Geotech Geol Eng (2011) 29:145–159
DOI 10.1007/s10706-009-9276-x

ORIGINAL PAPER

Shear Strength Criteria for Unsaturated Soils

Daichao Sheng · Annan Zhou · Delwyn G. Fredlund

Received: 9 May 2009 / Accepted: 24 September 2009 / Published online: 7 October 2009

Abstract Shear strength is one of the fundamental properties of unsaturated soils. It has been found to change with matric suction. Various shear strength equations have been proposed for predicting the shear strength versus suction relationship for unsaturated soils. Some of these equations are based on regression analysis of experimental data, while some are embodied in more complex stress–strain constitutive models. In this paper, a variety of shear strength equations are examined and compared with respect to their fit of experimental data. Data for specimens prepared from initially slurry conditions as well as data for initially compacted soil specimens are analysed. The advantages and limitations associated with various proposed shear strength equations are discussed in this paper.

Keywords Shear strength criteria · Unsaturated soil · Triaxial · Direct shear · Matric suction

D. Sheng (✉) · A. Zhou
Centre for Geotechnical and Materials Modelling, School of Engineering, The University of Newcastle, Newcastle, NSW 2308, Australia
e-mail: daichao.sheng@newcastle.edu.au

A. Zhou
e-mail: annan.zhou@newcastle.edu.au

D. G. Fredlund
Golder Associates Ltd., Saskatoon, SK, Canada
e-mail: del_fredlund@golder.com

1 Introduction

Shear strength is an important engineering property required in the design of numerous geotechnical and environmental structures such as foundations, earth dams, retaining walls, and pavement subgrades. The contribution of matric suction to the shear strength of unsaturated soils has attracted wide attention (Fredlund et al. 1978, 1996; Escario and Saez 1986; Vanapalli et al. 1996; Vanapalli and Fredlund 2000; Pham 2005). The contribution of matric suction to shear strength of soils generally results in a significant increase in bearing capacity and the factor of safety associated with slope stability calculations (Rassam and Williams 1999; Fredlund and Rahardjo 1993). Rahardjo et al. (1995) illustrated the importance of matric suctions with respect to the stability of residual soil slopes in Singapore. Oloo et al. (1997) concluded that matric suction could have a significant effect on the bearing capacity of thin pavement structures. Rassam and Williams (1997) demonstrated that the stability of tailing dams was enhanced by nearly 30% when the matric suction contribution to shear strength was taken into account in the slope stability calculations.

During the past two decades or so, numerous laboratory testing studies have been preformed in order to better understand the contribution of matric suction to the shear strength of unsaturated soils. These laboratory tests can be classified into two groups:

(1) Suction controlled tests using either a direct shear apparatus or a triaxial testing apparatus. Soil suction can be controlled using the axis-translation technique or the osmotic suction technique. The suction is usually kept constant during the application of shear stresses (Figs. 1a, 2a). In the suction controlled tests, the stress states (i.e., net stress and suction) and stress paths are usually pre-defined and controlled. The data analysed in this study were obtained using the axis-translation technique.
(2) Undrained triaxial tests where the gravimetric water content is kept constant while suction may change during the tests. Suction variation during the shear process is monitored (Figs. 1b, 2b). The undrained method is usually used for specific engineering applications or for the validation of constitutive models since the stress states are not controlled. Results from this type of tests are not analysed in this study.

Suction controlled tests are more commonly performed than undrained or constant water content tests because the stress states can be controlled. Suction controlled tests can also be considered as drained tests in a sense that water and air are allowed to flow in and out of the specimen in order to maintain the applied matric suction. Suction controlled triaxial tests can be used to determine both the shear strength and deformation characteristics of unsaturated soils. Suction controlled triaxial extension tests (Sun et al. 2000) and hollow cylinder tests (Toyota et al. 2001, Toyota et al. 2003) can be used to study the three-dimensional shear strength behaviour of unsaturated soils. Modified direct shear tests (Gan et al. 1988) are usually used for determining shear strength of unsaturated soils with increased plasticity because the length of the drainage path is reduced (Fig. 2a).

The methods for preparing soil specimens should be highlighted as well as drainage conditions since the soil fabric of the soil can affect the results. All published test results can be classified into the following categories with regard to specimen preparation:

(1) Slurry soil can be first consolidated to a specified applied pressure, i.e., a preconsolidation pressure. Then the soil specimens are allowed to air-dry to various suction levels. It is also possible to dry the specimens through the direct application of air pressure and water pressure, i.e., an applied matric suction. The specimens are then sheared to failure while controlling air and water pressures to maintain constant suction conditions. The results provide the relationship

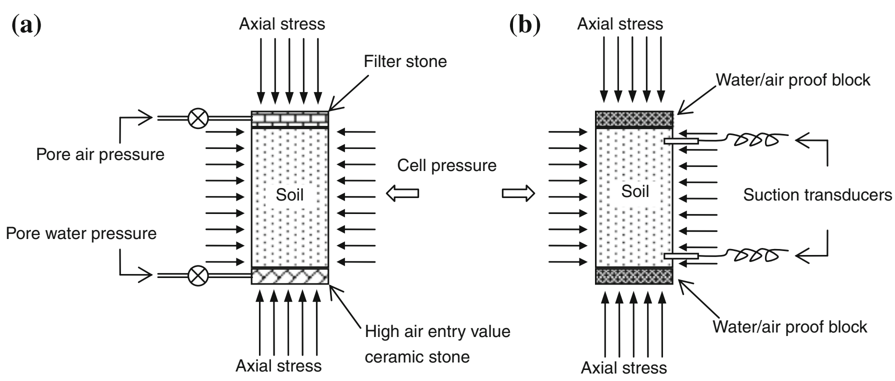

Fig. 1 Illustration of **a** suction controlled and **b** undrained triaxial shear test

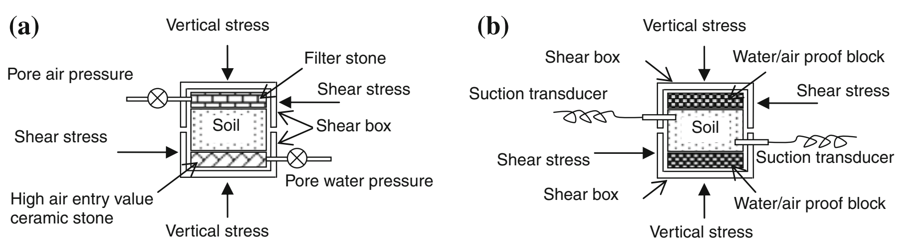

Fig. 2 Illustration of **a** suction controlled and **b** undrained direct shear test

between the shear strength and consolidation pressure (i.e., cell pressure) under different suctions conditions.

(2) Compacted specimens can be prepared by compacting soils that have been pre-mixed with water. Either static or dynamic compaction can be utilized. The water content of the soil specimens can be controlled during compaction. The matric suction is controlled through the application of various air and water pressures that allow the specimens to be tested under constant suction conditions.

(3) The third category soil specimens is that of undisturbed samples taken from the field. However, very few laboratory research studies have been conducted using undisturbed soil samples. This area is in need of further research.

Soil specimens air-dried from a slurry condition usually requires more time, particularly for soils with high clay content and hence a high air entry suction. However, these soil specimens can potentially provide more fundamental information on the shear strength of unsaturated soils. The stress state and stress path applied to a specimen are well-defined for specimens air-dried from slurry. Compacted specimens are relatively easy to prepare in laboratory, but the stress path and the preconsolidation pressure (i.e., yield stress) are not well defined. The difference in the yield stresses between air-dry slurry specimens and compacted specimen can be substantial (Zhou and Sheng 2009).

Some general past observations on the shear strength of unsaturated soils are as follows (Escario and Saez 1986; Fredlund and Rahardjo 1993; Vanapalli et al. 1996; Wheeler and Sivakumar 2000; Cunningham et al. 2003):

(1) Under the same vertical pressure (or confining pressure), higher matric suctions result in higher shear strengths;

(2) Under the same suction, higher vertical pressures (or confining pressures) result in higher shear strengths;

(3) The relationship between shear strength and the matric suction is nonlinear. The shear strength increases most rapidly at low matric suction levels, and then gradually flattens (or even decreases) at high suctions.

(4) It is generally more effective to increase the shear strength by increasing vertical stress (or confining pressure) than by increasing matric suction.

Several failure equations have been proposed in the literature to predict the shear strength of unsaturated soils. Some of these equations have been based on regression analysis of experimental data from direct shear and triaxial tests (e.g., Fredlund et al. 1996; Vanapalli et al. 1996; Toll and Ong 2003). Other shear strength equations are embodied within the so-called effective stress definitions and constitutive models for unsaturated soils (e.g., Alonso et al. 1990; Khalili and Khabbaz 1998; Sun et al. 2000; Sheng et al. 2008). The former usually contains additional material parameters as a result of being related to the soil–water characteristic curves. The latter shear strength equations are usually quite simple and form a part of a more complete elasto-plastic constitutive model. In addition, all these equations can be formulated based on two basic frameworks: the Bishop stress framework and the Fredlund independent stress framework, which is discussed below.

2 Shear Strength Criteria for Unsaturated Soils

2.1 Strength Criteria Based on Bishop Stress Variables

A group of shear strength equations for unsaturated soils are based on the effective stress equation by Bishop and Blight (1963). In this group, the shear strength was assumed to be governed by a single effective stress and the failure effective stress was assumed to be uniquely related to matric suction. The shear strength equations are formulated by extending Terzaghi's effective stress equation for saturated soils. The framework based on the effective stress by Bishop and Blight can be expressed as follows,

$$\begin{aligned}\tau &= c\prime + (\sigma_n - u_a)\tan\phi\prime + \chi(u_a - u_w)\tan\phi\prime \\ &= c\prime + \bar{\sigma}_n \tan\phi\prime + \chi s \tan\phi\prime\end{aligned} \tag{1}$$

or in terms of stress invariants, Eq. 1 can be written as,

$$q = 2c\prime + M(\bar{p} + \chi s) \quad (2)$$

In the above equations, τ is the shear strength of an unsaturated soil, ϕ' is the effective angle of internal friction, c' is effective cohesion, σ_n is the normal stress, $\bar{\sigma}_n$ is the net normal stress, u_a is the pore-air pressure, u_w is pore-water pressure, s is matric suction, χ is a material parameter that was originally assumed to be a function of the degree of saturation (S_r), q is the deviator stress, $\bar{p}$is the net mean stress, and M is the slope of the failure or critical state line in the $\bar{p} - q$ space and is a function of the effective angle of internal friction (ϕ').

Dependent on the determination of the parameter χ, there exist a number of different shear strength equations in this group. One of the popular forms is

$$\chi = S_r \quad (3)$$

The first to use the degree of saturation as χ in the effective stress definition for unsaturated soil modelling appears to be Schrefler (1984). The first to use $\chi = S_r$ and Eq. 1 to interpret shear strength data of unsaturated soils is perhaps Oberg and Sallfors (1997). This form of Bishop's effective stress has also been used by other researchers in modelling unsaturated soils (e.g., Sheng et al. 2003, 2004; Nuth and Laloui 2008). For simplicity, the shear strength Eq. 1 with $\chi = S_r$ is referred to as the equation of Oberg and Sallfors (1997) in this paper, even though many others have used the same equation.

Khalili and Khabbaz (1998) observed that Bishop's effective stress with $\chi = S_r$ did not appear to agree closely with measured shear strength data and proposed an alternative effective stress parameter:

$$\chi = \begin{cases} 1 & s < s_e \\ \left(\frac{s_e}{s}\right)^r & s \geq s_e \end{cases} \quad (4)$$

where s_e is the air entry suction for the soil and r is a material parameter. In this paper, r is set to 0.55 as suggested by Khalili and Khabbaz (1998).

Tarantino and Tombolato (2005) and Tarantino (2007) suggested that the ultimate shear strength of an unsaturated soil could be modelled in terms of the Bishop effective stress provided the degree of saturation considered only the macro pores, S_{rM}. Consequently, the conventional degree of saturation term, S_r, needed to be replaced with a term, S_{rM} that was defined as follows,

$$\chi = S_{rM} = \frac{\theta - \theta_{\mathrm{m}}}{\theta_s - \theta_{\mathrm{m}}} \quad (5)$$

where θ is the volumetric water content, θ_s is the volumetric water content at the fully saturated state, θ_{m} is defined as the microstructural water ratio and is usually assumed to correspond to the residual volumetric water content, θ_r. If θ_{m} is replaced by residual volumetric water content, θ_r in Eq. 5, the shear strength equation of Tarantino and Tombolato (2005) becomes identical to that proposed by Vanapalli et al. (1996).

The equation by Khalili and Khabbaz (1998) can directly be used to calculate shear strength, with only two additional material parameters to define unsaturated states. In the shear strength failure criteria proposed by Bishop and Blight (1963) and Tarantino and Tombolato (2005), shear strength is related to the degree of saturation (S_r) or the volumetric water content (θ). Both the degree of saturation (S_r) and volumetric water content (θ) are related to soil suction through the soil–water characteristic curves (SWCCs). Therefore, the number of material parameters associated with the failure criteria depends on the specific SWCC equation. In addition, it must be noted that there is no single unique SWCC. Rather, there is a bounding drying (or desorption) curve and a wetting (or adsorption) curve, as well as states that lie between the bounding curves. This complexity can be viewed as possibly incorporating further soil parameters or it can be viewed as incorporating further flexibility to the shear strength model.

2.2 Strength Criteria Based on Independent Stress Variables

Fredlund et al. (1978) proposed that the shear strength of an unsaturated soil can be formulated in terms of two independent stress state variables, namely the net stress and the suction. The Mohr–Coulomb criterion is then extended in the following form (Fredlund et al. 1978):

$$\tau = c\prime + (\sigma_n - u_a)\tan\phi\prime + (u_a - u_w)\tan\phi^b \quad (6)$$

where ϕ^{b} represents the contribution to the shear strength due to matric suction and can be a complex function of suction and other variables. The separation of the suction contribution from the stress contribution in the shear strength equation is more appealing to practical engineers (Fredlund and Rahardjo 1993).

Early data sets indicated that the ϕ^b angle might be relatively constant over a limited change in suction typical of what might be experienced in situ. However, subsequent data sets showed that there was considerable nonlinearity in the ϕ^b angle when the entire range of soil suction was considered. Consequently, this gave rise to nonlinear forms for the shear strength criterion. It should also be noted that the framework (6) can in general not be reduced to the single effective stress approach, i.e. Eq. 1, if $\tan \phi^b$ is not a linear function of $\tan \phi'$.

A number of shear strength equations based on the independent stress framework have been proposed since mid 1990s. Most of these equations were related to the soil–water characteristic curve and the nonlinearity of the SWCC would suggest that there was nonlinearity in the shear strength relationship. An example of the nonlinear shear strength equations was the following equation proposed by Fredlund et al. (1996). The equation takes the form of a nonlinear function for predicting the shear strength for an unsaturated soil.

$$\tan \phi^b = \left(\frac{\theta}{\theta_s}\right)^{\kappa} \tan \phi\prime = (S_r)^{\kappa} \tan \phi\prime \tag{7}$$

where θ is volumetric water content, θ_s is the volumetric water content at full saturation, and κ is a fitting parameter. Equation 7 recovers that of Oberg and Sallfors (1997) when κ is set to 1. It is also used in constitutive models for unsaturated soils, e.g., that by Sheng et al. (2003), with $\chi = (S_r)^{\kappa}$ and $\kappa = 1$ or 0.5.

Equation 7 by Fredlund et al. (1996) was simultaneously proposed by Vanapalli et al. (1996). More recently, Garven and Vanapalli (2006) related the parameter κ to the soil plastic index. Vanapalli et al. (1996) also suggested an alternative shear strength equation that involved a normalization of the soil–water characteristic curve between saturated conditions and residual conditions:

$$\tan \phi^b = \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right) \tan \phi\prime \tag{8}$$

Other examples belonging to this group of nonlinear shear strength functions have been reported by Bao et al. (1998), Rassam and Cook (2002), Miao et al. (2002), Tekinsoy et al. (2004), Xu (2004) and Lee et al. (2005). Some of the proposed functions contain more material parameters that need to be defined and are not included in the comparative study in this paper.Toll (1990) and Toll and Ong (2003) presented data for gravel and sandy clay, respectively, and suggested the following equation for the critical state line:

$$q = M_a \bar{p} + M_b s \tag{9}$$

where,

$$M_b = M\left(\frac{S_r - S_{r2}}{S_{r1} - S_{r2}}\right)^k \tag{10}$$

and,

$$M_a = M\left\{\left(\frac{M_a}{M}\right)_{\max} - \left[\left(\frac{M_a}{M}\right)_{\max} - 1\right]\left(\frac{S_r - S_{r2}}{S_{r1} - S_{r2}}\right)^k\right\} \tag{11}$$

In the above equations, S_{r1} and S_{r2} are two reference degrees of saturation, k is a fitting parameter, M is the slope of the critical state line for saturated states, and $\left(\frac{M_a}{M}\right)_{\max}$is the maximum value of $\frac{M_a}{M}$ which can be viewed as a material parameter. The authors suggested using full saturation as S_{r1} (i.e., $S_{r1} = 1$), and the residual suction as S_{r1} (i.e., $S_{r2} = S_{\text{re}}$, where S_{re} is the degree of saturation at residual suction).

2.3 Shear Strength Equations Used in Constitutive Models

In addition to the two categories of shear strength equations mentioned above, there are also a number of elastoplastic constitutive models that have also incorporated specific shear strength equations. Perhaps the simplest such equation is the one proposed by Alonso et al. (1990):

$$q = M(\bar{p} + \bar{p}_0) = M(\bar{p} + \alpha s) \tag{12}$$

where$\bar{p}_0$ is the apparent tensile strength, and α is a material parameter that was initially assumed to be a constant (Alonso et al. 1990), but can be a function of suction. In the above equation effective cohesion (c'), is assumed to be zero. This is a common assumption associated with the critical state line in $\bar{p} - q$ space.

Sun et al. (2000) suggested another equation to describe the failure envelope (i.e., critical state line) for unsaturated soils. The equation is as follows,

$$q = M(s)[\bar{p} + \bar{\sigma}_0(s)] \quad (13)$$

where,

$$M(s) = M(0) + M_s\bar{\sigma}_0(s) \quad (14)$$

and,

$$\bar{\sigma}_0(s) = \frac{s}{1 + s/a} \quad (15)$$

where, $M(0) \equiv M$, is the slope of the critical state line for saturated soils, M_s is a fitting parameter, and a is a constant equal to the maximum stress, $\sigma_0(s)$ when the soil is subjected to an infinite suction. The maximum stress, $\sigma_0(s)$ depends on the type of unsaturated soil. Sun et al. (2000) also extended the critical state equation for triaxial compression to 3-dimensional stress states using transform stress tensors based on the Matsuoka–Nakai criterion (Matsuoka et al. 1999).

An alternative shear strength equation is embodied in the recent SFG model proposed by Sheng et al. (2008). In this model, the following apparent tensile strength equation was proposed:

$$\bar{p}_0 = \left\{ \begin{array}{ll} -s & s < s_{sa} \\ -s_{sa} - (s_{sa} + 1)\ln\frac{s+1}{s_{sa}+1} & s \geq s_{sa} \end{array} \right\} \quad (16)$$

where s_{sa} is called the saturation suction. The saturation suction represents the unique transition suction between saturated and unsaturated states in the SFG model (Sheng et al. 2008). The air entry suction is defined as the meeting point between the line approximating the drying (or desorption) curve and the horizontal line representing full saturation. The saturation suction is defined as the meeting point between the line approximating the wetting (or adsorption) curve and the horizontal line representing full saturation. The saturation suction is also called the air expulsion suction. However, the name of air expulsion is avoided because it implies the existence of an air entry suction which is not used in the SFG model. The initial portion of the drying curve between the saturation suction and the straight line approximating the main drying curve is represented by the scanning line (Fig. 3). Therefore, the saturation suction is the unique transition suction between saturated and unsaturated states. In absence of the main wetting curve, the saturation suction is determined by fitting the soil–water characteristic curve with four straight lines: a horizontal line for fully

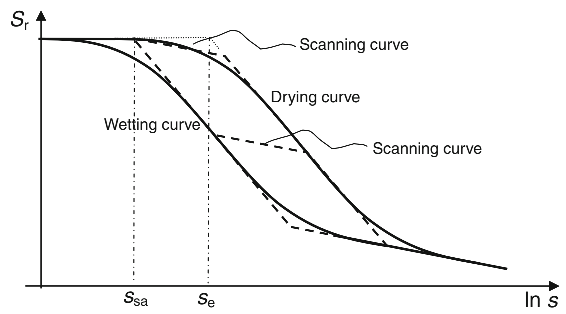

Fig. 3 Saturation suction and air entry suction

saturated states, a scanning line approximating the initial portion of the SWCC, a straight line approximating the main drying curve and a scanning line for suctions above the residual suction (Fig. 3).

The above-mentioned shear strength equations have been proposed as part of an elastoplastic model and can be directly used to calculate the shear strength of unsaturated soils. The model by Alonso et al. (1990) and Sheng et al. (2008) both use one material parameter (α or s_{sa}) in addition to the conventional saturated shear strength parameters (i.e., c' and ϕ'). The model by Sun et al. (2000) uses two additional material parameters (M_s and a).

2.4 Relationship Between Shear Strength and SWCC

The equations proposed by Fredlund et al. (1996), Vanapalli et al. (1996), Oberg and Sallfors (1997), Toll and Ong (2003) and Tarantino and Tombolato (2005) all make use of the degree of saturation or water content, and hence depend on SWCC. Consequently, this group of shear strength equations require the material parameters associated with the SWCC. Hysteresis associated with the SWCC would also suggest that there might be two shear strength envelopes; one corresponding to drying conditions and one corresponding to wetting conditions. A laboratory study reported by Melinda et al. (2004) suggests that there is some difference between the shear strength envelopes for drying and wetting; however, it has not been proven whether similar observed differences can be predicted from the drying and wetting SWCCs. In addition, the initial soil density and applied stress states (i.e., total stress applied when measuring the SWCC), are known to produce changes in the measured SWCC (Sun et al. 2007; Ng and Pang 2000).

The SWCC equation proposed by Fredlund and Xing (1994) is used in combination with the shear strength equations when shear strength data is compared. The Fredlund and Xing equation can be expressed as follows.

$$S_r = \frac{\theta}{\theta_s} = \frac{C(s)}{\left\{\ln\left[2.71828 + \left(\frac{s}{a}\right)^n\right]\right\}^m} \tag{17}$$

where

$$C(s) = 1 - \frac{\ln\left(1 + \frac{s}{s_{re}}\right)}{\ln\left(1 + \frac{10^6}{s_{re}}\right)} \tag{18}$$

s_{re} is the residual suction; a, n and m are three fitting parameters. Only the drying branch (i.e., the main drying curve), is used during the present analysis of shear strength results. This curve is assumed to be independent of soil density or stress state.

2.5 Relationship Between Different Shear Strength Equations

It should be mentioned that the strength equations in the three groups have no essential difference. They are all empirical and phenomenological in nature and all based on the basic assumption that the shear strength of a soil can be interpreted by fundamental properties like c' and ϕ'. One would hope that these soil properties are so fundamental that they are independent of the soil initial conditions and the testing conditions. However, the reality is perhaps not always so. For example, c' and ϕ' of a soil may depend on its initial condition (e.g., void ratio, water content, initial structure), leading to questions like "when a soil is a new soil?" (Fredlund 1989). On the other hand, there are some experimental evidence that the critical state shear strength of a soil is less dependent on the soil initial conditions and testing conditions (Muir Wood 1990). For unsaturated soils, the slope of the critical state line is usually assumed to be independent of suction and there are some experimental data supporting such an assumption (e.g., Toll 1990; Ng and Chiu 2001; Thu et al. 2006; Nuth and Laloui 2008). Indeed, in all the shear strength equations studied in this paper, the friction angle due to stress (ϕ') is assumed to be independent of suction and the friction angle due to suction (ϕ^b) is assumed to be independent of stress. All the equations mentioned above can be written either in the form of Eq. 1 as suggested by Bishop and Blight (1963), or in the form of Eq. 6 as suggested by Fredlund et al. (1978). The main differences between the various shear strength equations are the specific form of mathematical functions adopted and the material parameters used in the equations. These differences actually govern how well the equation can predict the shear strength for different unsaturated soils. It is also generally true that a function with more parameters tend to be more flexible in fitting different data sets.

The above-mentioned shear strength equations can all be formulated either in terms of stress invariants, q and $\bar{p}$, or in terms of net normal stress and shear stress. The latter formulation can also be used for the interpretation of direct shear test results. All equations can be converted into this form using the net normal stress and matric suction as suggested by Fredlund et al. (1978). The equations written in terms of the stress invariants q and $\bar{p}$ can be used for triaxial tests. All equations can be converted into the stress invariant form using the concept of an apparent cohesion, $\bar{p}_0$. The $\bar{p}-q$ form can also be converted to the effective stress form using $p\prime = \bar{p} + \bar{p}_0$. A summary of these equations is given in Table 1, where the number of material parameters associated with each shear strength equation is also given. The parameter $M(s)$ in the equation by Sun et al. (2000) and M_a in the equation by Toll and Ong (2003) are all assumed to be equal to the slope of the critical state line for fully saturated soils (M) in this paper.

3 Comparisons Between Experimental Results and Predictions

3.1 Prediction of Test Data for Soils Air-Dried from Slurry Condition

Cunningham et al. (2003) presented triaxial compression tests performed under various confining pressures on the reconstituted silty clay that comprised a mix of 20% pure Speswhite kaolin, 10% London clay and 70% silica silt. The slurry soil was isotropically preconsolidated to 130 kPa. The SWCC as well as the fitting parameters for Fredlund and Xing's equation (1994) are shown in Fig. 4.

Table 1 Two general frameworks of various strength equations

General frameworks	$q = M(\bar{p} + \bar{p}_0)$	$\tau = \bar{\sigma}_n \tan\phi\prime + s\tan\phi^b$		No. of parameters
No.	Equations*	$\bar{p}_0$	$\tan\phi^b$	
1	Oberg and Sallfors	sS_r	$S_r \tan\phi\prime$	SWCC†
2	Fredlund et al.	$(S_r)^{\kappa} s$	$(S_r)^{\kappa}\tan\phi\prime$	1, SWCC
3	Vanapalli et al.	$\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)s$	$\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)\tan\phi\prime$	2, SWCC
4	Toll and Ong	$\left(\frac{S_r-S_{r2}}{S_{r1}-S_{r2}}\right)^k s$	$\left(\frac{S_r-S_{r2}}{S_{r1}-S_{r2}}\right)^k \tan\phi\prime$	2, SWCC
5	Alonso et al.	αs	$\alpha\tan\phi'$	1
6	Sun et al.	$\frac{as}{s+a}$	$\frac{a}{s+a}\tan\phi\prime$	1
7	Khalili and Khabbaz	$\left(\frac{s_e}{s}\right)^r s$ or s	$\left(\frac{s_e}{s}\right)^r \tan\phi'$or $\tan\phi'$	2
8	Sheng et al.	$s_{sa} + (s_{sa}+1)\ln\frac{s+1}{s_{sa}+1}$ or s	$\tan\phi'\left(\frac{s_{sa}}{s} + \left(\frac{s_{sa}+1}{s}\right)\ln\frac{s+1}{s_{sa}+1}\right)$ or $\tan\phi'$	1

* c' the effective cohesion of the soil, is assumed to be zero for all equations here

† There are 4 parameters in the SWCC equation by Fredlund and Xing (1994)

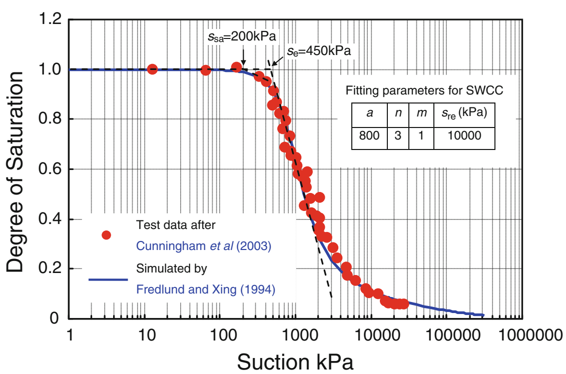

Fig. 4 Soil–Water characteristic curve of reconstituted silty clay

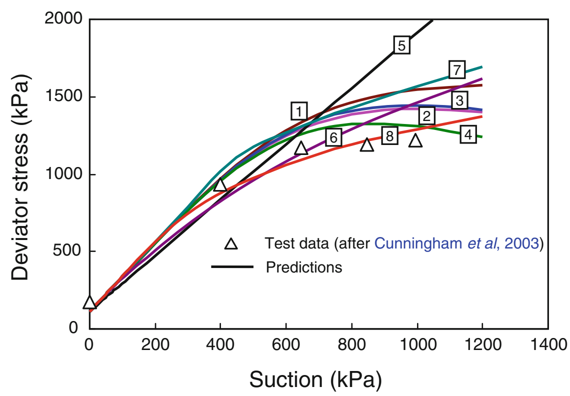

Fig. 6 Predictions of the triaxial test data on air-dry silty clay (confining pressure: 50 kPa)

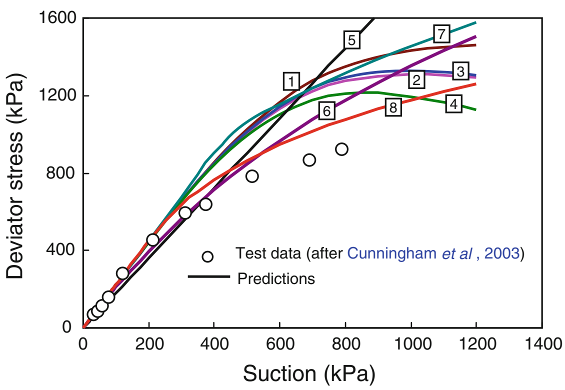

Fig. 5 Predictions of the unconfined triaxial test data on air-dry silty clay

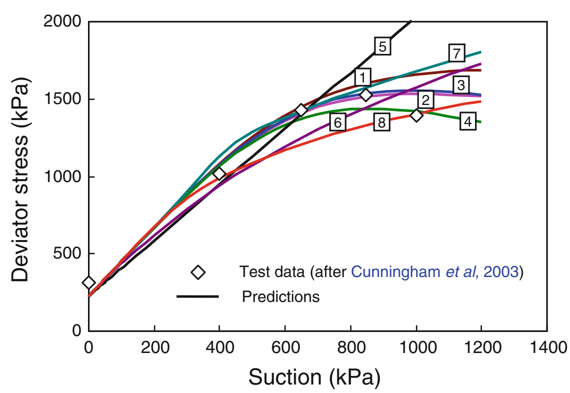

Fig. 7 Predictions of the triaxial test data on air-dry silty clay (confining pressure: 100 kPa)

The results from triaxial compression tests on the reconstituted silty clay are plotted for different confining pressures in Figs. 5, 6, 7 and 8. The parameters for the saturated soil are as follows: $\phi' = 32°$, c' kPa, as given in Cunningham et al. (2003). Except for the equations by Khalili and

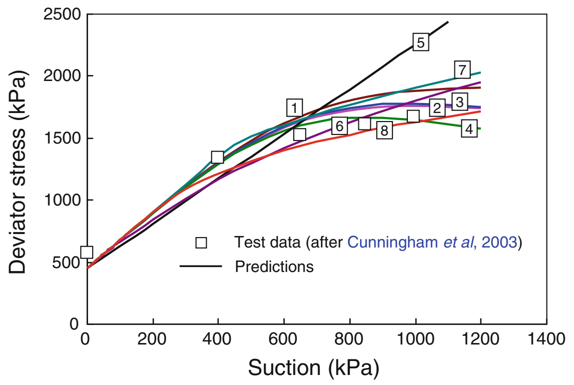

Fig. 8 Predictions of the triaxial test data on air-dry silty clay (confining pressure: 200 kPa)

Khabbaz (1998) and Sheng et al. (2008), all other equations require additional parameters. These additional parameters are best fit for the test results in Figs. 4, 5, 6, 7 and 8 and the values are listed in Table 2. The air entry value is set to 450 kPa and the saturation suction 200 kPa, according to Fig. 4. These values are consistent with the observation by Cunningham et al. (2003), where it was stated desaturation commences at a suction of about 250 kPa and begins to become significant at suctions in excess of 400 kPa. Both the air entry value and the saturation suction are assumed to be independent of the confining pressure.

Figure 5 shows the predictions of shear strength for the unsaturated soil for the case when the soil specimens are unconfined. All equations overestimate the shear strength data when the specimens are unconfined. The prediction closest to the experimental data is Eq. 8 of Sheng et al. (2008) in Table 1. An apparent difference between the simple equations, i.e. Eqs. 6–8 of Khalili and Khabbaz (1998), Sun et al. (2000) and Sheng et al. (2008), respectively, and the advanced ones based on the SWCC, Eqs. 1–4 of Oberg and Sallfors (1997), Fredlund et al. (1996), Vanapalli et al. (1996) and Toll and Ong (2003), respectively, is that the shear strength predicted by the former group seems to flatten out slower than that by the latter group. Equation (5) of Alonso et al. (1990) predicts a linear relationship between the shear strength and the suction and the parameter α is adjusted to fit the strength data at an intermediate suction.

Table 2 Additional parameters for shear strength equations for reconstituted silty clay

No.	Equations	Additional parameters
1	Oberg and Sallfors	SWCC
2	Fredlund et al.	$\kappa = 1.2$, $\theta_s = 0.6032$, SWCC
3	Vanapalli et al.	$\theta_s = 0.6032$, SWCC
4	Toll and Ong	$k = 1.2$, SWCC
5	Alonso et al.	$\alpha = 0.8$
6	Sun et al.	$a = 1{,}500$ kPa
7	Khalili and Khabbaz	$s_e = 450$ kPa, $r = 0.55$
8	Sheng et al.	$s_{sa} = 200$ kPa

Figure 6 shows the measured shear strength and the predictions of shear strength for the case when the cell pressure is equal to 50 kPa. The closest predictions of shear strength are given by Eq. 8 of Sheng et al. (2008) in Table 1, followed by Eq. 4 of Toll and Ong (2003). The shear strength equations based on the SWCC, i.e. Eqs. (1–4), seem to capture the pattern of the data, but somewhat overestimate the shear strength at low suctions. A possible reason for the overestimation by Eq. 3 of Vanapalli et al. (1996) and (4) of Toll and Ong (2003) is perhaps the residual suction (s_{re}), which is determined by fitting the soil–water characteristic data in Fig. 4. Because of the logarithmic scale used in the suction axis, a slight change in the fitting can result in a significant change in the residual suction. Using a larger residual suction in the equations by Vanapalli et al. (1996) and Toll and Ong (2003) would reduce the predicted shear strength.

Figure 7 shows the measured shear strength and the predictions of shear strength for the case when the cell pressure is equal to 100 kPa. The measured shear strength appears to reach a peak at suction around 800–900 kPa and then decreases a little. In this case, the shear strength equations based on the SWCC, i.e. Eqs. 1–4, give close predictions of shear strength, particularly Eq. 2 of Fredlund et al. (1996), Eq. 3 of Vanapalli et al. (1996) and Eq. 4 of Toll and Ong (2003). The shear strength equation by Sheng et al. (2008), i.e. Eq. 8 in Table 1, gives reasonable predictions at suctions of 400 and 1,000 kPa, but underestimates the strength at suctions of 600 and 800 kPa. Figure 8 shows the measured shear strength and the predictions of shear strength for the case when the cell pressure is equal to 200 kPa. In this case, the best prediction is given by Eq. 4 of Toll and Ong (2003), followed by Eq. 2 of Fredlund et al. (1996) and Eq. 3 of Vanapalli et al. (1996). Equation 7 of Khalili and

Khabbaz (1998) seems to give reasonable predictions for suctions up to around 800 kPa (or a suction ratio $\frac{s}{s_e} \approx 2$), and then significantly overestimates the shear strength at high suctions. The parameter r in Khalili and Khabbaz's equation is assumed to be constant in this paper, as suggested by the authors. If r was allowed to vary with the initial conditions of the soil or the confining pressure, the prediction would improve. In general, all shear strength equations that utilize the SWCC can fit well the strength data for high confining pressures, but are less successful for the data at low confining pressures. This is perhaps an indication that the dependence of the SWCC on confining pressure can not be neglected. The simpler equations can fit the strength data well over certain ranges of suctions, but tend to overestimate the strength at high suctions.

3.2 Prediction of Test Data of Compacted Soil Specimens

Thu et al. (2007) provided triaxial test data on the shear strength of a compacted kaolin under different soil suctions. The tested soil is a mixture of kaolin clay (15%) and silt (85%), compacted to the maximum dry density of 1.35 t/m^3 and the optimum water content of 22%. All specimens were statically compacted in 10 layers of equal thickness of 10 mm. The SWCC of the soil was obtained from a drying test under net vertical pressure of 100 kPa (Thu et al. 2008) and is shown in Fig. 9. The best-fit Fredlund and Xing's curve is also shown in Fig. 9.

The shear strength parameters for the saturated compacted kaolin were given by Thu et al. (2007) as: $\phi' = 31.1°$, $c' = 0$ kPa. Additional parameters used in various strength equations are listed in Table 3. The air entry value for the equation of Khalili and Khabbaz (1998) was set to 60 kPa according to Fig. 9. The saturation suction for the equation of Sheng et al. (2008) is set to 25 kPa according to the measured data in Fig. 9. All parameters associated with the SWCC, including the air entry and saturation suctions, are assumed independent of the confining pressure. The shear strength data under various confining pressures are shown in Figs. 10, 11 and 12, along with the predictions using various strength equations.

Figure 10 shows the measured and predicted deviator stresses under a net confining pressure of 100 kPa. The measured shear strength increases with matric suction in a nonlinear manner. The closest predictions of the shear strength data are given by Eq. 8 of Sheng et al. (2008) and (6) of Sun et al.

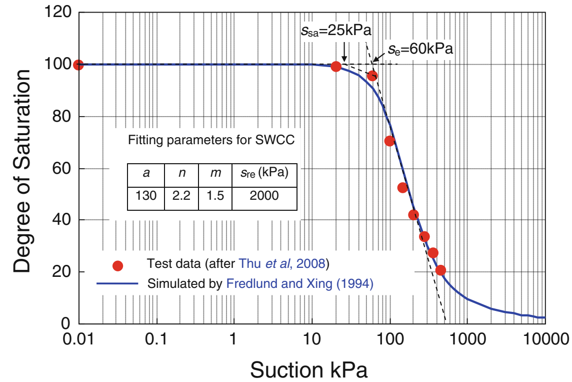

Fig. 9 Soil–Water characteristic curve for compacted kaolin clay

Table 3 Additional parameters for shear strength equations for compacted kaolin clay

No.	Equations	Additional parameters
1	Oberg and Sallfors	SWCC
2	Fredlund et al.	$\kappa = 1.3$, $\theta_s = 0.659$, SWCC
3	Vanapalli et al.	$\theta_s = 0.659$, SWCC
4	Toll and Ong	$k = 1.2$, SWCC
5	Alonso et al.	$\alpha = 0.4$
6	Sun et al.	$a = 110$ kPa
7	Khalili and Khabbaz	$s_e = 60$ kPa, $r = 0.55$
8	Sheng et al.	$s_{sa} = 25$ kPa

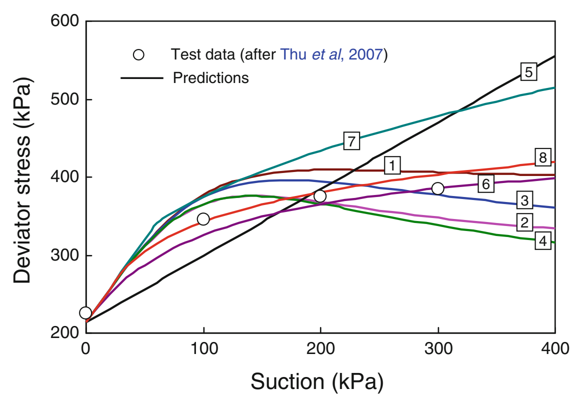

Fig. 10 Predictions of the triaxial test data on compacted kaolin clay (confining pressure: 100 kPa)

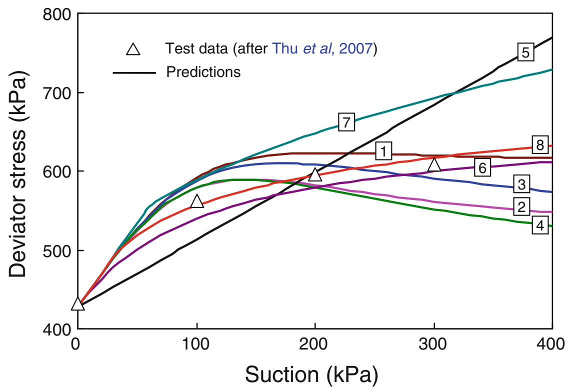

Fig. 11 Predictions of the triaxial test data on compacted kaolin clay (confining pressure: 200 kPa)

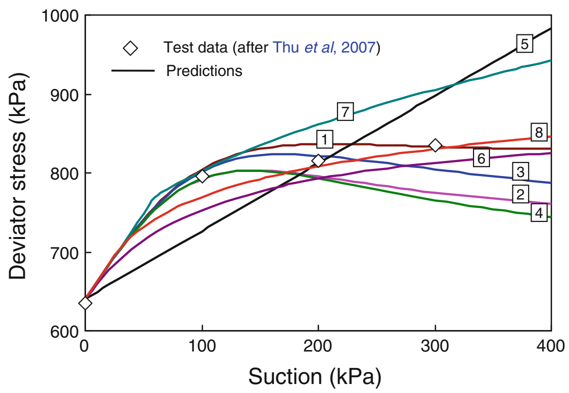

Fig. 12 Predictions of the triaxial test data on compacted kaolin clay (confining pressure: 300 kPa)

(2000). Equations 2 of Fredlund et al. (1996) and (4) of Toll and Ong (2003) give reasonable estimation of the shear strength at intermediate suctions. Equation 3 of Vanapalli et al. (1996) gives reasonable estimation at higher suctions. Equation 7 of Khalili and Khabbaz (1998) with an air entry value of 60 kPa predicts significantly higher shear strength than the measured data. Again, the predicted shear strength curves by the simpler equations (Alonso et al. 1990; Khalili and Khabbaz 1998; Sun et al. 2000; Sheng et al. 2008) do not flatten out as fast as those based on the SWCC (Fredlund et al. 1996; Vanapalli et al. 1996; Oberg and Sallfors 1997; Toll and Ong 2003).

The predictions of the shear strength data under net confining pressure of 200 kPa are shown in Fig. 11. The closest predictions of shear strength is given by Sheng et al. (2008), followed by Sun et al. (2000). The prediction by Eq. 1 of Oberg and Sallfors (1997) is reasonable at intermediate and high suctions. Equations 2 of Fredlund et al. (1996), (3) of Vanapalli et al. (1996) and (4) of Toll and Ong (2003) give reasonable predictions at intermediate suctions, but overestimate the shear strength at high suctions. Equations 5 of Alonso et al. (1990) and (7) of Khalili and Khabbaz (1998) also appear to overestimate shear strength at high suctions. The predictions of shear strength data under net confining pressure of 300 kPa are shown in Fig. 12. The prediction closest to the measured data is given by Sheng et al. (2008). The equations by Sun et al. (2000), Vanapalli et al. (1996) and Fredlund et al. (1996) all appear to give reasonable predictions.

The test data by Cunningham et al. (2003) and by Thu et al. (2007) are both limited to relatively lower suction values. Escario and Juca (1989) measured the direct shear strength for three types of clays under suctions up to 15 MPa. All the specimens were statically compacted (under identical water content and density conditions) and then allowed to come to equilibrium under pre-determined total normal stresses, applied air pressure and applied water pressure. The specimens were then sheared to failure under constant suctions in a modified direct shear apparatus. However, it was not mentioned if the recorded shear strength data refer to residual or peak shear stresses. Nevertheless, one set of data by Escario and Juca (1989) is used here for demonstration: the Madrid gray clay sheared under normal stress of 300 kPa.

The SWCC of the Madrid gray clay was obtained from drying tests and is shown in Fig. 13, where the best-fit Fredlund and Xing's curve is also shown. The shear strength parameters for saturated Madrid gray clay are as follows: $\phi' = 25.3°$, $c' = 30$ kPa (Escario and Juca 1989). The additional parameters used for comparing various strength equations are listed in the Table 4. The predictions of shear strength under net normal stress of 300 kPa are shown in Fig. 14. The measured shear strength appears to reach a peak around suction of 10,000 kPa and then decreases somewhat. Equations 2 of Fredlund et al. (1996), (6) of Sun et al. (2000) and (8) of Sheng et al. (2008) all give very good predictions. The prediction by Toll and Ong (2003), i.e. Eq. 4 in Table 1, is also reasonable. The prediction by Vanapalli et al. (1996), i.e. Eq. 3 in Table 1, is again sensitive to the residual suction. For the value used here (30,000 kPa), the prediction is not very good. Other

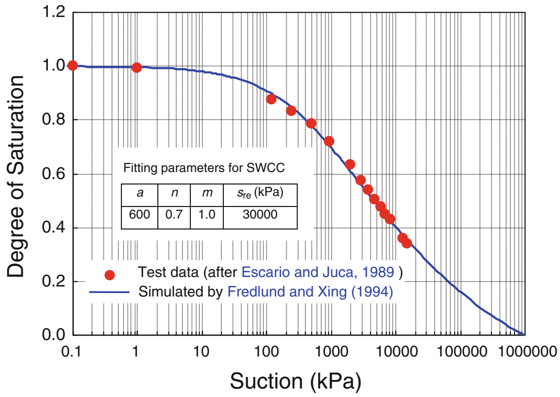

Fig. 13 Soil–Water characteristic curve for Madrid gray clay

Table 4 Additional parameters for strength equations for Madrid gray clay

No.	Equations	Additional parameters
1	Oberg and Sallfors	SWCC
2	Fredlund et al.	$\kappa = 2.8$, $\theta_s = 0.5074$, SWCC
3	Vanapalli et al.	$\theta_s = 0.5074$, SWCC
4	Toll and Ong	$k = 1.4$, SWCC
5	Alonso et al.	$\alpha = 0.1$
6	Sun et al.	$a = 800$ kPa
7	Khalili and Khabbaz	$s_e = 150$ kPa, $r = 0.55$
8	Sheng et al.	$s_e = 150$ kPa

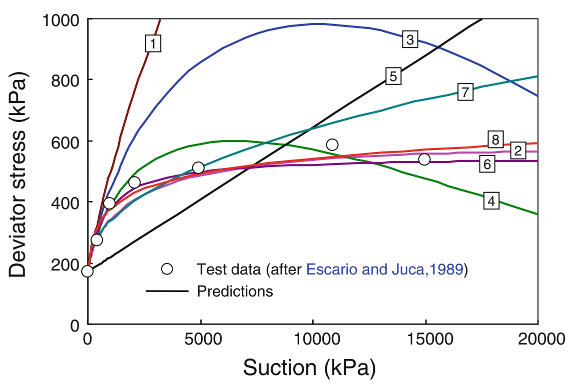

Fig. 14 Prediction of direct shear test data on Madrid gray clay (net normal stress: 300 kPa)

shear strength equations significantly overestimate the shear strength data. It is noted that Khalili and Khabbaz (1998) suggested their equation only applies to suction ratio of 15, which corresponds to suction of 2,250 kPa. For this data set, Eq. 7 of Khalili and Khabbaz (1998) seems to give reasonable predictions up to suction of 5,000 kPa.

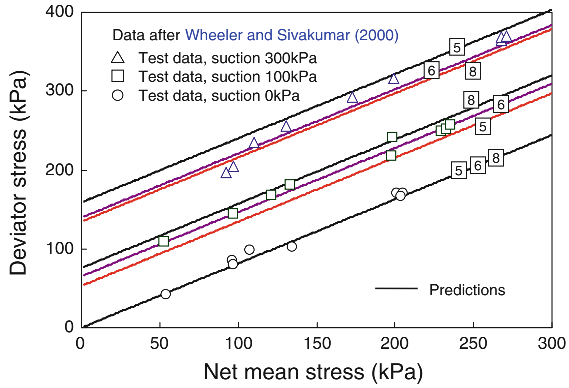

Fig. 15 Predictions of triaxial test data on compacted kaolin by Wheeler and Sivakumar (2000)

There is considerably more experimental data on the shear strength of unsaturated soils in the research literature. It is not surprising that the proposed shear strength equations will fit one data set better than another data set. For example, the simple equations by Alonso et al. (1990), Sun et al. (2000) and Sheng et al. (2008) appear to closely predict the triaxial data by Wheeler and Sivakumar (2000) on compacted specimens (see Fig. 15). It is expected that the other equations studied in this paper will also predict the data just as well as the three simple equations.

Another set of triaxial test data from Röhm and Vilar (1995) on a Brazilian sandy soil, is shown in Fig. 16. Again, the three simple equations appear to be able to closely predict the measured data. Indeed, it is expected that all the equations studied in this paper will closely predict the measured data provided the necessary parameters are carefully selected.

In summary, the shear strength equation by Oberg and Sallfors (1997) appears to capture the pattern of shear strength variation with suction, but somewhat overestimate the shear strength of unsaturated soils. The equations by Fredlund et al. (1996), Vanapalli et al. (1996) and Toll and Ong (2003) all give reasonable predictions, particularly for shear strength at intermediate and high suctions. These equations can all predict a peak shear strength at intermediate suction levels. The equations by Fredlund et al. (1996) and Toll and Ong (2003) offer additional flexibility in fitting experimental data through the parameter κ and k. It is noted that the predictions by Vanapalli et al. (1996) and Toll and Ong (2003) are sensitive to the

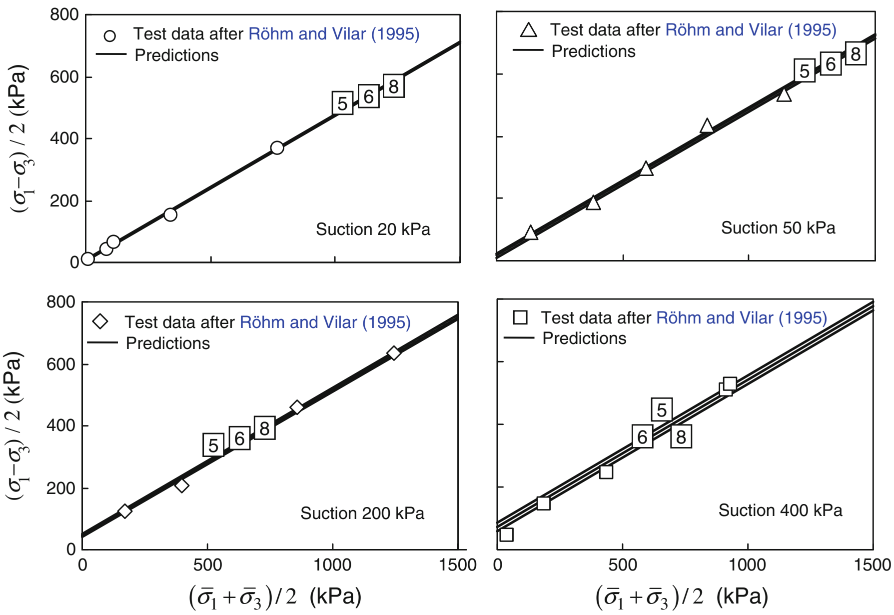

Fig. 16 Predictions of triaxial test data by Röhm and Vilar (1995)

selection of residual suction, which can be difficult to be accurate due to the logarithmic scale in the suction axis of soil–water characteristic curves.

Amongst the simpler equations, the equation by Sun et al. (2000) appears to provide a reasonable prediction of the shear strength for the compacted soils. The predictions for the air-dry soil are less successful. The equation by Sheng et al. (2008) seems to predict the shear strength for both compacted and air-dry unsaturated soil specimens with an acceptable degree of confidence. One special parameter used in Sheng et al. (2008) is the saturation suction. This parameter is not widely used and has a value smaller than the air entry suction. In absence of a SWCC, the saturation suction can be replaced by the air entry value as a first approximation. The equation by Khalili and Khabbaz (1998) appear to give reasonable predictions for small suction ratios ($\frac{s}{s_e}$), but tend to overestimate the shear strength at higher suctions. One common shortcoming of the simpler strength equations studied in this paper is that these equations do not predict a peak shear strength at intermediate suctions.

4 Conclusions

The shear strength equations proposed in the literature are presented in three groups; namely (i) equations based on Bishop's stress approach, (ii) equations based on the independent stress approach, and (iii) equations adopted in constitutive models. These shear strength equations are then used to predict laboratory data. One data set involves triaxial test results on reconstituted silty clay (Cunningham et al. 2003) and another data set involves triaxial test results on a compacted clay (Thu et al. 2007).

It should be pointed out that the comparative study in this paper is selective rather than comprehensive. Also, a number of shear strength equations have not been included, mainly due to difficulties in determining the necessary material parameters. The experimental data sets used in the comparative study are only representative of a larger amount of data available in the literature. The procedure of best-fitting material parameters is somewhat subjective. The determination of the residual suction can have some effect on the performance of some strength equations. Nevertheless, some general conclusions can be drawn from this comparative study:

1. The shear strength of a soil is a nonlinear function of suction. At low suction values (i.e., suctions below the air entry value), the shear strength increases in accordance with the effective angle of internal friction. The rate of shear strength increase with suction gradually

decreases as residual suction conditions are approached. The shear strength may even decrease somewhat after a peak value is attained near residual suction conditions.

2. The shear strength equations studied in this paper are empirical and phenomenological. These equations can all be formulated in the form of Bishop's effective stress approach or in the form of Fredlund's independent stress approach. The performance of these equations in predicting experimental data depends on the careful determination of material parameters and even on the specific data set. A shear strength equation may predict one data set better than other data sets.
3. The shear strength equations that incorporate material parameters from the soil–water characteristic curve require more parameters to define shear strength. This group of equations seem to provide reasonable prediction of shear strength for unsaturated soils. However, as the plasticity of the soil increases it may become somewhat difficult to determine the residual suction accurately.
4. Simpler shear strength equations that do not directly use the soil–water characteristic curve appear to be able to provide reasonable predictions of shear strength. These equations are usually embodied in more complex constitutive models for the mechanical behaviour of unsaturated soils and there is typically only one or two soil parameters required. One common shortcoming for the equations in this group is that the predicted shear strength does not flatten out until very high suctions.

References

Alonso EE, Gens A, Josa A (1990) A constitutive model for partially saturated soils. Géotechnique 40(3):405–430

Bao CG, Gong B, Zhan L (1998) Properties of unsaturated soils and slope stability of expansive soil. Keynote lecture. In: Proceedings of the second international conference on unsaturated soils—UNSAT 98, vol 1. Beijing, pp 71–98

Bishop AW, Blight GE (1963) Some aspects of effective stress in saturated and partly saturated soils. Géotechnique 13:177–197

Cunningham MR, Ridley AM, Dineen K, Burland JB (2003) The mechanical behaviour of a reconstituted unsaturated silty clay. Géotechnique 53(2):183–194

Escario V, Juca JFT (1989) Strength and deformation of partly saturated soils. In: Proceedings of the 12th international conference on soil mechanics and foundation engineering (ICSMFE), vol 1. Rio de Janeiro, pp 43–46

Escario V, Saez J (1986) The shear strength of partly saturated soils. Géotechnique 36(3):453–456

Fredlund DG (1989) The character of the shear strength envelope for unsaturated soils. In: Proceedings of the 12th international conference on soil mechanics and foundation engineering (ICSMFE). The Victor de Mello Volume, Rio de Janeiro, pp 142–149

Fredlund DG, Rahardjo H (1993) Soil mechanics for unsaturated soils. Wiley, New York

Fredlund DG, Xing A (1994) Equations for soil–water characteristic curve. Can Geotech J 31:521–532

Fredlund DG, Morgenstern NR, Widger A (1978) Shear strength of unsaturated soils. Can Geotech J 15:313–321

Fredlund DG, Xing A, Fredlund MD, Barbour SL (1996) The relationship of unsaturated soil shear strength to the soil–water characteristic curve. Can Geotech J 33:440–448

Gan JK-M, Fredlund DG, Rahardjo H (1988) Determination of the shear strength parameters of an unsaturated soil using the direct shear test. Can Geotech J 25(3):500–510

Garven EA, Vanapalli SK (2006) Evaluation of empirical procedures for predicting the shear strength of unsaturated soils. In: Proceeding of the fourth international conference of unsaturated soil—unsaturated soil 2006. Carefree, pp 2570–2581

Khalili N, Khabbaz MH (1998) A unique relationship for the determination of the shear strength of unsaturated soils. Géotechnique 48(5):681–687

Lee IM, Sung SG, Cho GC (2005) Effect of stress state on the unsaturated shear strength of a weathered granite. Can Geotech J 42:624–631

Matsuoka H, Yao YP, Sun DA (1999) The cam-clay models revised by the SMP criterion. Soils Found 39(1):81–95

Melinda F, Rahardjo H, Han KK, Leong EC (2004) Shear strength of compacted soil under infiltration condition. J Geotech Geoenviron Eng ASCE 130(8):807–817

Miao L, Liu S, Lai Y (2002) Research of soil–water characteristics and shear strength features of Nanyang expansive soil. Eng Geol 65:261–267

Muir Wood D (1990) Soil behaviour and critical state soil mechanics. Cambridge University Press, Cambridge

Ng CWW, Chiu ACF (2001) Behaviour of loosely compacted unsaturated volcanic soil. J Geotech Geoenviron Eng ASCE 127(12):1027–1036

Ng CWW, Pang YW (2000) Influence of stress state on soil–water characteristics and slope stability. Journal of Geotechnical and Geoenviromental Engineering, ASCE 126(2):157–166

Nuth M, Laloui L (2008) Effective stress concept in unsaturated soils: clarification and validation of a unified framework. Int J Numer Anal Methods Geomech 32:771–801

Oberg A, Sallfors G (1997) Determination of shear strength parameters of unsaturated silts and sands based on the water retention curve. Geotech Test J GTJODJ 20(1):40–48

Oloo SY, Fredlund DG, Gan JKM (1997) Bearing capacity of unpaved roads. Can Geotech J 34:398–407

Pham QH (2005) A volume-mass constitutive model for unsaturated soil. PhD thesis, University of Saskatchewan, Saskatoon, Saskatchewan, Canada

Rahardjo H, Lim TT, Chang MF, Fredlund DG (1995) Shear strength characteristics of a residual soil. Can Geotech J 32:60–77

Rassam DW, Cook FJ (2002) Predicting the shear strength envelope of unsaturated soil. Geotech Test J 28:215–220

Rassam DW, Williams DJ (1997) Shear strength of unsaturated gold tailings. In: Proceedings of the eighth Australia-New Zealand conference on geotechnics, vol 1. Hobart, pp 329–335

Rassam DW, Williams DJ (1999) A relationship describing the shear strength of unsaturated soils. Can Geotech J 36: 363–368

Röhm SA, Vilar OM (1995) Shear strength of an unsaturated sandy soil. In: Proceedings of the first international conference on unsaturated soils, vol. 1, September 1995. Paris. A. A. Balkema, Rotterdam, pp 189–193

Schrefler BA (1984) The finite element method in soil consolidation (with applications to surface subsidence). Ph.D. Thesis, University College of Swansea, C/Ph/76/84, 1984

Sheng D, Sloan SW, Gens A, Smith DW (2003) Finite element formulation and algorithms for unsaturated soils. Part I: theory. Int J Numer Anal Methods Geomech 27(9):745–765

Sheng D, Sloan SW, Gens A (2004) A constitutive model for unsaturated soils: thermomechanical and computational aspects. Computational Mechanics 33(6):453–465

Sheng D, Fredlund DG, Gens A (2008) A new modelling approach for unsaturated soils using independent stress variables. Can Geotech J 45:511–534

Sun DA, Matsuoka H, Yao YP, Ichihara W (2000) An elastoplastic model for unsaturated soil in three-dimensional stresses. Soils Found 40(3):17–28

Sun DA, Sheng D, Cui HB, Sloan SW (2007) A density dependent elastoplastic hydro-mechanical model for unsaturated compacted soil. International Journal for Numerical and Analytical Method in Geomechanics 31(11):1257–1279

Tarantino A (2007) A possible critical state framework for unsaturated compacted soils. Géotechnique 57(4):385–389

Tarantino A, Tombolato S (2005) Coupling of hydraulic and mechanical behaviour in unsaturated compacted clay. Géotechnique 55(4):307–317

Tekinsoy MA, Kayadelen C, Keskin MS, Soylemaz M (2004) An equation for predicting shear strength envelope with respect to matric suction. Comput Geotech 31(7):589–593

Thu TM, Rahardjo H, Leong EC (2006) Shear strength and pore water pressure characteristics during constant water content triaxial tests. J Geotech Geoenviron Eng ASCE 136(3):411–419

Thu TM, Rahardjo H, Leong EC (2007) Critical state behavior of a compacted silt specimen. Soils Found 47(4):749–755

Thu TM, Rahardjo H, Leong EC (2008) Soil–water characteristic curve and consolidation behavior for a compacted silt. Can Geotech J 44:266–275

Toll DG (1990) A framework for unsaturated soil behaviour. Géotechnique 40(1):31–44

Toll DG, Ong BH (2003) Critical state parameters for an unsaturated residual sandy clay. Géotechnique 53(1): 93–103

Toyota H, Sakai N, Nishimura T (2001) Effects of stress history due to unsaturation and drainage condition on shear properties of unsaturated cohesive soil. Soils Found 41(1):13–24

Toyota H, Nakamura K, Sakai N (2003) Mechanical properties of unsaturated cohesive soil in consideration of tensile stress. Soils Found 43(2):115–122

Vanapalli SK, Fredlund DG (2000) Comparison of different procedures to predict unsaturated soil shear strength. In: ASTM proceedings, unsaturated soils, Geo-Denver 2000, August 3–8

Vanapalli SK, Fredlund DG, Pufahl DE, Clifton AW (1996) Model for the prediction of shear strength with respect to soil suction. Can Geotech J 33:379–392

Wheeler SJ, Sivakumar V (2000) Influence of compaction procedure on the mechanical behaviour of an unsaturated compacted clay. Part 2: shearing and constitutive modelling. Géotechnique 50(4):369–376

Xu YF (2004) Fractal approach to unsaturated shear strength. J Geotech Geoenviron Eng 130(3):264–273

Zhou AN, Sheng D (2009) Yield stress, volume change and shear strength behaviour of unsaturated soils: validation of the SFG model. Can Geotech J 46(9):1034–1045