the finite location of the outer boundary used during the time of interest.

4. A multiple-well system

The fundamental approach of a PAT system is to locate and operate the discharge well(s) at the most contaminated area in order to have the maximum impact. However if a single discharge well [9] is used for the remediation of the contaminated groundwater, environmental problems such as lowering of the groundwater level and the resultant settlement could be created by the long term operation of an overdrawn discharge well. Therefore the installation of recharge well(s) may be

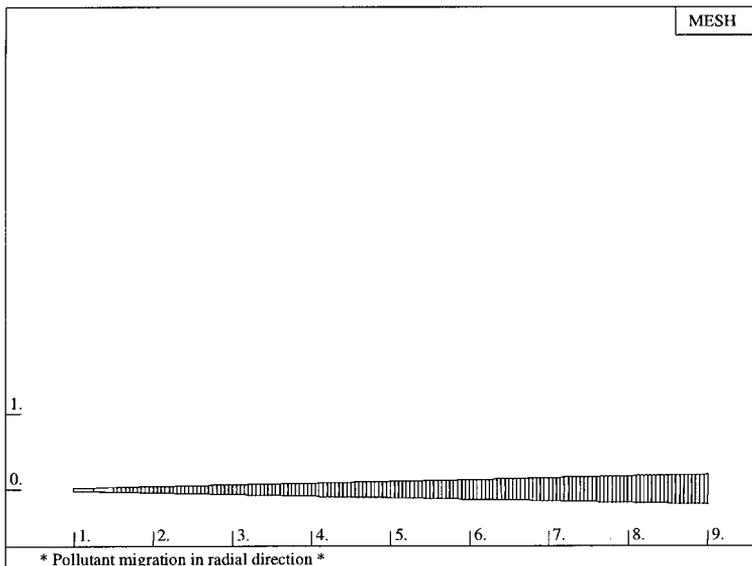


Fig. 2. Finite element mesh of the dispersion problem due to a radial flow.

necessary in a PAT system to avoid this happening. The installation of recharge well(s) can also be expected to provide an additional advantage by restraining the contaminant from entering the area near the recharge well and preventing further spreading of contaminants to unpolluted regions. In this section, the effects of recharge well(s) in a multi-well system are investigated.

4.1. Location of a recharge well in a two-well system

In order to understand the interaction of discharge and recharge wells, a two-well system including a discharge well and a recharge well, under the influence of a natural regional groundwater velocity of $V_{Ri} = 0.5$ m/h, will be examined. Fig. 4 shows the contaminated zone and locations of wells being considered. Both discharge and recharge wells are assumed to pump at a rate of $18 \text{ m}^3/\text{h}$. The uniform thickness of

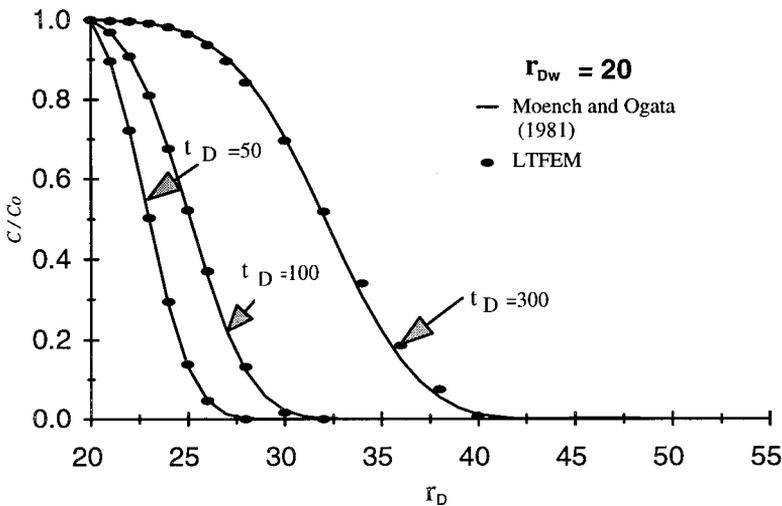


Fig. 3. Comparison of results of LTFEM and Moench and Ogata [19].

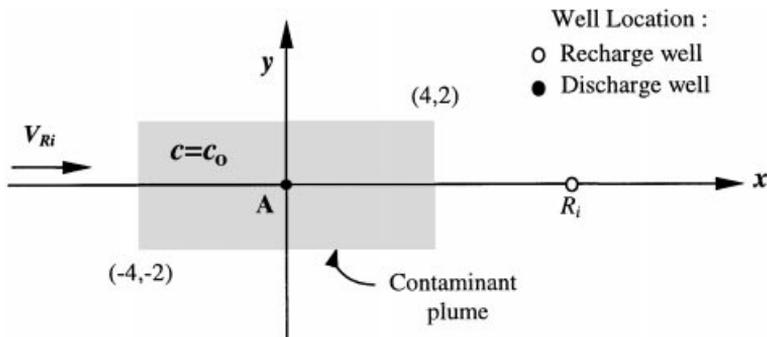


Fig. 4. Schematic of a two-well (discharge-recharge) system.

the aquifer is assumed to be 2 m and the radius of both discharge and recharge wells is 0.1 m. It is assumed that fresh water is injected at the recharge well.

The location of the discharge well (A) is assumed to be placed at the centre of the contaminated area, and since the uniform initial regional groundwater flow is assumed parallel to the x axis, the recharge well is placed on the downstream side of the contaminated area. Three different positions (R_1 , R_2 , and R_3) along the centre line (x axis) of the contaminated zone are considered and they are located at 6, 8 and 10 m away from the centre of the contaminated region, respectively.

The numerical simulation was conducted using the finite element mesh shown in Fig. 5. Fig. 6 shows a plan view of the half plane of streamlines of groundwater flow created by the combination of an initial natural regional groundwater flow ($V_{Ri} = 0.5$ m/h) and a two-well (discharge–recharge) system, in which the initial contaminated region is indicated by a rectangle and the recharge well (R_3) is located at the position which is 10 m away from the discharge well.

The variation of the mass of contaminant removed with time for these three cases ($A&R_1$, $A&R_2$ and A and R_3) is presented in Fig. 7, and the predictions along $y = 0$ m for these cases are also shown in Figs. 8 and 9. Inspecting both Figs. 8 and 9, it is found that some contaminant moves past the recharge well. This indicates the operation of the recharge well can not completely stop the contaminant transport. However the most effective remediation for the two-wells cases considered here is $A&R_3$. It is interesting to note that if the recharge well is placed too close to the contaminated area, as in the case of the two wells $A&R_1$, serious spreading of the contaminant will occur and the remedial well system will have difficulty in capturing

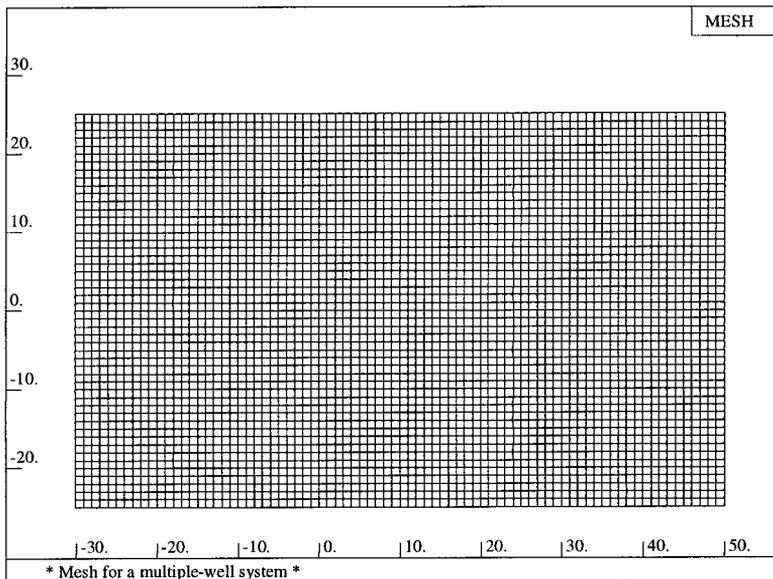


Fig. 5. Finite element mesh used to model a well system.

much of the contamination, even if it is in operation for a long time. This can be readily seen by examining Figs. 8 and 9.

In order to provide a general picture of the extent of the contamination, the concentration contours of the two-well case $A&R_3$ are presented in Figs. 10 and 11. The

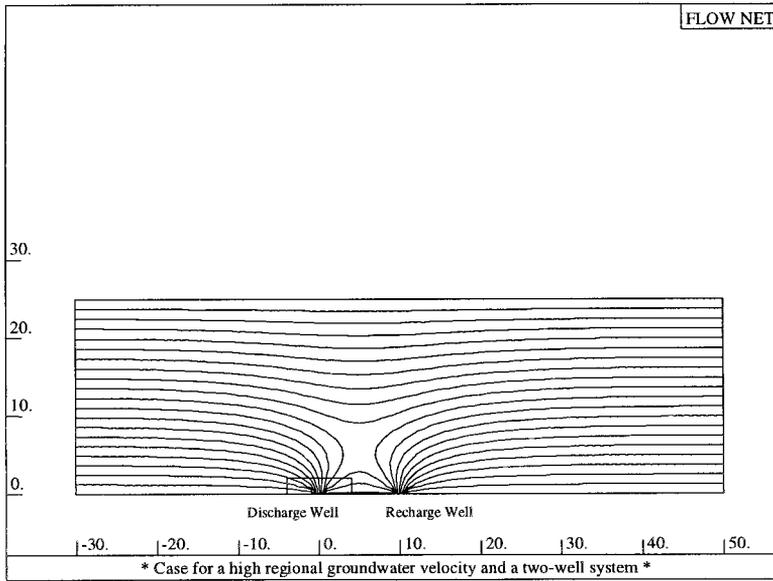


Fig. 6. Streamlines of groundwater flow created by a uniform natural regional groundwater flow and a two-well system.

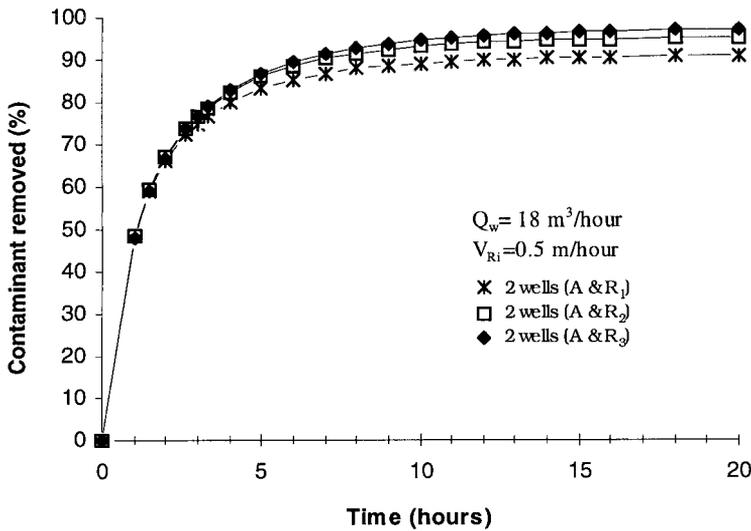


Fig. 7. Total removed mass of contaminant with time for the two-well (discharge-recharge) system.

concentration contours of the case of two wells $A&R_1$ are also presented in Fig. 12. The initial contaminated region is indicated by a rectangle while the recharge well is indicated by a small square in these figures. By inspecting Figs. 11 and 12, the disadvantage of a case where the recharge well is placed too close to the contaminant plume can be readily detected.

However, if the recharge well is placed at an appropriate location (R_3 in this case), further spreading of the contaminant plume can be minimized (See Figs. 8 and 9 and Figs. 11 and 12). This means that more effective remediation will be achieved by the employment of an appropriately located recharge well.

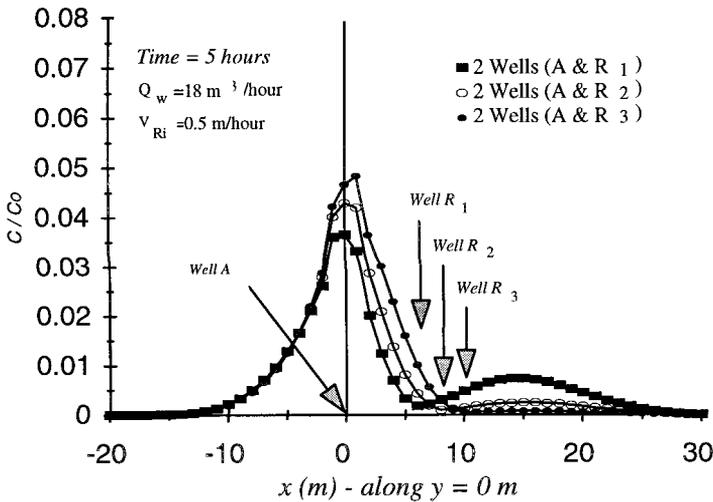


Fig. 8. Concentration profile along x axis of a two-well (discharge–recharge) system at time = 5 h.

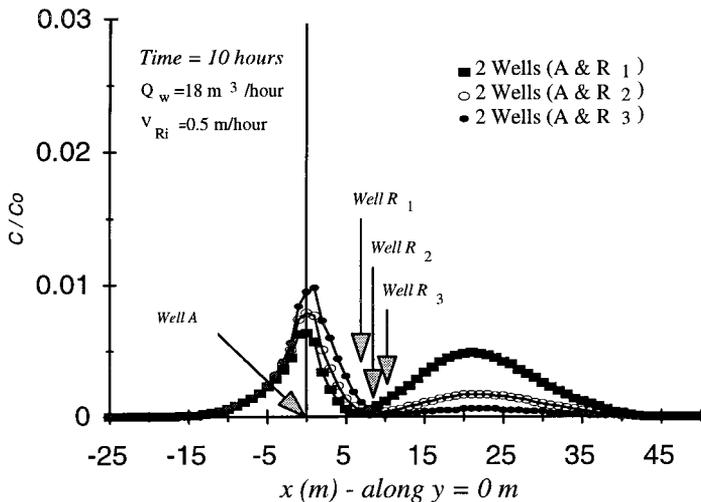


Fig. 9. Concentration profile along x axis of a two-well (discharge–recharge) system at time = 10 h.

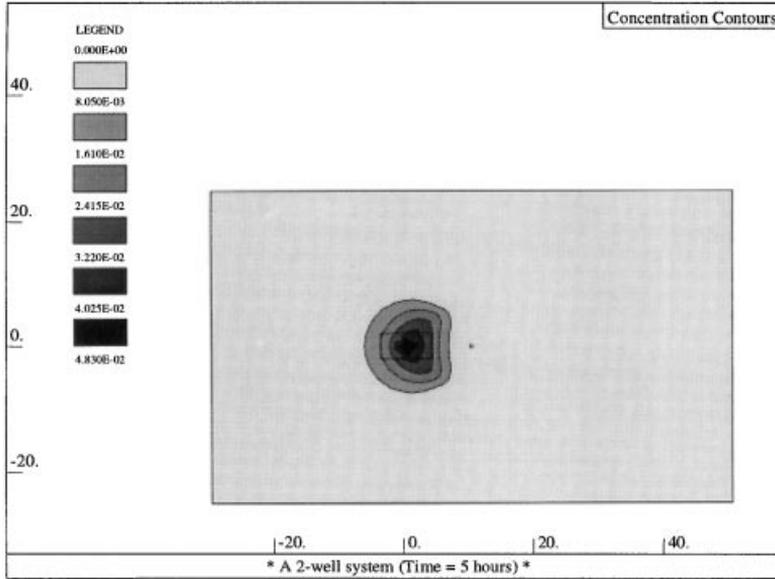


Fig. 10. Concentration contours at time = 5 h of the two-well case ($A&R_3$).

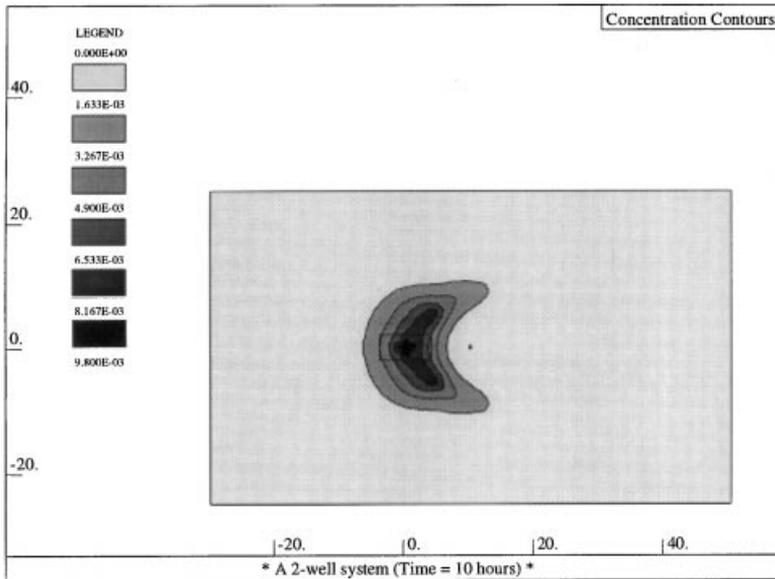


Fig. 11. Concentration contours at time = 10 h for the two-well case ($A&R_3$).

4.2. A multiple-well system

The results of the two-well case presented in Fig. 11 indicate that some of the contaminant is transported downstream around the recharge well (R_3). It is therefore of interest to examine a four-well system (Fig. 13), which includes a discharge

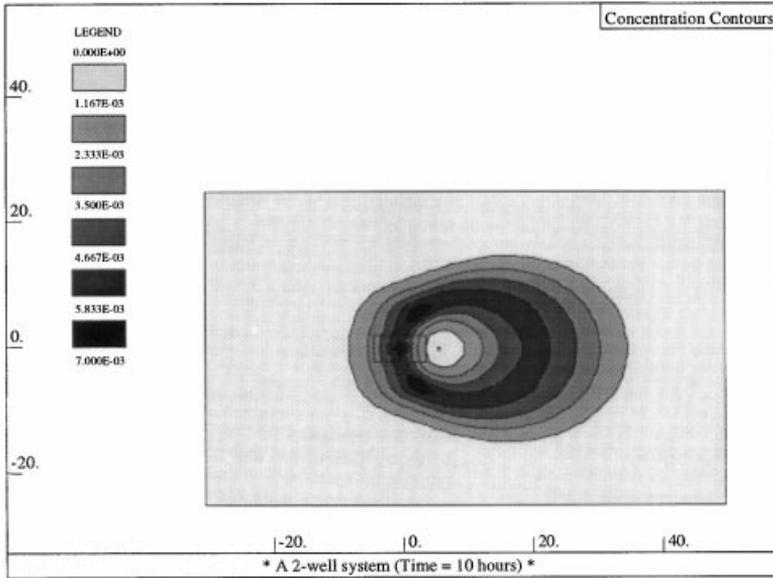


Fig. 12. Concentration contours at time = 10 h for the two-well case (A & R_1).

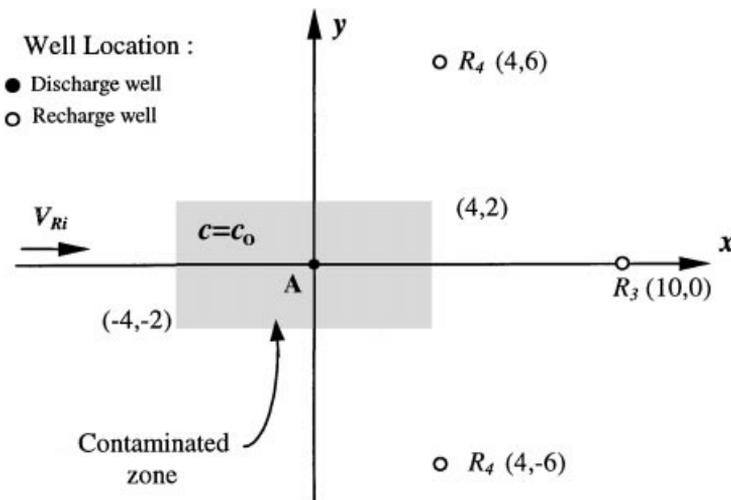


Fig. 13. Schematic of a multiple-well system.

well located at the centre of contaminated area and three recharge wells which are located around the contaminant plume, to examine whether the migration of the contaminant plume can be restrained.

The positions of these wells and the original contaminated region are shown in Fig. 13. Since the results of the previous case (Wells A & R_3 , Fig. 11) showed that the contaminant passes around the recharge well (R_3), two additional recharge wells (R_4 & R_5), placed at the sides of the contaminant zone, were considered.

The recharge rates of wells R_3 , R_4 and R_5 were assumed to be 10, 4, and 4 m^3/h , respectively, in order to keep a balance with the discharge rate of the discharge well ($Q_w = 18 \text{ m}^3/\text{h}$). Fig. 14 shows a representation of the velocity field of groundwater flow created by this four-well system, which includes the discharge well A and recharge wells R_3 , R_4 and R_5 .

The predictions for this four-well case are also compared to the most effective two-well case (A & R_3). Table 1 shows that the location and pumping rate of each well

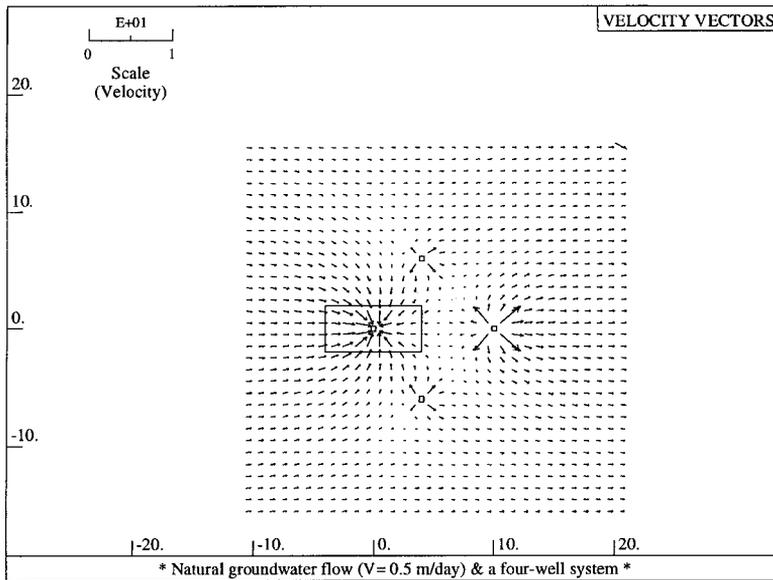


Fig. 14. Velocity field of groundwater flow for the four-well case.

Table 1
Location and pumping rate of each well

| Case | Location of well ($Q_w : \text{m}^3/\text{h}$) | |
|------------|--|----------------------------------|
| | Discharge well | Recharge well |
| Two wells | A (18) | R_3 (18) |
| Four wells | A (18) | R_3 (10), R_4 (4), R_5 (4) |

being considered. Fig. 15 shows the mass of contaminant removed in these two cases. The concentration contours for four-well case are presented in Figs. 16 and 17.

It can be seen from Fig. 15 that there is no significant difference in the remedial effect observed for these two cases (two wells and four wells). This is because the

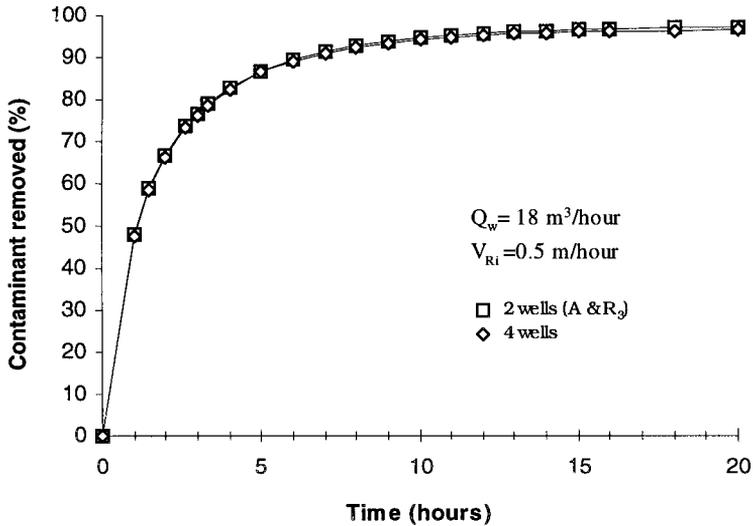


Fig. 15. Removal mass of contaminant with time for the two-well and four-well systems.

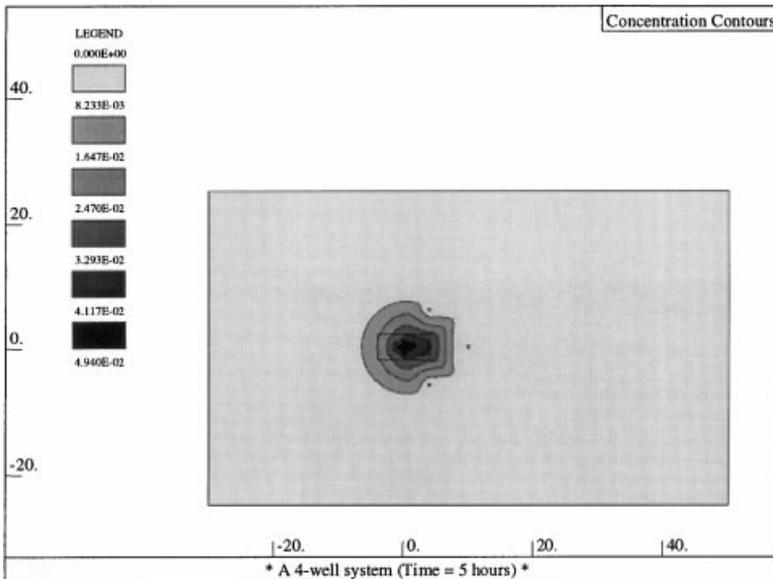


Fig. 16. Concentration contours at time = 5 h for the four-well case.

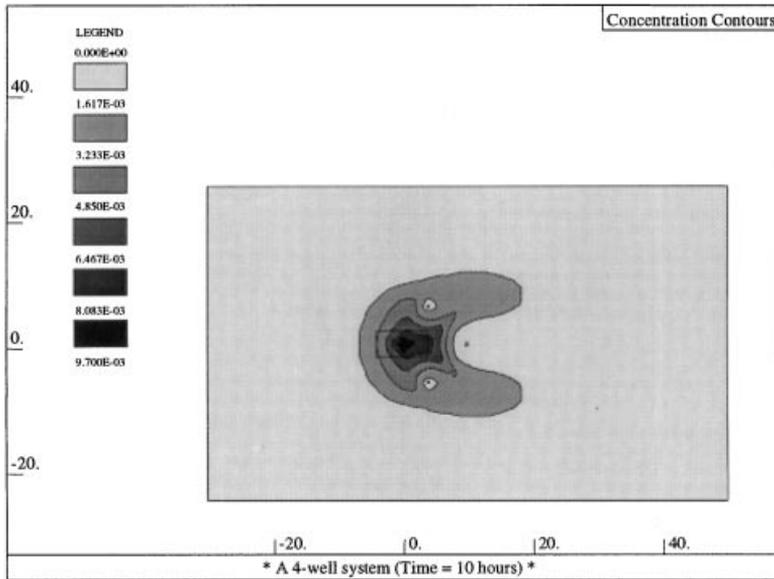


Fig. 17. Concentration contours at time = 10 h for the four-well case.

same effective discharge rate ($Q_w = 18 \text{ m}^3/\text{h}$) was used in each case. Although the four-well case did perform better in restraining the contaminant plume (see Figs. 11 and 17), the two-well system is only slightly better in the removal of the mass of contaminant (Fig. 15). Inspection of Figs. 11 and 17 shows that the installation of recharge well(s) may create an artificial barrier for the movement of the major part of the contaminant, although it can not completely stop the migration of contamination. From the viewpoint of being cost-effective and avoiding settlement problems, the two-well system ($A\&R_3$) seems to be the better remedial strategy to be adopted in this case.

5. Conclusions

In this paper, the analysis of the effectiveness of a multiple-well system for the remediation of a contaminated aquifer by the Laplace transform finite element method has been presented. Groundwater contamination problems are site specific and contaminant specific. In the case of a very large contaminated area multiple discharge wells and recharge wells may be required. The number of wells, pumping rate(s) and the location(s) of well(s) can be simulated and the optimum combination can be determined using the method illustrated in this paper.

Although groundwater contamination problems are generally site specific and contaminant specific, some general conclusions may be drawn from the cases investigated in this paper.

- A pump and treat system (PAT) does have a significant effect on the decontamination of a polluted aquifer and can preclude the further spreading of a contaminant plume.
- A recharge well does help in preventing the further spreading of a contaminant, provided the recharge well is located appropriately. However, if it is located too close to the contaminant plume, some part of the contaminant will be difficult to capture by the well system.

However, it needs to be kept in mind that a PAT system with groundwater extraction wells may not work efficiently for some contaminants, such as non-aqueous phase liquids including benzene and other petroleum products [10,21,22]. In these cases, other effective alternatives need to be used. In addition, the source of contamination must first be considered in the design of a remediation strategy for a contaminated aquifer. If no action is taken to deal with the source of the contamination, the clean-up of a contaminated aquifer by a PAT system might not be achievable.

Appendix A. Well modelling

In this paper, a discharge well is defined as a well which extracts groundwater from the aquifer and a recharge well is defined as a well which injects water into the aquifer. Both discharge and recharge wells may be used in a PAT system for the cleanup of a polluted aquifer. The discharge well is used to remove contaminants and the recharge well is used to recharge clean water into the aquifer to provide an artificial barrier for the contaminant movement or to avoid the settlement of the soil deposit. It is thus necessary to be able to model the functioning of both discharge and recharge wells when using a numerical model to examine the effect due to the remedial action of a PAT system.

A.1. Discharge well

A schematic diagram of a discharge well is shown in Fig. A.1.

If the concentration in the well is assumed to be spatially uniform and the aquifer has a uniform thickness, the mass balance equation may be written as:

$$m(t) = \int_0^t 2\pi R_w H f(c, \tau) d\tau - \int_0^t Q_w c_w d\tau + c_{w0} A_w H n_w \quad (\text{A1})$$

where,

- $m(t)$ is the mass of contaminant in the well at time t ,
- $f(c, \tau)$ is the mass flux entering the well at time τ ,
- H is the thickness of the aquifer,
- R_w is the radius of well,

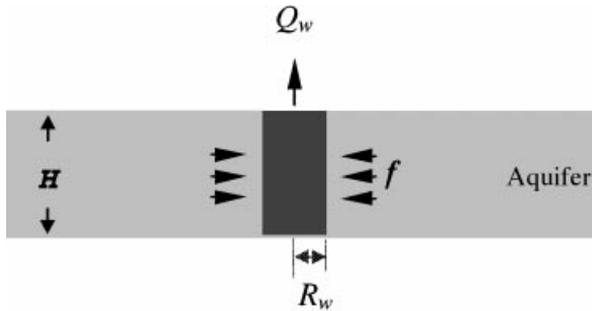


Fig. A1. Boundary condition for modelling a discharge well.

- c_w is the concentration in the well at time τ ,
- $c_w = m(\tau)/(\pi R_w^2 H n_w)$
- in which n_w is the porosity of the part of the well in the aquifer,
- Q_w is the discharge rate of discharge well,
- A_w is the area of well section ($A_w = \pi R_w^2$),
- c_{w0} is the initial concentration in the well.

If Eq. (A1) is divided by the volume ($\pi R_w^2 H n_w$), then it becomes:

$$c_w = \int_0^t \frac{2f(c, \tau)}{n_w R_w} d\tau - \int_0^t \frac{Q_w c_w}{n_w \pi R_w^2 H} d\tau + c_{w0} \tag{A2}$$

By taking the Laplace transform, Eq. (A2) can be expressed as:

$$\bar{f} = \left(\frac{sn_w R_w}{2} + \frac{Q_w}{2\pi R_w H} \right) \bar{c}_w - \frac{n_w R_w c_{w0}}{2} \tag{A3}$$

A.2. Recharge well

In describing the recharge well it will also be assumed that a uniform thickness of aquifer and a spatially uniform concentration in the well are considered. Fig. A2 shows the schematic diagram of a recharge well.

If Q_r represents the recharge rate of the well and c_r represents the concentration of contaminant in the water recharged, the mass balance equation for the condition of a recharge well may be written as:

$$m(t) = \int_0^t Q_r c_r d\tau - \int_0^t 2\pi R_w H f(c, \tau) d\tau + c_{w0} A_w H n_w \tag{A4}$$

If it is assumed that fresh (clean) water is injected and thus the concentration of contaminant in the water recharged is zero ($c_r = 0$), by taking the Laplace transform, Eq. (A4) becomes:

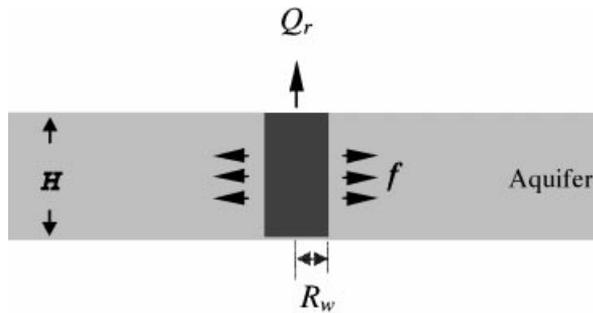


Fig. A2. Boundary condition for modelling a recharge well.

$$\bar{f} = \frac{n_w R_w c_{w0}}{2} - \left(\frac{sn_w R_w}{2} \right) \bar{c}_w \quad (\text{A5})$$

References

- [1] Booker JR. and Leo CJ. A FLT boundary integral equation method for analysis of contaminant leakage. In: Valliappan, Pulmeno, Tin-Loi, editors. Computational Mechanics—from Concepts to Computations, Proceedings of the Second Asian–Pacific Conference on Computational Mechanics, Sydney, Australia, 1993.
- [2] Huyakorn PS, Lester BH, Mercer JW. An efficient finite element technique for modeling transport in fractured porous media 1. Single Species transport. *Water Resources Research* 1983;V19(3):841–54.
- [3] Huyakorn PS, Lester BH, Mercer JW. An efficient finite element technique for modeling transport in fractured porous media 2. Nuclide decay chain transport. *Water Resources Research* 1983;19(5):1286–96.
- [4] Leo CJ. Boundary element analysis of contaminant transport in porous media. Ph.D. thesis, The University of Sydney, 1994.
- [5] Noorishad J, Mehran M. An upstream finite element method for solution of transient transport equation in fractured porous media. *Water Resources Research* 1982;18:588–96.
- [6] Rowe RK, Booker JR. A finite layer technique for calculating three-dimensional pollutant migration in soil. *Geotechnique* 1986;36(2):205–14.
- [7] Sudicky EA. The Laplace transform Galerkin technique: a time-continuous finite element theory and application to mass transport in groundwater. *Water Resource Research* 1989;25(8):1833–46.
- [8] Sun NZ, Yeh WW-G. A proposed upstream weight numerical method for simulating pollutant transport in groundwater. *Water Resources Research* 1983;19(6):1489–500.
- [9] Wang JC. and Booker JR. A Laplace transform finite element method (LTFEM) for the analysis of contaminant transport in porous media. Research Report (R756), Department of Civil Engineering, University of Sydney, 1997.
- [10] Travis CC, Doty CB. Can contaminated aquifers at Superfund sites be remediated? *Environ Sci Technol* 1990;V24(10):1464–6.
- [11] Javandel I, Tsang CF. Capture-zone type curves: a tool for aquifer cleanup. *Groundwater* 1987;24(5):616–25.
- [12] Bredehoeft JD. Hazardous waste remediation: a 21st century problem. *Ground Water Monitoring Review*. 1994;winter:95–100.
- [13] Istok J. Groundwater modeling by the finite element method. American Geophysical Union, 1989.
- [14] Stehfest H. Algorithm 368, numerical inversion of Laplace transforms. *Commun Assoc Comput Mach* 1970;13(1):47–9.

- [15] Stehfest H. Remark on Algorithm 368, numerical inversion of Laplace transforms. *Commun Assoc Comput Mach* 1970;13(10):624–5.
- [16] Ogata A. Dispersion in porous media. Ph.D. dissertation, North-western University, Evanston, (IL), 1958.
- [17] Lau LK, Kaufman WJ, Todd DK. Dispersion of a water tracer in radial laminar flow through homogeneous porous media. *Hydraul. Lab. Progress Rep. 5*, Univ. of California, Berkeley, 1959.
- [18] Hoopes JA, Harleman DRF. Waste water recharge and dispersion in porous media. Report No. 75. Cambridge (MA): Hydrodynamics Laboratory, Department of Civil Engineering, Massachusetts Institute of Technology, 1965.
- [19] Moench AF, Ogata A. A numerical inversion of the Laplace transform solution to radial dispersion in a porous media. *Water Resour Res* 1981;17(1):250–2.
- [20] Bear J. *Hydraulics of groundwater*. New York: McGraw–Hill, 1979.
- [21] Mackay DM, Cherry JA. Groundwater contamination: pump-and-treat remediation. *Environ Sci Technol* 1989;23(6):630–6.
- [22] Haley JL, Hanson B, Enfield C, Glass J. Evaluating the effectiveness of ground water extraction systems. *Ground Water Monit Rev* 1991;11(1):119–24.