ELASTIC ANALYSIS OF BURIED PIPES UNDER SURFACE PATCH LOADINGS

By N. S. M. Fernando[1] and J. P. Carter[2]

ABSTRACT: A parametric study was undertaken to assess the behavior of buried pipes under uniformly distributed vertical patch loads applied to the surface of a layer of soil. Predictions of the maximum circumferential and longitudinal bending moments and hoop thrusts acting on the buried pipes are presented in the form of nondimensionalized design charts. Fourier transforms were used to model the three-dimensional behavior under the patch loading, and conventional finite-element analysis was used to approximate the field quantities in the two-dimensional transform plane. The soil has been idealized as a homogeneous linear elastic continuum, and the buried pipe has been represented by a linear elastic plate bending element capable of transmitting axial thrusts. The results of the parametric study are relevant to the design of buried pipes under the action of vertical live loads applied to the surface of a soil layer.

INTRODUCTION

Buried pipes, culverts, and arches are generally designed to withstand construction loads, dead loads due to any surrounding fill material plus the weight of the structure itself, and any traffic loadings that may be imposed upon it during its lifetime. These vehicular loads are generally approximated as patches of vertical stress (equivalent in area to the contact area of the tires) acting at the surface of the soil above the pipe. Because of the finite dimensions of the patch, these vehicle loads induce a three-dimensional (3D) stress pattern, especially on shallow buried structures.

It is possible to approximate the loading for such problems using an analytical solution, such as the Boussinesq solution for a vertical load at the surface of an elastic half-space, to convert the 3D loading to an equivalent line load, which can then be examined using conventional two-dimensional (2D) finite-element analysis. However, this approach disregards the interaction between the structure and the surrounding soil. Alternatively, a 3D finite-element analysis could be used to analyze the problem, but accurate 3D finite-element analysis [e.g., Zienkiewicz and Taylor (1977)] is extremely time consuming and requires a lot of computer memory. To overcome these difficulties, integral transform techniques can be used to analyze this type of 3D problem whenever the material behavior is linear, by transforming it into an equivalent 2D problem. Small and Wong (1988), Moore and Brachman (1994), and Fernando et al. (1996) have applied these Fourier transform techniques to a number of geotechnical problems.

Fourier analysis techniques have also been used in this study to assess the behavior of buried pipes under a surface patch load (such as a tire loading above a culvert or buried pipe). A parametric study has been carried out to assess the effects of the various geometric and material parameters on the behavior of pipes under square, centrally placed, surface patch loads. Fig. 1 shows schematic illustration of the problem considered here. Predictions of the maximum circumferential bending moments and hoop thrusts in the cross-sectional plane of the pipe, and the longitudinal bending moments induced in the buried pipe, are presented in the form of nondimensionalized design charts.

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IDEALIZATIONS

In this analysis it is assumed that the pipe may be represented adequately as an elastic tube of constant diameter and thickness. It is also assumed that the soil behaves as an elastic continuum with no plastic failure. Given that in practice, the soil surrounding the pipe is usually well compacted prior to vehicular loading, this is a reasonable first assumption. It is also assumed that the pipe and the surrounding soil remain in intimate contact, so there is no slip between the pipe and the adjacent soil when deformation occurs. This assumption is also likely to be realistic in many cases as the pipe tends to settle with the soil unless the soil is very soft relative to the pipe. The soil is assumed to be homogeneous throughout, hence the effect of any stiffer surface pavement is specifically ignored. The omission of the stiffer pavement generally results in more conservative (i.e., larger) estimates of forces and moments, and it may be prudent in practice to design the pipe to withstand these higher effects. Indeed, during construction, the buried structure may actually experience the effects of surface loading from vehicular traffic prior to the completion of a pavement course.

All field quantities and the 3D loading are transformed (in the z-direction) by the use of a Fourier transform. It is assumed that the global z-coordinate is aligned parallel to the longitu-

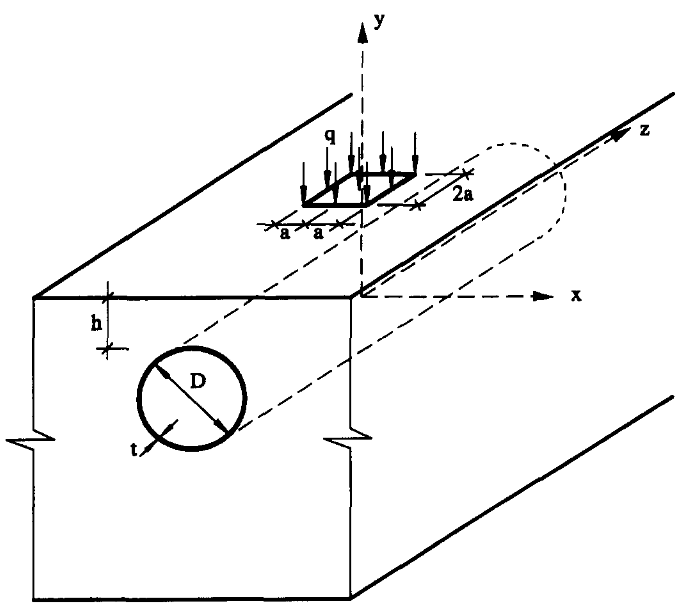

FIG. 1. Definition of Problem and Geometric Parameters

dinal axis of the pipeline or culvert, as shown in Fig. 1. A 2D finite-element mesh is used to model the transformed displacement field in the x, y-plane. To evaluate the response in the z-direction, the Fourier transform is inverted to determine the actual displacements and stresses. Numerical integration of the transformed field quantities is used to determine values such as displacements and stresses in real x, y, z-space. Direct output of bending moments (both longitudinal and transverse), shear forces, and axial forces for the pipe is obtained.

THEORETICAL BACKGROUND

Small and Wong (1988) have previously used an integral transform technique to analyze 3D geotechnical problems. In cases where the soil profile is constant in one coordinate direction, a Fourier series can be used to represent the field quantities in that direction, simplifying the problem from three dimensions to two. In general, this type of analysis has been termed the finite strip or finite prism method. Small and Wong extended the finite strip method to elastic continuum elements, which are applicable to geotechnical problems. Their method relied upon the use of finite-element techniques to approximate the transformed field quantities in a plane containing two of the coordinate axes.

Fernando et al. (1996) extended the method of Small and Wong (1988) to infinitely long plate elements. This allows modeling of problems such as those involving flexible culverts at shallow depths under vehicular loading. The transverse behavior of the culvert can be examined using the infinitely long plate elements (an extension of the finite strip method) while the surrounding soil is modeled using the Fourier transform continuum elements developed by Small and Wong. This enables direct calculation of shear and axial forces and moments (both longitudinal and transverse) for the culvert. The combined Fourier and finite-element analysis has been encoded in a program called AFENA (Carter and Balaam 1995). Details of the formulation have been given previously by Fernando et al. (1996) and so they will not be repeated here.

CASES CONSIDERED

For the parametric study, a buried pipe of diameter D was analyzed with a square patch of vertical stress applied to the surface of the soil. In most cases this loading was applied directly above the centerline of the buried pipe. Fig. 2 shows a typical mesh used in the analysis. Only one-half of the pipe and soil was modeled in the analysis because of symmetry considerations. The height of cover above the pipe, h, the thickness of the pipe wall, t, the Young's moduli of the pipe, E_p, and the surrounding soil, E_s, and the area over which the load was applied (half side a) were all varied in the parametric study.

Solutions were evaluated for ratios of height of cover to pipe diameter (h/D) of 0.1, 0.25, and 0.5. It was found that further increases in relative cover height resulted in almost negligible forces and moments being induced in the pipe by the surface loading, compared to those that would be induced by the self weight of the cover soil. Different Young's moduli of the fill were also adopted, corresponding to ratios of pipe modulus to soil modulus (E_p/E_s) of 100,000, 10,000, and 1,000. These ratios are typical of tubular metal, plastic, and concrete pipes in a variety of soils, but they will also be relevant to other types of pipe installation, as explained later in this paper. The thickness of the pipe was varied from 0.05 to 10% of the pipe diameter, i.e., (t/D) = 0.0005 to 0.01. As will be demonstrated, these are the two extremes beyond which any further variation in the modulus ratio has negligible effect. The area over which the loads acted was also varied. Three values of the ratio of the width of the square load to the pipe diameter ($2a/d$) were chosen: 0.1, 0.5, and 1.0.

RESULTS

The results presented here are in the form of changes in thrust or moment due to the surface loading, i.e., they do not include the effects of self weight of the pipe or the effects of dead load of the cover soil, or the influence of the method of placement of the pipe. Most of the results are for symmetric cases where the loading is applied centrally above the pipe. A limited study has also been made of a case where the surface

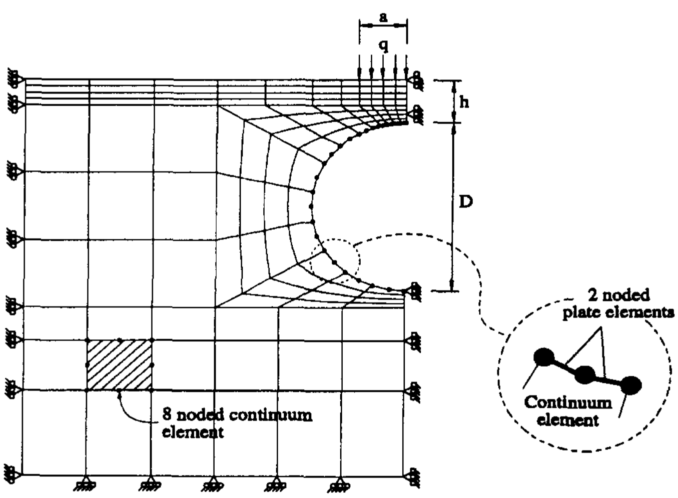

FIG. 2. Typical Finite-Element Mesh Used in Analysis

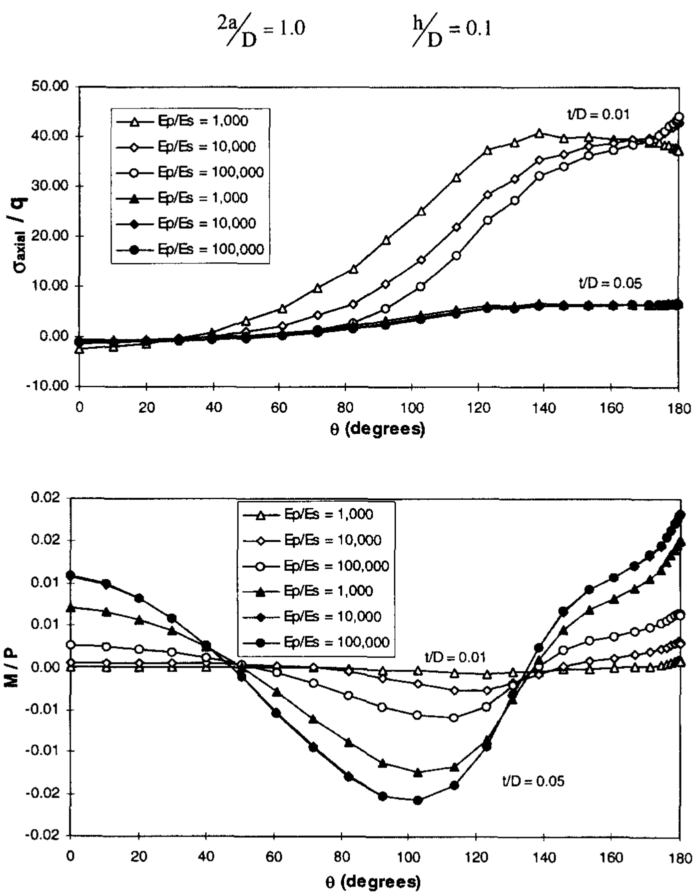

FIG. 3. Typical In-Plane Forces and Moments ($2a/D$ = 1.0: h/D = 0.1)

loading is located nonsymmetrically with respect to the pipe position.

Symmetric Loading

Figs. 3 and 4 show typical plots of normalized in-plane moments and average hoop stresses in the pipe acting under the center of the load. Fig. 3 corresponds to the case of a relatively large loaded area and relatively shallow cover, i.e., $(2a/D) = 1$ and $h/D = 0.1$, while Fig. 4 corresponds to relatively concentrated load with relatively greater cover, i.e., $(2a/D) = 0.1$ and $h/D = 0.5$. The graphs in Figs. 3 and 4 show the maximum increments of the average hoop stress, σ_{axial}, normalized by the applied surface pressure, q, and the in-plane moment per unit length, M, normalized by the total surface load, $P = 4a^2q$, acting on the pipe at any particular angle θ. For all cases, the angle θ is measured from the pipe invert (hence the crown corresponds to $\theta = 180°$). The hoop stresses are largest at or near the crown of the pipe (generally within 30° of the crown) and become almost negligible in the lower portion of the pipe ($\theta < 45°$). Positive hoop stresses imply compression. Positive bending moments, indicating inward movement or "sagging" flexure, occur near the crown and invert of the pipe, while in the region of the springline the pipe flexes outward ("hogging"). The influence of the modulus ratio is reduced as the relative thickness of the pipe is increased. The analyses also showed that the in-plane moments and forces reduce rapidly with distance along the pipe from the center of the load.

Fig. 5 shows typical variations in the z-direction of the longitudinal moments in the pipe (for the case where $h/D = 0.1$, i.e., relatively shallow cover). As seen in this figure, the longitudinal moments become almost negligible at a distance of one to two pipe diameters from the center, when the patch loading is very concentrated ($2a/D = 0.1$). Where the patch of load is less concentrated, the reduction in the moments is more gradual.

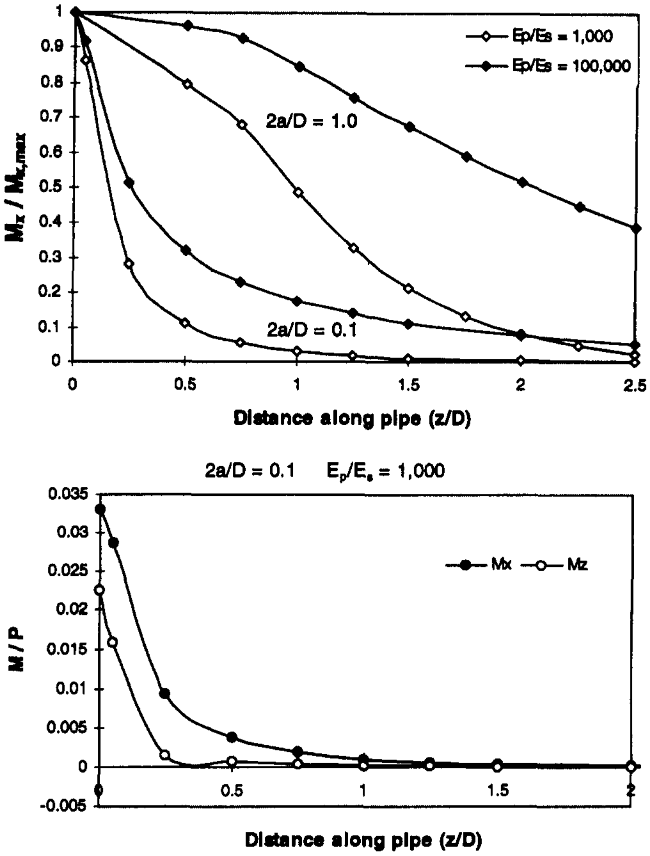

FIG. 5. Typical Variation of Moments along Pipe ($h/D = 0.1$)

Fig. 6 illustrates typical variations of the vertical stress in the soil around the pipe for selected locations along the pipe for the case where $2a/D = 0.5$ and $h/D = 0.25$. Again, this figure indicates a quite rapid reduction in the maximum vertical stresses transmitted to the pipe as the distance from the center of the loading is reduced.

Figs. 7–9 summarize the maximum in-plane moments and average hoop stresses induced in buried pipes by the surface patch loads. These are presented in the form of nondimensionalized moments per unit length and normalized axial stresses plotted against the nondimensional parameter $t^3/(12D^3)$, which is the normalized second moment of area of the pipe section about a neutral axis through the pipe wall. These figures clearly show that the bending moments increase and the hoop stresses reduce as the relative thickness of the tube is increased. It should be noted that a reduction in the hoop stress with $t^3/(12D^3)$ does not necessarily imply a reduction of the hoop force, since for a given diameter pipe (D) the thickness (t) increases as $t^3/(12D^3)$ increases, and the force is calculated as the product of the hoop stress and the pipe thickness. Figs. 7–9 also show that the moments and the average hoop stresses are also functions of the modulus ratio, E_p/E_s.

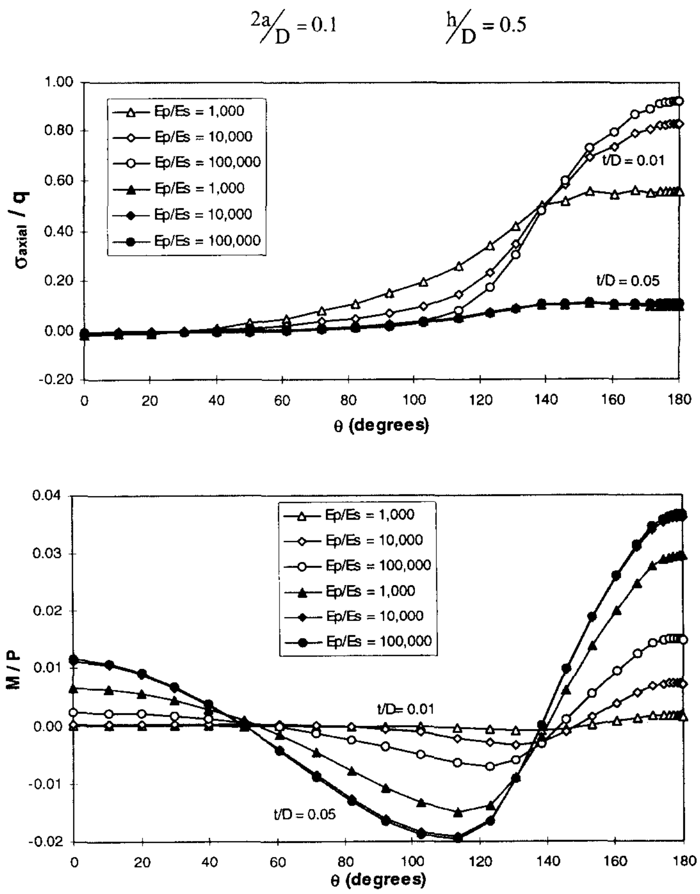

FIG. 4. Typical In-Plane Forces and Moments ($2a/D = 0.1$; $h/D = 0.5$)

Nonsymmetric Loading

A further analysis was carried out with nonsymmetric loading to assess whether the maximum stresses and moments were indeed induced when the load was directly above the crown of the pipe. Fig. 10 shows a typical set of results for a case where the load is not acting centrally. In this case the load acts at the surface of the soil at a distance of one pipe

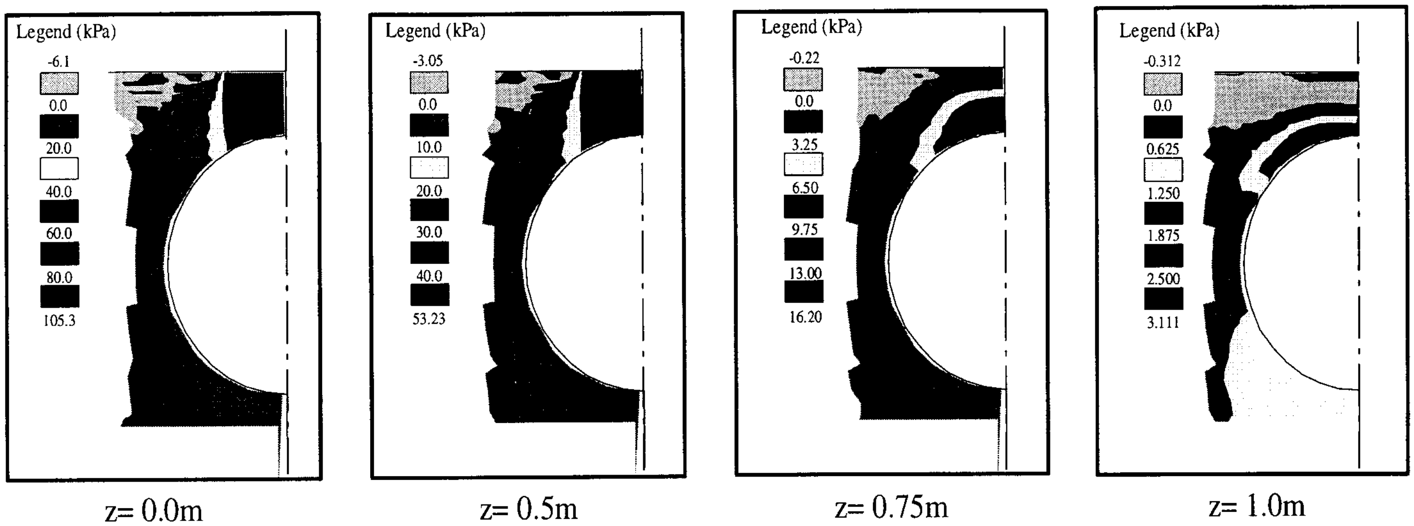

FIG. 6. Variation of Vertical Stress along *z*-Axis

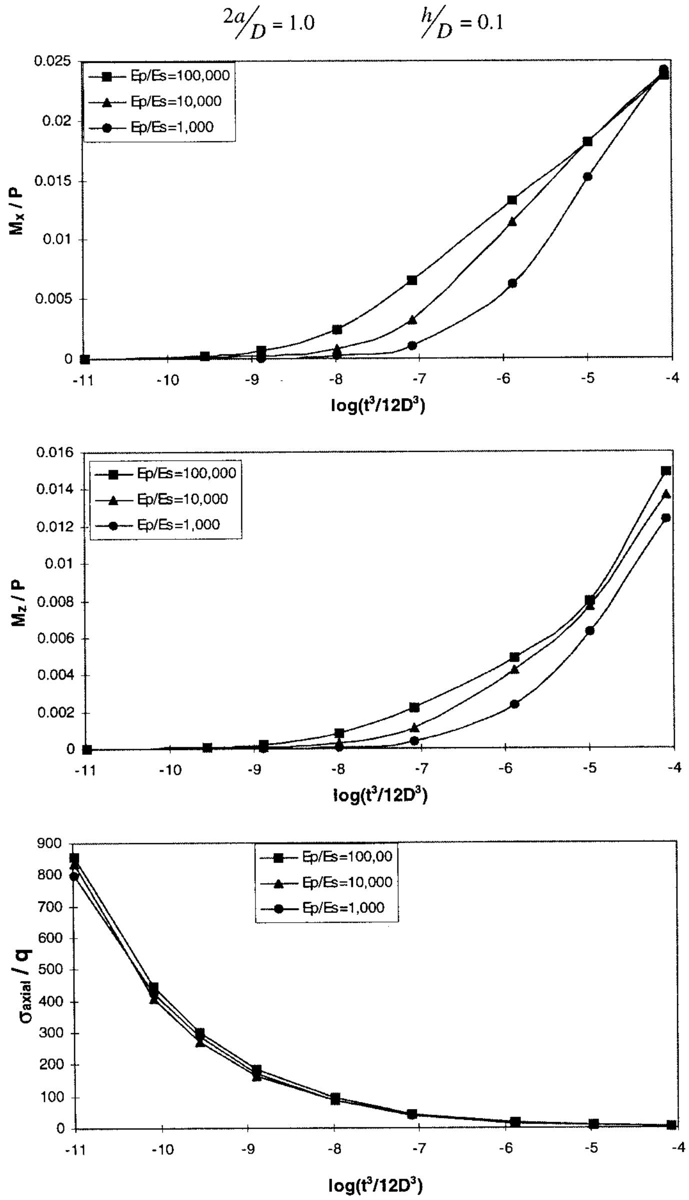

FIG. 7(a). Nondimensionalized Axial Stress and Moment ($2a/D = 1.0$; $h/D = 0.1$)

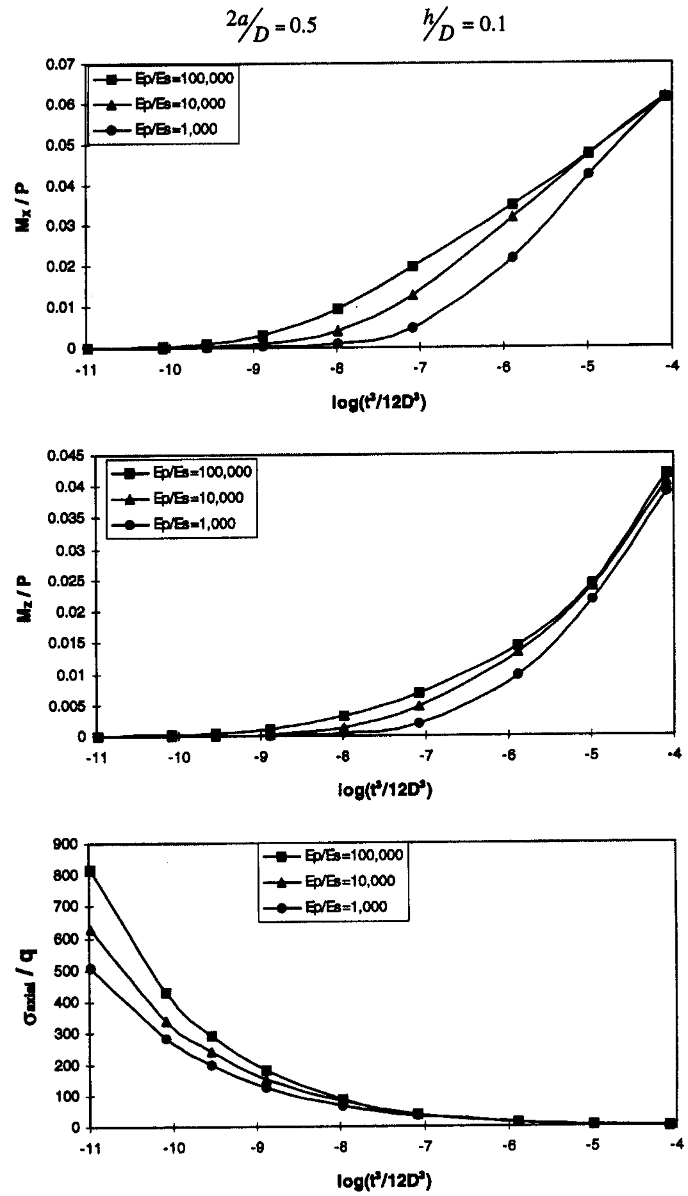

FIG. 7(b). Nondimensionalized Axial Stress and Moment ($2a/D = 0.5$; $h/D = 0.1$)

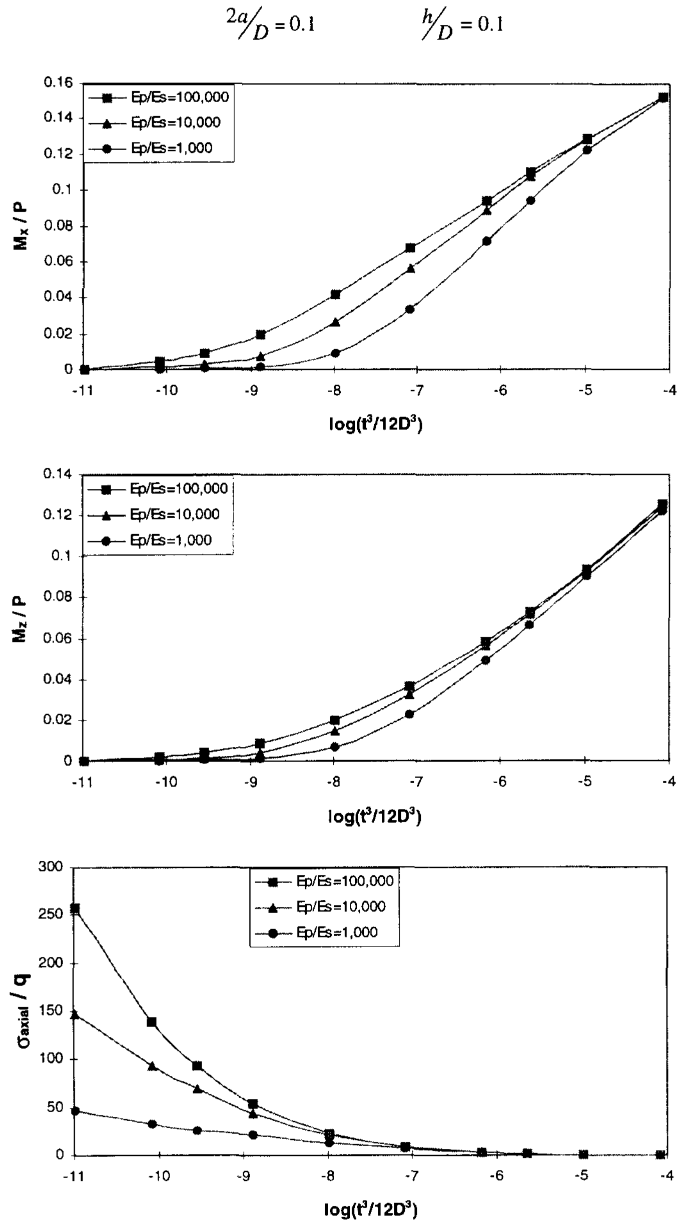

FIG. 7(c). Nondimensionalized Axial Stress and Moment ($2a/D = 0.1$; $h/D = 0.1$)

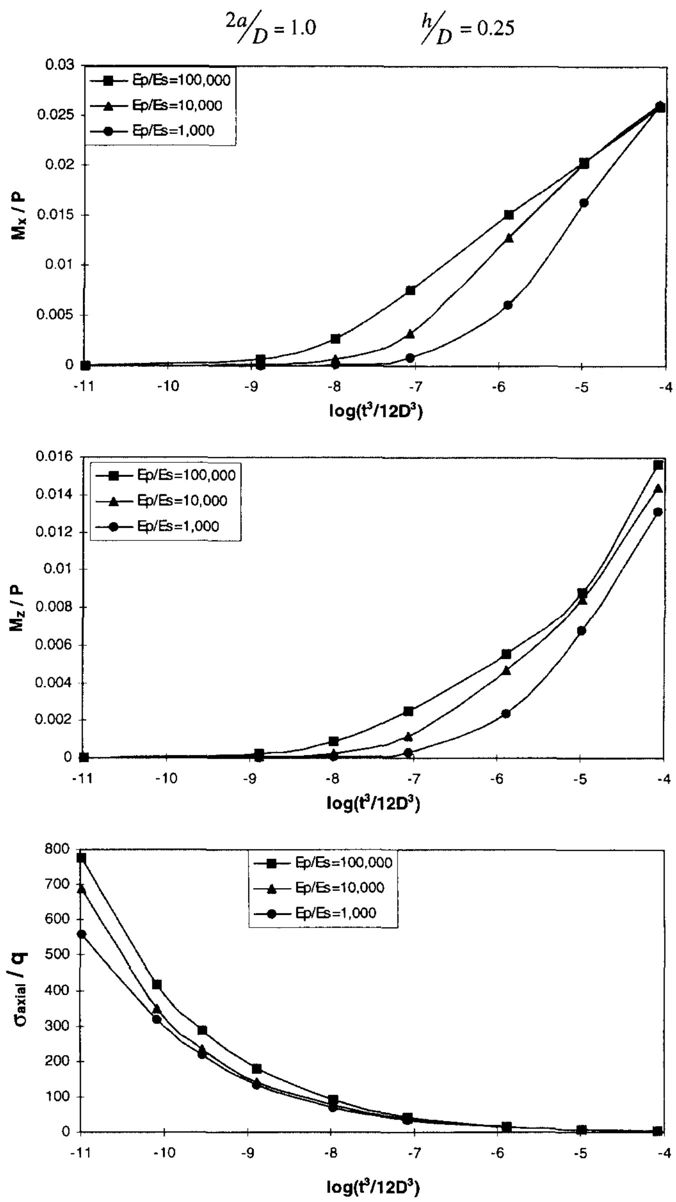

FIG. 8(a). Nondimensionalized Axial Stress and Moment ($2a/D = 1.0$; $h/D = 0.25$)

radius from the center. As might have been expected, the in-plane hoop stresses and moments are less than those induced when the load acts centrally above the pipe. However, the respective maximum thrust and moment do not occur at the same locations for the symmetric and nonsymmetric cases. Similar trends have been observed by Katona et al. (1979) for plane strain analyses of live loading.

ANALYSIS

The parametric study of the effects of the patch load has highlighted several important trends, as seen in Figs. 7–9. In general, for a given modulus ratio and surface load, the stiffer the pipe (larger t), the less it will deflect and the higher the moments acting on it. This is because a stiffer pipe (relative to the surrounding fill) will generally attract more load and hence will have higher moments. However, it is interesting that the average hoop stress decreases as the relative pipe stiffness $t^3/(12D^3)$ increases. As noted previously, this is probably because the increase in hoop force borne by a stiffer pipe is not large enough to offset the decrease in the average hoop stress due to an increase in pipe thickness

Conversely, for a given pipe stiffness, an increase in soil modulus results in lower deflections, forces, and moments. This is because the stiffer the surrounding fill material, the more load borne by it and hence the lower the displacements and stresses of the pipe. The parametric study demonstrates that for practical cases, where t/D is greater than 0.01 or less than 0.0005, the effect of the soil modulus is negligible over the range of modulus ratios investigated. As would be expected, the average hoop stresses and moments acting on the pipe generally decrease with increasing cover height. In addition, more concentrated loads tend to generate higher peak moments than the same loads distributed over a larger area, but the reduction in moments with increasing distance from the load is more pronounced for the concentrated loads.

Since the plate bending element used to model the pipe has isotropic properties, it will represent the behavior of a uniform tubular pipe with good accuracy. However, this element cannot model directly the corrugations in a corrugated

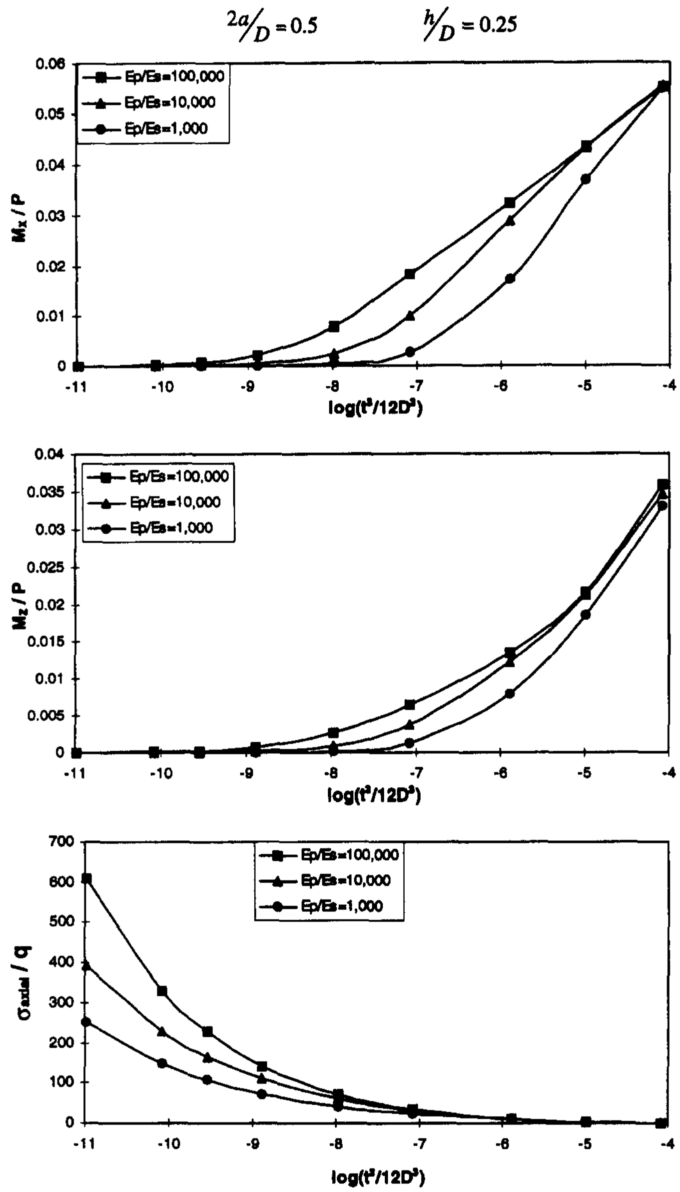

FIG. 8(b). Nondimensionalized Axial Stress and Moment ($2a/D = 0.5$; $h/D = 0.25$)

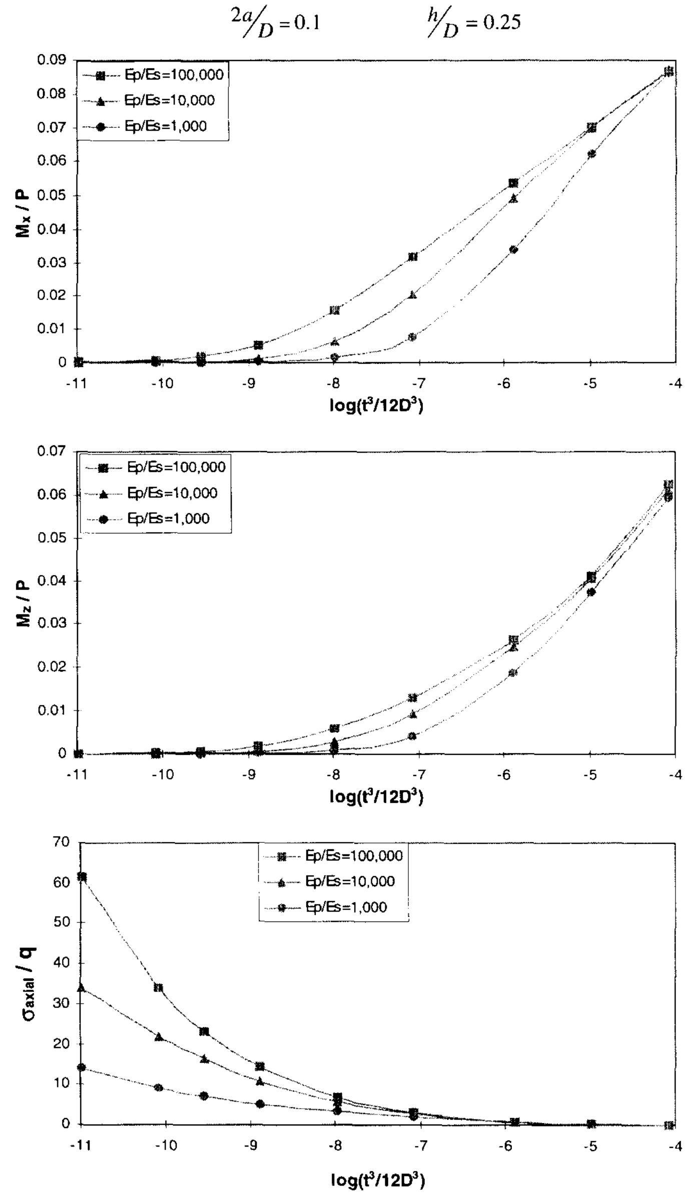

FIG. 8(c). Nondimensionalized Axial Stress and Moment ($2a/D = 0.1$; $h/D = 0.25$)

metal pipe. Hence, the results of these analyses may be applied only approximately to corrugated pipes by calculating an effective constant thickness for the corrugated pipe. This can be done by either assuming the same area as the actual pipe (to model the correct axial stiffness) or the same second moment of area (to model the correct bending stiffness), but not both. Usually it is the correct in-plane bending stiffness that is chosen. Although this approach models the correct in-plane bending stiffness of the corrugated pipe, it will overestimate the axial stiffness and the out-of-plane bending stiffness. However, this approach should be conservative in the prediction of the axial stress acting on the pipe as a stiffer structure will generally attract more load to it than would the actual structure.

The influence of the rigid bottom boundary of the finite-element mesh on the predicted structural behavior of the pipe was also evaluated. In general, it was observed that the mesh adopted for the parametric study adequately replicated the effects of a distant boundary. The predicted bending moments and hoop thrusts are within 0.5% (or less) of those calculated using a boundary two or more pipe diameters away from the pipe. The mesh adopted for the parametric study was chosen for efficiency of computer solution.

EXAMPLE APPLICATIONS

Two examples are included here to demonstrate the use and relevance of the charts. The first example is of a steel culvert of diameter 4.0 m, with an "effective" thickness of 20 mm and a cover of 400 mm. The load is relatively concentrated and spread over an area equivalent to a typical tire load. The second example is of a steel pipe of diameter 0.8 m, with an "effective" thickness of 20 mm and a cover of 400 mm. The load in this instance is spread over the entire width of the pipe.

Example 1

The parameters assumed for example 1 are listed in Table 1, from which the following nondimensional values may be calculated: $h/D = 0.4/4.0 = 0.1$; $2a/D = 0.4/4.0 = 0.1$; $E_p/E_s = 200{,}000/20 = 10{,}000$; Log $(t^3/12d^3) = -7.982$.

Given these values, the nondimensional moments and axial

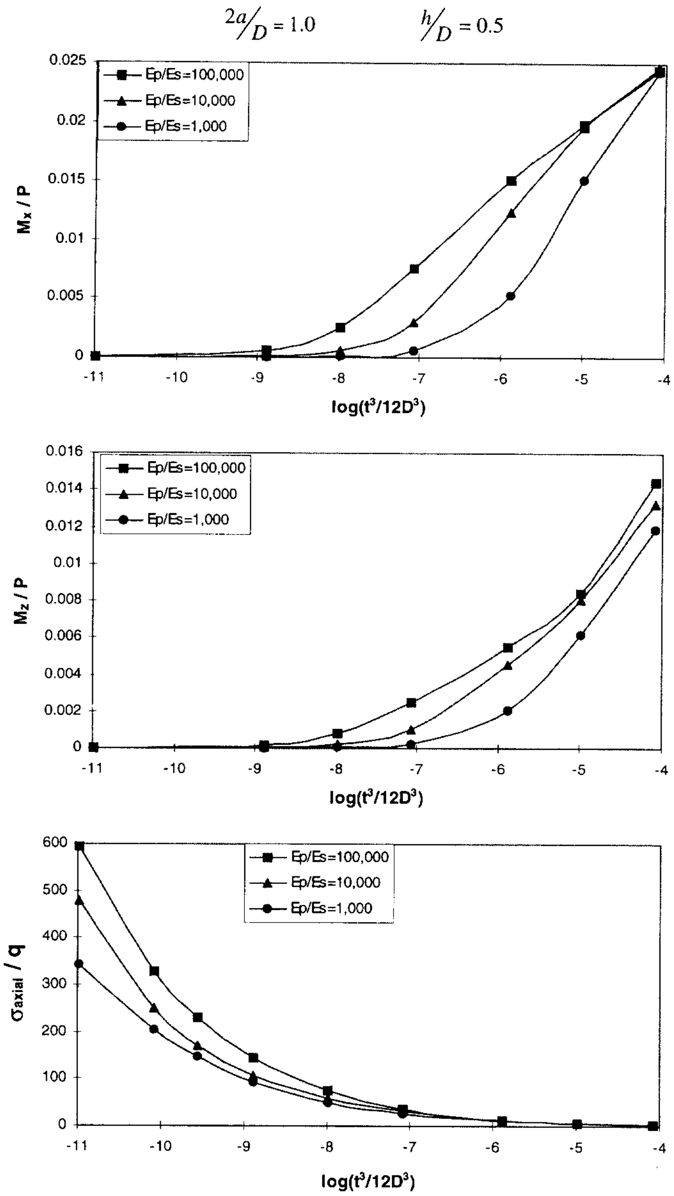

FIG. 9(a). Nondimensionalized Axial Stress and Moment ($2a/D = 1.0$; $h/D = 0.5$)

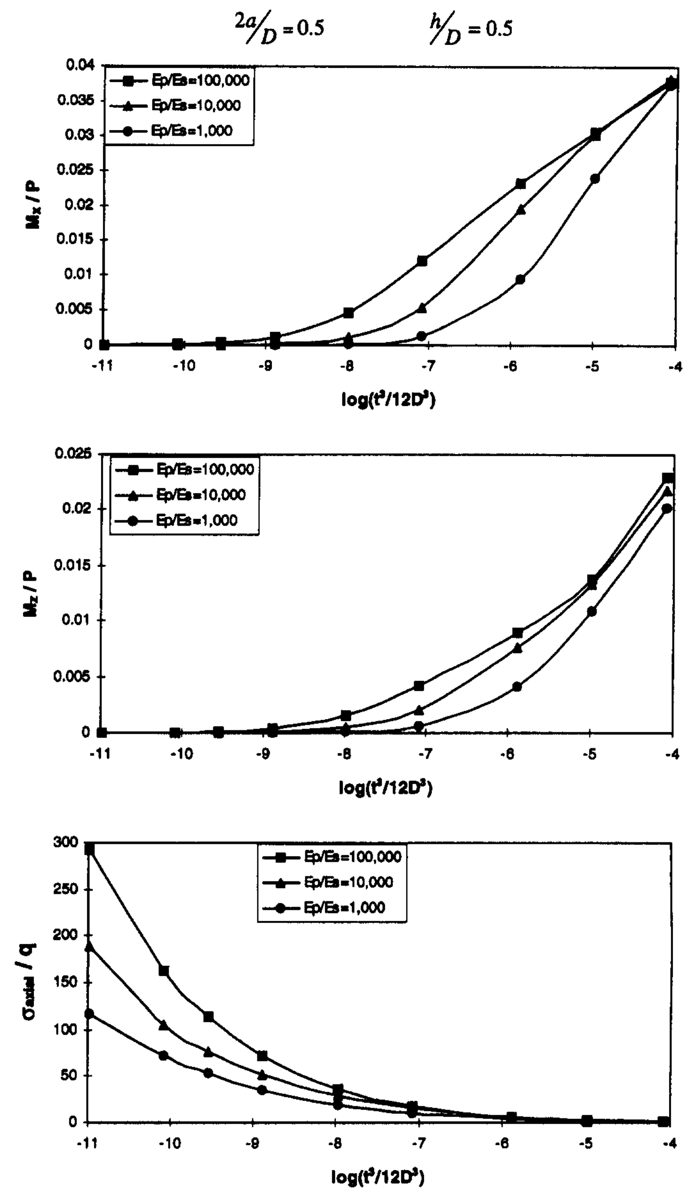

FIG. 9(b). Nondimensionalized Axial Stress and Moment ($2a/D = 0.5$; $h/D = 0.5$)

stresses can be obtained from Fig. 7(c), i.e., $M/P \sim 0.026$; $\sigma_{axial}/q \sim 20.5$. From these nondimensional values, the actual effects can be calculated once P and q are known. If $P = 9.81 \times 40 = 392.4$ kN, and $q = 392.4/(0.4 \times 0.4) = 2{,}453$ kPa, then the maxiumum in-plane moment, $M_x = 0.026 \times 392.4 = 10.2$ kN·m/m width, and the axial thrust, $\sigma_{axial} = 20.5 \times 2.453 = 50.3$ MPa.

Example 2

The parameters assumed for example 2 are listed in Table 1 from which the following nondimensional values may be calculated: $h/D = 0.4/0.8 = 0.5$; $2a/D = 0.8/0.8 = 1.0$; $E_p/E_s = 200{,}000/20 = 10{,}000$; $\text{Log}(t^3/12d^3) = -5.885$.

Given these values, the nondimensional moments and axial stresses can be obtained from Fig. 9(a), i.e., $M/P \sim 0.0123$; $\sigma_{axial}/q \sim 11.3$. From these nondimensional values, the actual effects can be calculated once P and q are known. If $P = 9.81 \times 40 = 392.4$ kN, and $q = 392.4/(0.8 \times 0.8) = 613$ kPa, then the maximum in-plane moment, $M_x = 0.0123 \times 392.4 = 4.8$ kN·m/m width, and the axial thrust, $\sigma_{axial} = 11.3 \times 613 = 6.9$ MPa.

CONCLUSIONS

A parametric study has been undertaken to assess the behavior of buried pipes under centrally located vertical patch loads applied to the surface of a layer of soil. The maximum bending moments and hoop thrusts acting on the buried pipes, as predicted by the analysis, have been presented in the form of nondimensionalized design charts. Predictions of moments in both the plane of the pipe section and longitudinal moments in the pipe were included.

Fourier transforms were used to model the 3D behavior of the patch loading, and conventional finite-element analysis was used to approximate the field quantities in the 2D transform plane. The soil has been idealized as a homogeneous linear elastic continuum and the buried pipe has been represented by a linear elastic plate bending element capable of transmitting axial thrusts.

The parametric study has highlighted the different parameters that affect the magnitude of the forces and moments induced in buried pipes subject to surface loads. For the range of parameters investigated, it was found that the relative thickness of the annular pipe had an important influence on the

TABLE 1. Parameters for Examples 1 and 2

Example (1)	Pipe diameter, D (m) (2)	Pipe modulus, E_p (MPa) (3)	Effective pipe thickness, t (m) (4)	Soil modulus, E_s (MPa) (5)	Load (t) (6)	Loaded area, $2a \times 2a$ (m) (7)	Cover height, h (m) (8)
1	4.0	200,000	0.02	20	40	0.4 × 0.4	0.4
2	0.8	200,000	0.02	20	40	0.8 × 0.8	0.4

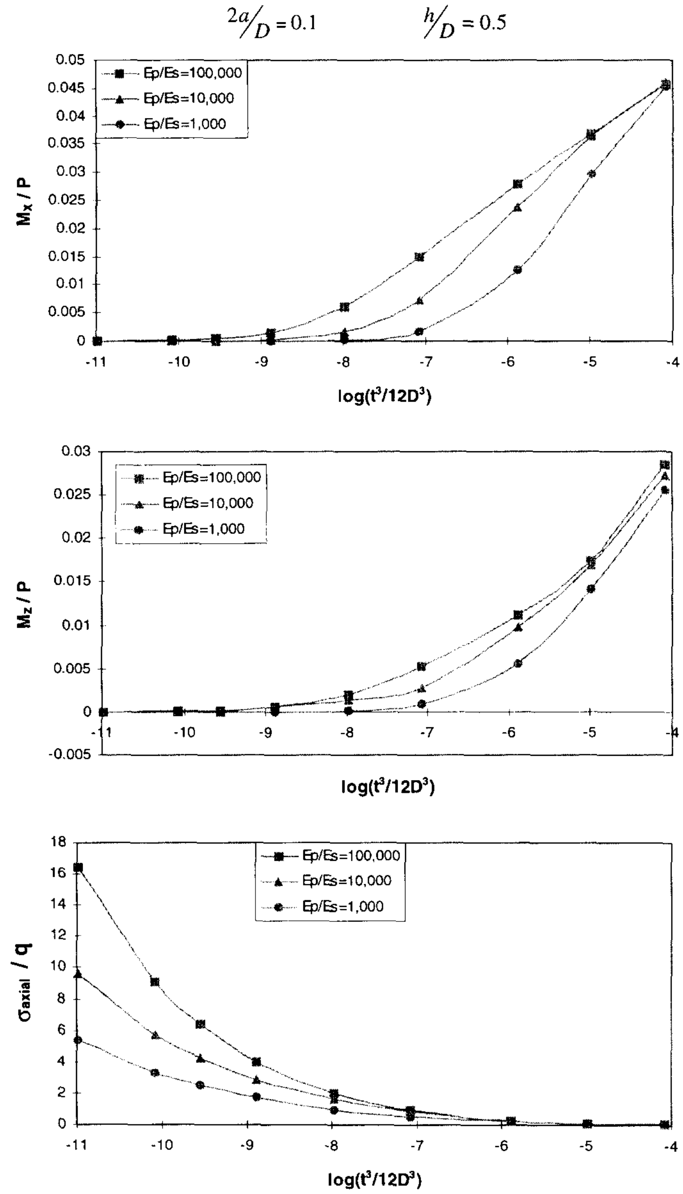

FIG. 9(c). Nondimensionalized Axial Stress and Moment ($2a/D = 0.1$; $h/D = 0.5$)

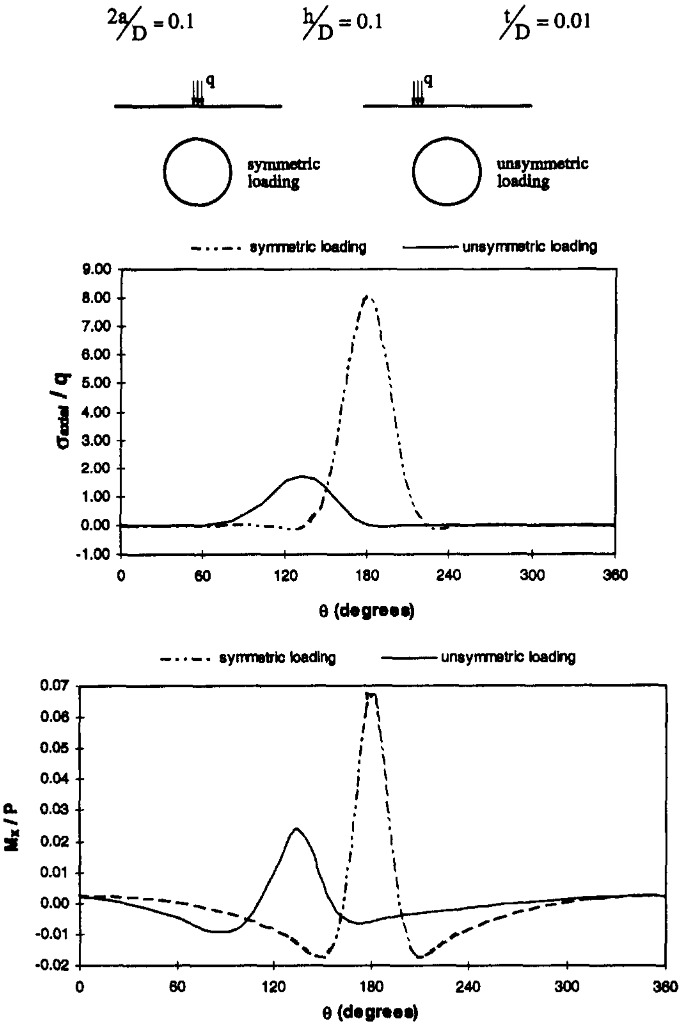

FIG. 10. Typical Axial Stress and Bending Moment for Symmetric and Nonsymmetric Loading

magnitude of force and moment that the pipe will be required to carry. The ratio of the pipe to soil moduli also influenced the forces and moments acting in the pipe, but over the range investigated this ratio appeared to be not as important as the relative thickness. The effects of concentrated loading reduce rapidly with distance along the pipe, and the forces and moments due to the surface loading reduce with increasing soil cover.

APPENDIX I. REFERENCES

Carter, J. P., and Balaam, N. P. (1995). *AFENA—a general finite element program for geotechnical engineering*, School of Civil and Mining Engineering, University of Sydney, Sydney, Australia.

Fernando, N. S. M., Small, J. C., and Carter, J. P. (1996). "Elastic analysis of buried structures subject to three dimensional surface loadings." *Int. J. Numer. and Analytical Methods in Geomech.*, 20, 331–349.

Katona, M. G., Meinhert, D. F., Orvillac, R., and Lee, C. H. (1979). "Structural evaluation of new concepts for long-span culverts and culvert installations." *Rep. No. FHWA-RD-79-115*, prepared for U.S. Federal Highway Administration, Washington, D.C.

Moore, I. D., and Brachman, R. W. I. (1994). "Three-dimensional analysis of flexible circular culverts." *J. of Geotech. Engrg.*, ASCE, 120(10), 1829–1844.

Small, J. C., and Wong, H. K. W. (1988). "The use of integral transforms in solving three dimensional problems in geomechanics." *Comp. and Geotechnics*, 6, 99–216.

Zienkiewicz, O. C., and Taylor, R. L. (1977). *The finite element method.* McGraw-Hill Book Co., London, U.K.

APPENDIX II. NOTATION

The following symbols are used in this paper:

a = half-width of surface patch loading;
D = diameter of circular pipe;

E_p = effective Young's modulus of pipe material;
E_s = Young's modulus of elastic soil;
H = height of soil cover over circular pipe;
M = bending moment in pipe;
M_x = maximum in-plane bending moment in pipe section;
M_z = maximum longitudinal bending moment in pipe;
P = total surface load;
q = intensity of uniform surface pressure;
t = effective thickness of pipe section;
x = horizontal Cartesian coordinate in cross section of circular pipe;
y = vertical Cartesian coordinate;
z = horizontal Cartesian coordinate along pipe axis;
θ = angle around pipe section, measured from pipe invert; and
σ_{axial} = maximum hoop thrust circumferential stress in pipe section.