Unsaturated Soils – Alonso & Gens (eds)

Constitutive modelling of unsaturated soils: Discussion of fundamental principles

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ABSTRACT: An unsaturated soil is not a special type of soil, rather a state of the soil. All soils can be partially saturated with water. Therefore, constitutive models for soils should ideally represent the soil behaviour over entire ranges of possible pore pressure and stress values and allow arbitrary stress and hydraulic paths within these ranges. This paper attempts to present an overview of constitutive modelling for unsaturated soils. In particular, it focuses on the fundamental principles that govern the volume change, shear strength, yield surfaces, water retention and hydro-mechanical coupling. Alternative forms of these principles are critically examined in terms of their advantages and disadvantages. The paper also presents a short summary of the finite element implementation of unsaturated soil models.

1 INTRODUCTION

Soils that are partially saturated with water are often referred to as unsaturated soils. It should be stressed that all soils can be partially saturated with water. An unsaturated soil is just a state of the soil, not a special type of soil, as pointed out by Gens *et al.* (2006). Some soils may exhibit distinctive volume, strength and hydraulic properties when become unsaturated. For these soils, a change in the degree of saturation can cause significant changes in the volume, shear strength and hydraulic properties. Nevertheless, the distinctive volume, strength and hydraulic behaviour for unsaturated states only represent a form of material non-linearity and should therefore be handled within a continuous and coherent framework. In other words, a constitutive model for a soil should represent the soil behaviour over entire ranges of possible pore pressure and stress values and should allow arbitrary stress and hydraulic paths within these ranges.

Soil mechanics principles are more established for soils at saturated states. Generalisation of these principles to unsaturated soils requires careful consideration of the following fundamental issues: (1) volume change behaviour associated with suction or saturation changes, (2) shear strength behaviour associated with suction or saturation changes, and (3) hydraulic behaviour associated with suction or saturation changes. Soils can experience significant volume changes upon changes of the degree of saturation or suction. Some soils expand upon wetting, some collapse and some do both depending on the stress level. The large volume changes associated with saturation change can lead to severe damages to foundations and structures. Shear strength of soils can also change dramatically as the degree of saturation changes, and a related engineering problem is slope failures caused by rainfall. Unsaturated soils also have distinctive hydraulic behaviour which has profound implications in designing cover and containment systems for various industrial and municipal wastes. These fundamental issues are indeed the main concerns of unsaturated soil mechanics and its engineering applications (Fredlund & Rahardjo 1993; Houston 2002).

Constitutive modelling of unsaturated soils generally involves the generalisation of constitutive models for saturated states to unsaturated states, by incorporating the fundamental issues mentioned above. The research was pioneered by Alonso *et al.* (1990) and it has since attracted extensive interest. A large number of constitutive models can be found in the literature. There are several state-of-the-art reports and review papers over the last 15 years or so, e.g., Gens (1996), Wheeler & Karube (1996), Kohgo (2003), Gens *et al.* (2006), Wheeler (2006), Gens (2008), Sheng & Fredlund (2008), Sheng *et al.* (2008c), Gens (2009), Cui & Sun (2009) and Gens (2010). These papers may serve as good references for studying the topic. They usually contain (1) a thorough discussion of stress state or constitutive variables used to establish various models, (2) an in-depth analysis of specific constitutive models and their advantages and disadvantages, and (3) latest developments in the area of unsaturated soil modelling. The papers by Gens (2009, 2010) also provide interesting discussions on the physical significance of different suction components and their roles in constitutive modelling.

Instead of a comprehensive review of constitutive models for unsaturated soils, this paper devotes its main attention to a number of specific issues: (1) volume change behaviour, (2) variation of yield stress and shear strength with suction, (3) water retention behaviour and hydro-mechanical coupling. These issues represent the most fundamental components of constitutive models. Alternative methods for tackling these fundamental issues are scrutinised, particularly against the principle that partial saturation is a state of soil. The issue of stress state variables, a seemingly unavoidable topic in unsaturated soil mechanics, is not specifically discussed in this paper. In other words, constitutive models are not judged based on the stress state variables they use, rather on their qualitative predictions of observed soil behaviour. In addition, a number of advanced topics are excluded:

1. Thermodynamics of unsaturated soils. This topic often leads to interesting discussion of thermodynamically consistent stress-state variables and models. Readers may refer to Houlsby (1997), Hutter *et al.* (1999), Gray & Schrefler (2001), Li (2007), Samat *et al.* (2008), Coussy *et al.* (2010) and Zhao *et al.* (2010).
2. Micromechanical modelling of unsaturated soils. This approach can often lead to insights into soil behaviour and sometimes also validation of macroscopic (continuum) constitutive equations. Some good examples of this approach include the work by Gili & Alonso (2002), Jiang *et al.* (2004), Katti *et al.* (2007) and Scholtès *et al.* (2009). A closely related approach is the micro-macro double-structure models by Gens & Alonso (1992), Alonso *et al.* (1999), Sánchez *et al.* (2006) and Cardoso & Alonso (2009).
3. Thermo-hydro-chemo-mechanical modelling of unsaturated soils. In this approach, additional environmental variables such as temperature and chemical concentrations are introduced to study soil behaviour. Some recent work in this area refers to Loret *et al.* (2002), Guimarães *et al.* (2007), Cleall *et al.* (2007), Kimoto *et al.* (2007) and Gens (2006, 2008, 2010).
4. Miscellaneous topics such as non-isothermal behaviour, anisotropy, rate-dependent behaviour and liquefaction of unsaturated soils. These topics are usually related to specific soils or specific problems. Some representative work on these topics are Cui & Delage (1996), Stropeit *et al.* (2008) and D'Onza *et al.* (2010) for anisotropy; Modaressi & Modaressi (1995) and Cui *et al.* (2000) for non-isothermal behaviour; Cardoso & Alonso (2009) for degradation modelling; Arson & Gatmiri (2008) and Yang *et al.* (2008) for damage modelling; Oldecop & Alonso (2007) and Pereira & de Gennaro (2010) for rate-dependent behaviour; Unno *et al.* (2008) and Bian & Shahrour (2009) for liquefaction.

The paper is organised as follows. It first presents a brief discussion of two basic concepts used in constitutive modelling, i.e. the net stress and suction. The fundamental issues on volume change, yield stress, shear strength, water retention and hydro-mechanical coupling are then discussed. The paper finally outlines the challenges and possible solution strategies for implementing unsaturated soil models into the finite element method.

2 NET STRESS AND SUCTION

Net stress is commonly used in interpreting unsaturated soil behaviour and in constitutive modelling. It is defined as

$$\bar{\sigma}_{ij} = \sigma_{ij} - \delta_{ij} u_{\mathrm{a}} \tag{1}$$

where $\bar{\sigma}_{ij}$ is the net stress tensor, σ_{ij} the total stress tensor, δ_{ij} the Kronecker delta, and u_{a} the pore air pressure. The net stress is often used to analyse laboratory data, particularly those based on the axis-translation technique where the air pressure is not zero. It is sometimes perceived to recover the effective stress when soils become saturated. Such a perception should however be avoided. Under natural ground conditions where the air pressure is atmospheric, the net stress is equivalent to the total stress. Indeed, we never use the net stress concept to describe the behaviour of dry sand. In other words, the atmospheric pore air pressure should be considered zero. Net stress is different from total stress only when the air pressure is not atmospheric.

The concept of net stress can be useful in interpreting experimental data based on axis-translation technique, if the technique is indeed valid for applying suction (see discussions on its validity in e.g. Ng *et al.* 2007; Baker & Frydman 2009). In this case, the air pressure is used as a reference value for stress measures and the net stress is the total stress in excess of the pore air pressure.

The soil suction in the literature of unsaturated soil modelling usually refers to the matric suction and is usually expressed as:

$$s = u_{\mathrm{a}} - u_{\mathrm{w}} \tag{2}$$

where s is called the matric suction in soil physics terminology and also called the matrix suction (Baker & Frydman 2009), and u_{w} the pore water pressure. The matric suction is used interchangeably with the matric potential in soil-water potential. The latter is a measurement of energy and consists of two parts: capillary and adsorptive potentials. When pore water exists as capillary water at relatively high degrees of saturation, the capillary potential is dominant in the matric potential and the definition by (2) is then considered to be valid. However, when pore water exists as adsorbed water films at low degrees of saturation, the adsorptive potential (ψ_a) becomes dominant in the matric potential. Consequently, questions have been raised regarding whether equation (2) is still valid for the matric potential (Baker & Frydman 2009). In the case when water exists as adsorbed films to solid particles, the true water pressure is not well defined. It is not unique at one material point and is dependent on the proximity to the solid particle surface (Marshall *et al.* 1996). However an apparent water pressure can be introduced: $u_w = u_a - \psi_a$, i.e. the apparent water pressure represents the negative adsorptive potential measured in excess of air pressure. Such an apparent water pressure is then unique at one material point. With such a definition of u_w, the matric potential can be expressed by equation (2) over the full range of saturation. Nevertheless, the matric suction should be differentiated from capillary phenomena. Its two components may not easily be separable in a soil with double-porosity. As pointed out in Gens (2010), it is more appropriate to think of matric suction as a variable that expresses quantitatively the degree of attachment of water to solid particles that results from the general solid/water/interface interaction.

In constitutive modelling, the matric suction is often treated as an additional variable in a stress space for establishing constitutive laws. This approach was pioneered in the Barcelona Basic Model (BBM) by Alonso *et al.* (1990) and is then followed in most existing models, with a few exceptions where the suction is treated as an internal variable (e.g. Bolzon *et al.* 1996; Loret & Khalili 2002). Again, these approaches are not differentiated in this paper, even though there are arguments against treating suction as an internal variable (e.g. Laloui & Nuth 2009). Since suction can vary independently of stress, it is treated as an independent axis in the stress space. In addition, the matric suction is considered to coincide with negative pore water pressure for fully saturated states and thus the suction axis goes from negative infinity to positive infinity in the stress space.

3 VOLUME CHANGE BEHAVIOUR

The volume change behaviour is one of the most fundamental properties of soils. For unsaturated soils, the large volume changes associated with suction change can cause severe damages to foundations and structures. The volume change equation

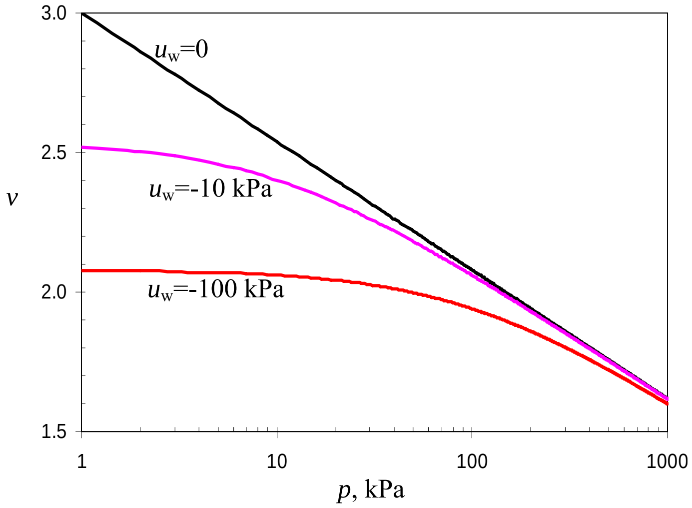


Figure 1. Normal compression lines for saturated clay under constant pore water pressures (λ=0.2, N=3).

also underpins the yield stress – suction and shear strength – suction relationships (Sheng *et al.* 2008c). It is indeed the only absolutely necessary component that is needed to add in order to extend a saturated soil model to unsaturated states. The model that defines the volume change caused by stress and suction changes should again be applicable to the entire range of possible pore pressure or suction values. The discussions below are limited to isotropic stress states. The volume change associated with changes of deviator stress has to be considered in a three-dimensional constitutive framework, which depends on the specific model used for saturated soils, and is not discussed in this paper.

For saturated soils, a common starting point is the linear relationship between the specific volume (v) and the logarithmic effective mean stress ($\ln p'$) for normally consolidated clays:

$$v = N - \lambda \ln p' = N - \lambda \ln\left(p - u_w\right) \tag{3}$$

where p is the mean stress, λ is the slope of the $v - \ln p'$ line, and N is the intercept on the v axis when $\ln p' = 0$. Equation (3) is only valid for positive increments of the effective stress. For unloading and reloading, the volume change depends on the specific plasticity framework adopted in the constitutive model. For example, hypoplasticity and bounding surface plasticity adopts different volume change mechanisms than classical elastoplasticity. However, for normally consolidated soils subject to positive stress increments, equation (3) is usually used independently of the theoretical framework.

It should be noted that equation (3) represents a straight line in the $v - \ln p$ space only if the pore water pressure is zero. If the pore water pressure were kept at a negative value (suction), equation (3) would predict a smooth curve in the $v - \ln p$ space, as shown by Figure 1. The air entry suction for the soil in Figure 1 is assumed to be larger than 100 kPa, so that the soil remains saturated. Indeed, these compression lines look very much like those for overconsolidated soils. However, the curvature of the normal compression lines is purely due to the nature of the logarithmic function and the translation from the effective stress space ($v - \ln p'$) to the total stress space ($v - \ln p$), not due to overconsolidation.

Equation (3) can also be written in an incremental form as follows:

$$\mathrm{d}v = -\lambda \frac{\mathrm{d}p}{p - u_{\mathrm{w}}} - \lambda \frac{\mathrm{d}(-u_{\mathrm{w}})}{p - u_{\mathrm{w}}} \qquad (4)$$

It is clear that a negative increment in pore water pressure has exactly the same effect on the volume of a saturated soil as an equal positive increment in mean stress.

In the literature, equation (3) is extended to unsaturated states in one of the three approaches:

- Approach A: Separate stress and suction (the net stress and suction) approach.
- Approach B: Combined stress-suction approach (the effective stress approach).
- Approach C: SFG approach (middle ground between Approach A and B).

These approaches are discussed below in detail.

3.1 *Separate Stress and Suction Approach*

In Approach A, the volume change due to stress change is separated from that due to suction change. A typical example of the volume change equations in this approach is:

$$v = N - \lambda_{\mathrm{vp}} \ln \bar{p} - \lambda_{\mathrm{vs}} \ln\left(\frac{s + u_{\mathrm{at}}}{u_{\mathrm{at}}}\right) \qquad (5)$$

where $\bar{p}$ is the net mean stress, N is the specific volume when $\ln \bar{p} = 0$ and $s = 0$, λ_{vp} is the slope of an assumed $v - \ln \bar{p}$ line or the compressibility due to stress change, λ_{vs} the slope of an assumed $v - \ln s$ line or the shrinkability due to suction change, and u_{at} the atmospheric pressure and is added to avoid the singularity when s=0. Again, equation (5) is only used for increasing mean stress or increasing suction. Indeed, λ_{vs} is usually replaced by the elastic compression index κ_{vs} in most applications, unless the suction increases above the suction-increase yield surface (see section below on yield stress).

Equation (5) has been used in many models like Alonso *et al* (1990), Wheeler & Sivakumar (1995), Cui & Delage (1996), Chiu & Ng (2003), Georgiadis *et al*. (2005) and Thu *et al.* (2007a). The main advantage of equation (5) is that the compressibility due to stress and suction changes can be dealt with

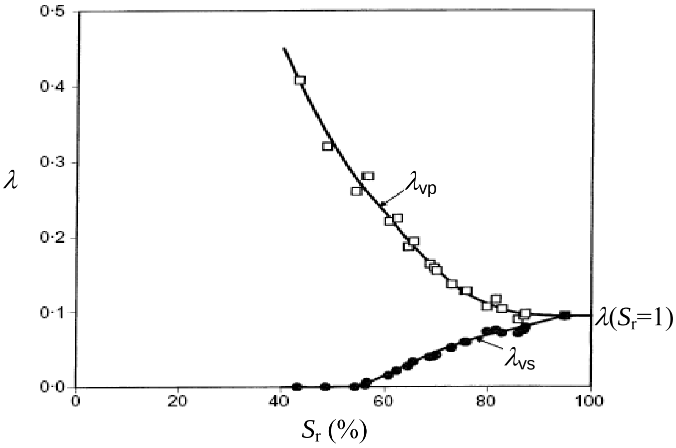


Figure 2. Variation of compressibilities with degree of saturation (after Toll 1990).

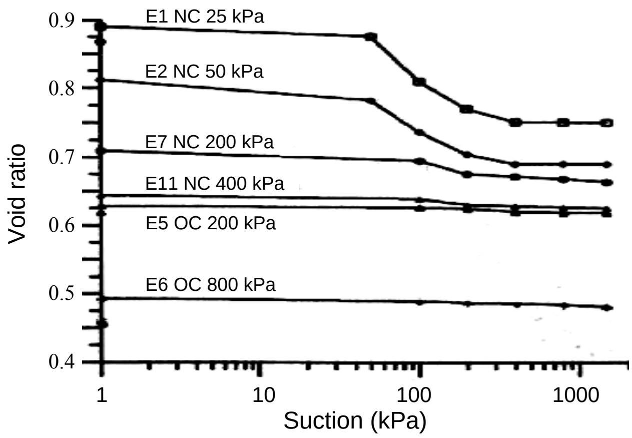
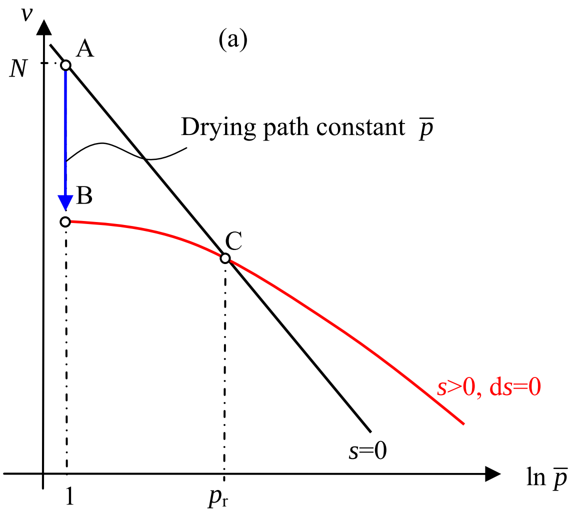


Figure 3. Variation of shrinkability with stress (after Delage and Graham 1996).

separately. This does not only provide extra flexibility for modelling soil behaviour, but is also supported by experimental data. Toll (1990) and Toll & Ong (2003) showed that the two compressibilities λ_{vp} and λ_{vs} can be totally different (Figure 2). It is usually true that the suction shrinkability (λ_{vs}) decreases with decreasing degree of saturation. On the other hand, the stress compressibility (λ_{vp}) can increase with decreasing degree of saturation, particularly for compacted soils where highly compressible macropores (inter-aggregate pores) are formed (Romero *et al.* 1999; Gallipoli *et al.* 2003a).

However, there are also a few disadvantages about equation (5). First, equation (5) does not recover equation (3) when the soil becomes saturated. It represents a linear $v - \ln \bar{p}$ relationship for constant suctions, unless λ_{vp} is assumed to be a function of stress (as in Georgiadis *et al.* 2005). This is not consistent with the saturated soil model (Figure 1). As such, the volume change becomes undefined at the transition suction between saturated and unsaturated states. Second, the volume change caused by suction changes is independent of stress. This con-



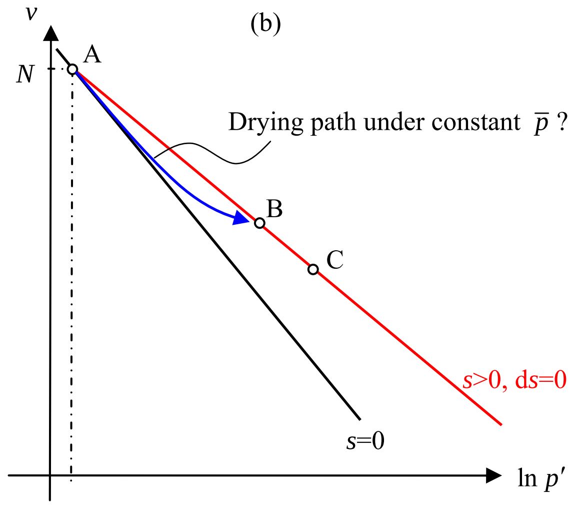


Figure 4. Normal compression lines according to the effective stress approach – constraint on $\lambda(s)$ with a constant N: (a) net mean stress space, (b) effective mean stress space.

tradicts with experimental observation shown in Figure 3. In addition, the atmospheric pressure (u_{at}) in (5) makes the suction change insignificant when $s<u_{at}$, which is not consistent with the drying of a slurry soil. The first point, i.e. the discontinuity at the transition between saturated and unsaturated states, was also one of the reasons that some researchers turned to the effective stress approach (e.g. Sheng *et al.* 2003a). A simple numerical example will illustrate this problem. Let a soil be compressed at the transition suction (s_{sa}) from mean stress 1 kPa to 100 kPa. Let the air pressure remain atmospheric. In the saturated zone, the volume changes according to equation (3):

$$\Delta v\big|_{s_{sa}^-} = -\lambda_{vp} \ln \frac{100 + s_{sa}}{1 + s_{sa}}$$

In the unsaturated zone, the volume changes according to equation (5):

$$\Delta v\big|_{s_{sa}^+} = -\lambda_{vp} \ln 100$$

These two volume changes can be quite different, dependent on the value of the transition suction. The transition suction is either the air-entry or the air-expulsion value, dependent on the hydraulic path.

3.2 *Combined Stress-Suction Approach*

In Approach B, the matric suction and the net mean stress are combined into one single variable, i.e., an effective stress, to define their effects on soil volume. A general form of the effective stress is

$$p' = \bar{p} + f(s) \tag{6}$$

where f is either a function of suction or a function of suction and degree of saturation. Obviously such a definition of effective stress is very general and covers most existing definitions in the literature. With such an effective stress, equation (3) is assumed to be valid for unsaturated states:

$$v = N - \lambda \ln p' = N - \lambda(s) \ln\left(\bar{p} + f(s)\right) \tag{7}$$

where N is the specific volume when $\ln p' = 0$. If the effective stress is indeed effective in controlling soil volume, v should remain constant under constant p'. As such, parameters N and λ should be independent of suction. However, this is seldom the case in reality. In the literature, λ is usually assumed to be function of s, while N is treated either as a constant or a variable. We first discuss the case where N is constant and then explore the possibilities with a varying N.

Equation (7) is widely used in the literature and in fact most models based on the effective stress approach adopt it as the volume change equation (Kohgo *et al.* 1993; Bolzon *et al.* 1996; Jommi 2000; Loret & Khalili 2002; Sheng *et al.* 2003a; Sheng *et al.* 2004; Pereira *et al.* 2005; Santagiuliana & Schrefler 2006; Sun *et al.* 2007a, c; Kohler & Hofstetter 2008; Nuth & Laloui 2008; Buscarnera & Nova 2009). Equation (7) generally recovers equation (3) when the soil becomes saturated. This is one of the greatest advantages of using the effective stress.

However, there are also some disadvantages with equation (7). The obvious issue is the difficulty in addressing the different compressbilities due to stress and suction changes, as shown in Figure 2. The second issue is related to a constraint on the compressibility λ. Let a saturated slurry soil be dried from zero suction to an arbitrary suction under constant mean stress of 1 kPa, i.e. the stress path AB in Figure 4. As will be shown in the section on yield

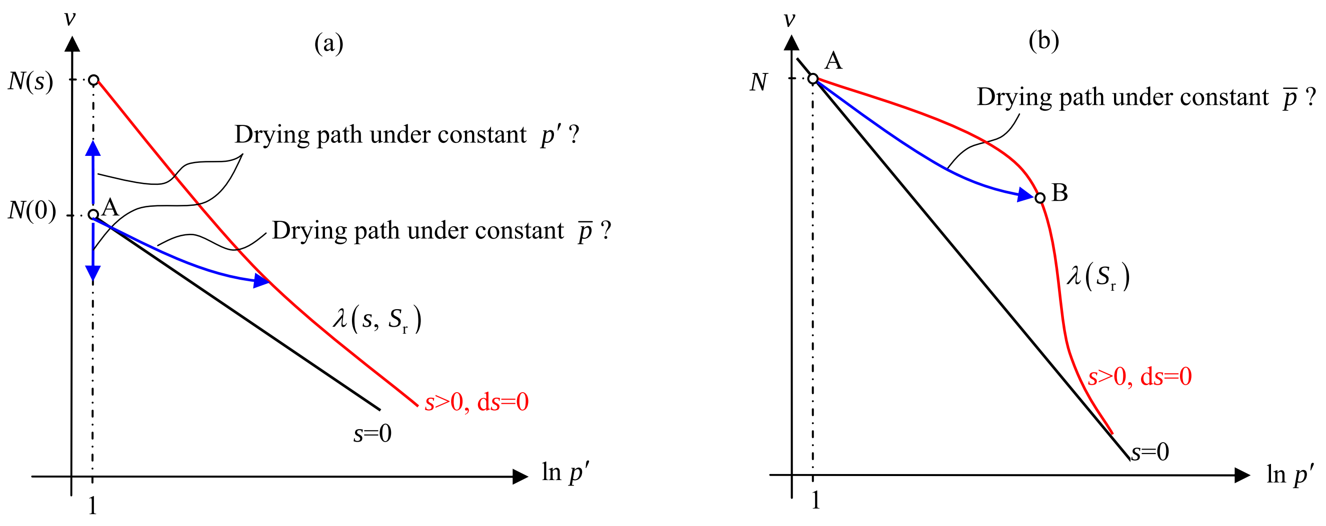


Figure 5. Normal compression lines according to the effective stress approach – possible augmentations: (a) N increases with increasing suction. (b) λ is a function of degree of saturation (S_r).

stress (Figure 8), this drying path is elastoplastic, not purely elastic. The volume of the soil then changes according to:

$$v_B = N - \lambda(s)\ln\left(1 + f(s)\right) \tag{8}$$

Now compress the soil under constant suction, i.e. stress path BC in Figure 4. The compression line will be curved in the $v - \ln \bar{p}$ space, due to the $f(s)$ term. If the suction at point B is above the air entry value, this compression line is expected to intersect with the initial compression line for saturated states. Let the intersection be point C (at net mean stress of $\bar{p}_r$). The volume at C is then:

$$v_C = v_B - \lambda(s)\ln\left(\frac{\bar{p}_r + f(s)}{1 + f(s)}\right) \tag{9}$$

Along path AC, the volume changes according to:

$$v_C = N - \lambda(0)\ln\left(\bar{p}_r + f(0)\right) = N - \lambda(0)\ln \bar{p}_r \tag{10}$$

Replacing (8) into (9) and then equating (9) with (10) lead to:

$$\frac{\lambda(s)}{\lambda(0)} = \frac{\ln(\bar{p}_r)}{\ln(\bar{p}_r + f(s))} < 1 \tag{11}$$

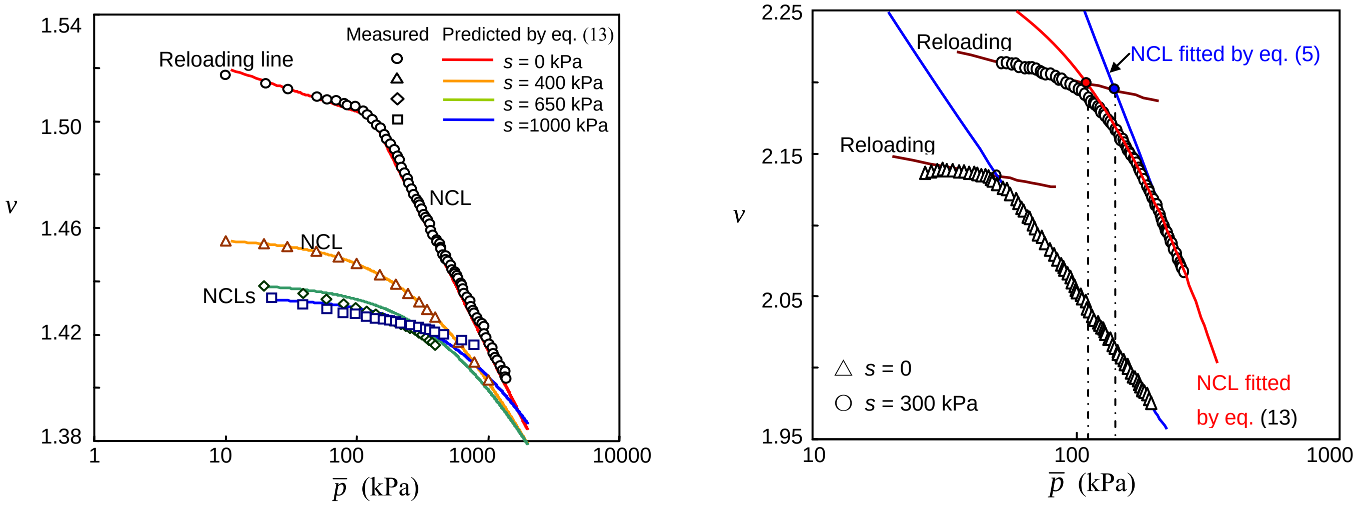
We usually anticipate the effective stress increases with increasing suction, at least at low suction values. Therefore, we have: $\lambda(s) < \lambda(0)$, meaning that the slope of the compression line decreases with increasing suction. Such a constraint on λ is however not supported by experimental data. In the data by Jennings & Burland (1962) for air-dry soils, the slope of the compression lines is more or less constant. In the data by Sivakumar & Wheeler (2000) for compacted soils, the slope of the compression lines increases with increasing suction. In addition, experimental data on wetting-induced collapse (e.g. Sun *et al.* 2007d) do not support an ever increasing collapse volume with increasing mean stress.

In some constitutive models that use the combined stress-suction approach, parameter N is assumed to vary with suction (Kohgo *et al.* 1993; Kikumoto *et al.* 2010). If N decreases with increasing suction, the same constraint on λ, i.e. equation (11) would apply. To avoid this constraint, N has to increase with increasing suction. An increasing N with suction basically implies that drying a slurry soil under constant effective mean stress will cause the yield surface to expand, which is not consistent with the stress path. This inconsistency will be discussed below in association with Figure 8.

Gallipoli *et al.* (2003a) proposed the following volume change equation:

$$v = \left(N - \lambda \ln p'\right)\left(1 - a(1 - \exp(b\xi)\right) \tag{12}$$

where N and λ are the two parameters of the normal compression line for saturated states, a and b fitting parameters, and ξ a positive variable representing the bonding effects of suction. The bonding variable (ξ) is a function of both s and S_r. Equation (12) is hence equivalent to equation (7) with N and λ being functions of s and S_r. Gallipoli *et al.* (2003a, 2008) showed that equation (12) is able to predict the volume change at both normal compression and critical states for a variety of compacted soils. One challenge in using equation (12) is that the yield stress is likely to be functions of both suction and degree of saturation. Because the $s - S_r$ relationship is usually not unique due to hydraulic hysteresis, the resulting loading-collapse surface may not be well defined. However, Gallipoli (personal communication) sug-



(a) Air-dry silty clay

(b) Compacted Kaolin

Figure 6. Predicted isotropic compression curves for (a) air-dry silty clay (data by Cunningham *et al.* 2003), (b) compacted kaolin (data by Sivakumar and Wheeler 2000).

gests that it is sufficient to define the bonding variable (ξ) in terms of S_r only (e.g. ξ=1-S_r), which would then resolve the non-uniqueness problem and lead to a unique loading-collapse surface in the $S_r - p'$ space.

To avoid the constraint defined by equation (11), another possible augmentation to equation (7) is to assume that the compressibility λ is a function of degree of saturation, i.e. $\lambda(S_r)$, while keeping N constant. This is similar to the approach by Gallipoli *et al.* (2003a) with ξ being function of S_r only. Because S_r changes with soil volume even if the suction is kept constant, the slope of the compression line will then change and will most likely increase with increasing mean stress. Therefore, it is possible to have $\lambda(S_r)$ decreasing with decreasing S_r, but the slope of the compression line for constant suction increases with increasing stress (Figure 5b). Al-Badran & Schanz (2009) used an approach similar to Figure 5b, but they formulated their volume change equation in the net stress space. The same issue of non-uniqueness of the loading-collapse yield surface in the $s - p'$ space may arise, due to hydraulic hysteresis. Instead, the yield surface may have to be defined in the $S_r - p'$ space.

Clearly further research is required if equation (7) is adopted for the volume change, particularly in terms of consistent explanation of the suction-caused volume change for reconstituted soils. A worthwhile endeavour in this direction is perhaps to explore the possibilities of using $\lambda(S_r)$ in equation (7) while keeping parameter N constant.

3.3 *SFG Approach*

Sheng *et al.* (2008a) proposed a third way to model the volume change for unsaturated soils under isotropic stress states. The new model, referred to as the SFG model, represents a middle ground between Approach A and Approach B and is expressed in an incremental form as follows:

$$dv = -\lambda_{vp}\frac{d\bar{p}}{\bar{p}+f(s)} - \lambda_{vs}\frac{ds}{\bar{p}+f(s)} \tag{13}$$

Equation (13) is in the same form as equation (4). Similar to Approach A, equation (13) is defined in terms of net stress and suction and separates the compressibilities due to the two variables, i.e. λ_{vp} and λ_{vs}. Similar to Approach B, it combines the suction with the net mean stress in the denominator, i.e. the term $\bar{p}+f(s)$, and recovers equation (4) for saturated states. The term $\bar{p}+f(s)$ represents the interaction between stress and suction and makes the normal compression lines for non-zero suction curved in the $v-\ln\bar{p}$ space. However, there is no constraint on parameter λ_{vp}. As a first approximation, λ_{vp} can be assumed to be independent of suction, as indicated by the data of Jennings & Burland (1962) for air-dry soils. More realistically it should depend on suction. For example, the data of Sivakumar & Wheeler (2000) shows that λ_{vp} increases with increasing suction for compacted soils. Parameter λ_{vs} must equal λ_{vp} when the soil is fully saturated, because of equation (4). It generally decreases with increasing suction and approaches zero. Sheng *et al.* (2008a) suggested the following simple function for λ_{vs}

$$\lambda_{\rm vs} = \begin{cases} \lambda_{\rm vp}, & s \le s_{\rm sa} \\ \lambda_{\rm vp} \dfrac{s_{\rm sa}+1}{s+1}, & s > s_{\rm sa} \end{cases} \tag{14}$$

where $s_{\rm sa}$ is the transition suction and was also called the saturation suction in Sheng *et al.* (2008a). It is the unique transition suction between saturated and unsaturated states in the SFG model.

We note that the number '1' in equation (14) is used to avoid the singularity when $s_{\rm sa}=0$ and is not truly needed if $s_{\rm sa}$ is not absolutely zero. A better expression would be:

$$\lambda_{\rm vs} = \begin{cases} \lambda_{\rm vp}, & s \le s_{\rm sa} \\ \lambda_{\rm vp} \dfrac{s_{\rm sa}}{s}, & s > s_{\rm sa} \end{cases} \tag{15}$$

The difference between equation (14) and (15) is minimal, but equation (15) is preferred. Equation (15) can be applied as long as the transition suction is not absolutely zero. Kurucuk *et al.* (2009) used equation (13), but a different function for $\lambda_{\rm vs}$ than (15). Again, both $\lambda_{\rm vp}$ and $\lambda_{\rm vs}$ can vary with stress path and take different values on a loading and unloading path respectively.

The function $f(s)$ in equation (13) can also take different forms. Sheng *et al.* (2008a) initially used the following function:

$$f(s) = s \tag{16}$$

This is perhaps the simplest form possible for $f(s)$ and yet guarantees the continuity between saturated and unsaturated states. Even with this simplest form, Zhou & Sheng (2009) showed that the SFG model is able to predict a good set of experimental data on volume change and shear strength, both for reconstituted soils prepared from slurry states and for compacted soils. Due to the $f(s)$ term in equation (13), the normal compression lines will be curved in the $v - \ln \bar{p}$ space (Figure 6). In Figure 6a, the compression curves for s=400, 650 1000 kPa are all normal compression lines that do not involve any unloading or reloading. The SFG predictions for these curves were obtained with one single $\lambda_{\rm vp}$ value. Figure 6b shows the difference in the estimated yield stress for s=300kPa by equations (5) and (13), respectively. The yield stress is indicated by the meeting points of the unloading-reloading line and the normal compression lines. The parameter $\lambda_{\rm vp}$ was allowed to change with suction in Figure 6b.

One shortcoming of equation (16) is that the soil compressibility approaches zero as suction increases to infinite. There is also a theoretical discontinuity between unsaturated states and completely dry state ($S_{\rm r}$=0). To avoid these problems, an alternative form of $f(s)$ could be used, for example:

$$f(s) = S_{\rm r}\, s \tag{17}$$

This equation will not only guarantee the continuity between saturated and unsaturated states, but also the continuity between unsaturated and the completely dry state ($S_{\rm r}$=0). More interestingly, both equations (16) and (17) will lead to the same shear strength – suction relationship. The degree of saturation in (17) can also be replaced by the effective degree of saturation ($S_{\rm r}^{\rm e}$), as suggested by Pereira & Alonso (2009) when discussing Bishop's χ parameter. However, the performance of equation (17) is yet to be validated against experimental data, and the loading-collapse yield surface may become non-unique due to hydraulic hysteresis.

The SFG approach seems to be able to overcome some disadvantages of Approach A and Approach B. The main disadvantage of the SFG approach is that it exists only in an incremental form and its integration depends on stress path (Zhang & Lytton 2008; Sheng *et al.* 2008b). The stress path dependency requires special treatment in the stress integration of the constitutive model (Sheng *et al.* 2008d).

4 YIELD STRESS VERSUS SUCTION

In the literature of unsaturated soil mechanics, the yield stress of an unsaturated soil is usually assumed to be a function of soil suction. The concept of yield stress in classical elastoplasticity theory refers to the stress level that causes plastic deformation. In other plasticity frameworks such as bounding surface plasticity (Dafalias 1986; Russell & Khalili 2006; Morvan *et al.* 2010), hypoplasticity (Kolymbas 1991; Mašín & Khalili 2008) and generalised plasticity (Pastor *et al.* 1990; Sánchez *et al.* 2006; Manzanal *et al.* 2009), plastic deformation occurs along all loading paths including reloading. In this case, a loading function or a bounding surface is used to differentiate unloading from loading. In the discussion below, the yield surface concept is based on the classical elastoplasticity theory, but can be adopted to the loading or bounding surface concepts.

Under isotropic stress states, the yield stress is also called the preconsolidation stress. For unsaturated soils, the yield net mean stress, denoted here by $\bar{p}_{\rm c}$, is conventionally determined from isotropic compression curves obtained at constant suctions. These compression curves are usually plotted in the space of void ratio versus logarithmic net mean stress. The initial portion of the curve is typically flatter than the ending portion of the curve, if the suction is larger than zero. Such a curve is then usu-

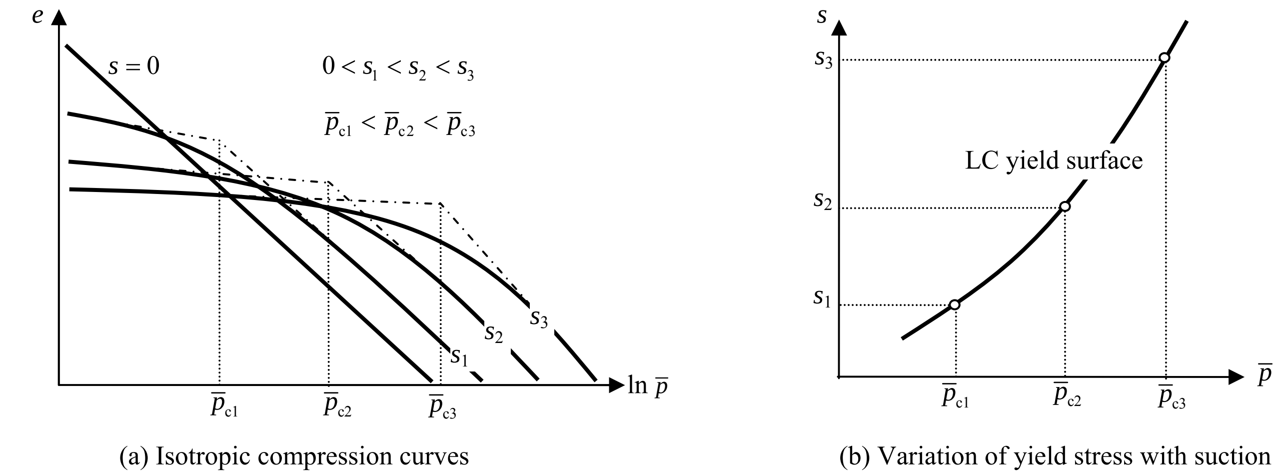


Figure 7. Isotropic compression curves under constant suction (s) and derived yield stresses.

ally approximated by two straight lines, one representing the elastic unloading-reloading line and the other the elastoplastic normal compression line. The intersection point of the two straight lines is used to define the preconsolidation (yield) stress (Figure 7a). The yield stress so determined is found to increase with increasing suction generally, irrespective of samples prepared from slurry states or from compacted soils, leading to the so-called loading-collapse yield surface that is widely used in constitutive models for unsaturated soils (Figure 7b).

The procedure outlined above for determining the yield stress is based on the assumption that the $e - \ln \overline{p}$ relationship for normally consolidated soils at s>0 is linear and may not be consistent with the definition of yield stress. To demonstrate this inconsistency, we should first realise that the isotropic compression curves shown in Figure 7 are typical of unsaturated soils reconstituted from slurry (e.g. Jennings & Burland 1962) as well as of compacted soils. Because it is difficult to define the yield stress for a compacted soil and is relatively easy to understand the preconsolidation stress for a slurry soil, we use a slurry soil as an example here. Let us assume that the slurry soil has not been consolidated (with a zero preconsolidation stress). The initial yield stress for the soil is then zero (Point A in Figure 8a). We also note that the effective stress for saturated states is constant along the 135° line in the $\overline{p} - s$ space. Drying the slurry soil to suction B under zero mean stress is similar to consolidating the soil to stress E under zero suction (Figure 8a). Indeed, if the air entry value of the soil is larger than suction at B, the length AB is exactly the same as AE, due to the effective stress principle for saturated soils. However if the air entry value is lower than suction at B, the length AB should generally be larger than AE, because a suction increment is generally less effective than an equal stress increment in terms of consolidating the unsaturated soil. Once the soil becomes unsaturated, the yield stress does not necessarily change with suction along the 135° line (denoted by the dashed line). However, the new yield surface always go through the current stress point, as shown in Figure 8b. The elastic zone expands as the suction increases from A to B. The stress points (e.g. B and C) are on the current yield surfaces. Let now the soil be isotropically compressed under the constant suction at Point C (i.e., stress path CD in Figure 8a). The isotropic compression path (CD) is again outside the current elastic zone and the soil at point C is normally consolidated. Therefore, the isotropic compression path (CD) as well as the drying path from B to C is elastoplastic and does not involve a purely elastic portion as Figure 7 indicates, suggesting that the method for determining the yield stress in Figure 7 be conceptually inconsistent. Indeed, the suction-induced apparent consolidation effect should refer to the increase of the preconsolidation stress at zero suction ($\overline{p}_c(0)$ moves from E to F as suction increases from B to C in Figure 8a), not the preconsolidation stress at the current suction.

The same analysis can be done in the effective stress – suction space (Figure 8b). In the effective stress space, the initial yield stress for a slurry soil follows the vertical line that goes through point A. This is also the zero shear strength line, which is commonly assumed to be vertical. However, the stress path for suction increase under constant net mean stress is initially inclined to horizontal by 45° for saturated states. Therefore, the stress path will cross the current yield surface and the drying path AB for a slurry soil is elastoplastic, not purely elastic. Once the soil becomes unsaturated, the stress path will drift away from the 45° line and the yield surface will also drift away from the vertical line (the dashed lines in Figure 8b). However, stress points A, B, C, and D stay on the current yield sur-

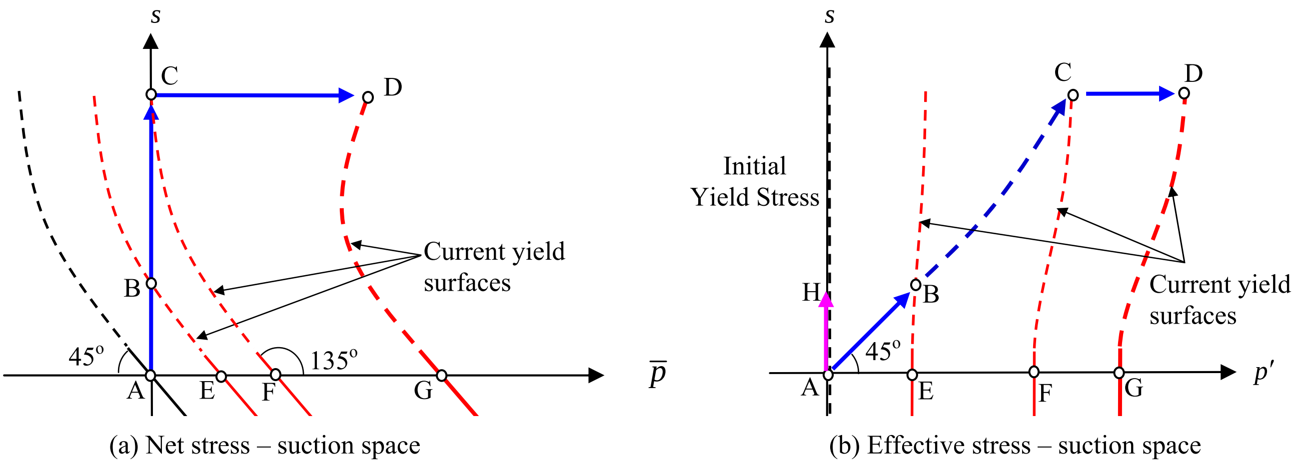


Figure 8. Evolution of the elastic zone during drying and compression of a slurry soil (ABB': constant net mean stress).

faces and the stress path ABCD causes elastoplastic volume change, in consistency with Figure 8a. On the other hand, increasing suction under constant effective mean stress, i.e. stress path AH in Figure 8b, does not lead to the expansion of the elastic zone. The expansion of elastic zone is a prerequisite for shifting the normal compression lines towards higher specific volume in $v - \ln p'$ space. Therefore, parameter N in equation (7) should generally not be a function of suction.

In the literature, the yield stress variation with suction is a rather confusing point. Many models adopt three yield surfaces in the net stress – suction space, namely the loading-collapse yield surface, the suction-increase yield surface and the apparent tensile strength surface. Figure 9 shows some examples of these models. The loading-collapse yield surface is used to model the volume collapse when an unsaturated soil is first loaded under constant suction and then wetted under constant stress. The suction-increase yield surface is used to capture the plastic volume change when an unsaturated soil is dried to a historically high suction. The apparent tensile strength surface defines the zero shear strength or the apparent tensile strength due to suction increase. These yield surfaces are usually defined separately in models based on Approach A. For example, setting the preconsolidation stress ($\bar{p}_{c0}$) to zero in the loading-collapse yield function does not recover the apparent tensile strength function. The suction-increase yield surface is usually horizontal or gently sloped (Figure 9) and is not related to the loading-collapse surface or the apparent tensile strength surface in models based on Approach A.

Sheng *et al.* (2008a) showed that the loading-collapse surface, the apparent tensile strength surface and the suction-increase yield surface are related to each other. In the SFG model, the yield stress – suction relationship, the apparent tensile strength – suction relationship and the shear strength – suction relationship are all derived from the volume change equation, i.e. equation (13). In this model, the yield stress for a slurry soil that has never been consolidated or dried varies with suction in a unique function. This function also defines the apparent tensile strength surface or the zero shear strength surface in the stress – suction space (the curve through point A in Figure 10). The curve approaches the 45° line as the suction becomes zero or negative (positive pore water pressure). Drying this slurry soil under zero stress (stress path ABC) causes the expansion of the yield surface to point C in Figure 10. Therefore, the suction-increase yielding is already included in the yield function and there is no need to define a separate function. If the unsaturated soil at point C is then compressed under constant suction (stress path CD in Figure 10), the yield surface will evolve to the loading-collapse surface that passes through point D in Figure 10. The yield surface in the stress space represents the contours of the hardening parameter, which is usually the plastic volumetric strain. The stress path CD will change the initial shape of the yield surface, because the plastic volumetric strain along CD depends on the suction level. The loading-collapse yield function recovers the apparent tensile strength function when the preconsolidation stress at zero suction ($\bar{p}_{c0}$) is set to zero. All these yield surfaces are continuous and smooth in the stress – suction space.

As pointed out by Wheeler & Karube (1996) and shown by Sheng *et al.* (2008a) and Zhang & Lytton (2009b), the apparent tensile strength function, the suction-increase yield function and the loading-collapse yield function are all related to the volumetric model that defines the elastic and elastoplastic volume changes caused by stress and suction changes. If Approach A, i.e. equation (5), is adopted

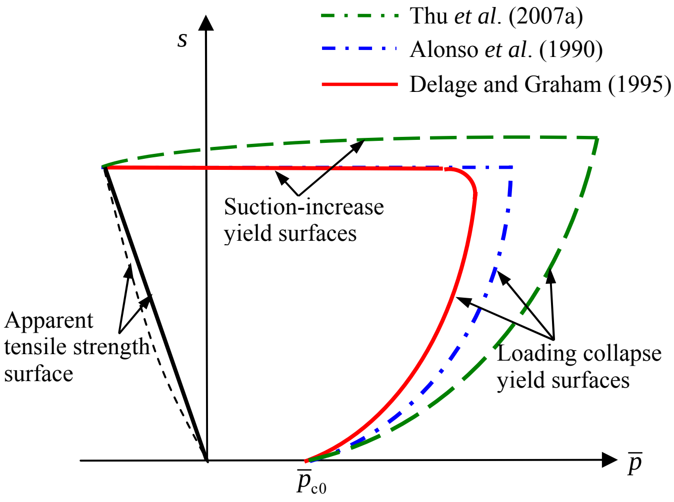


Figure 9. Loading-collapse, suction-increase and apparent tensile strength surfaces in various models.

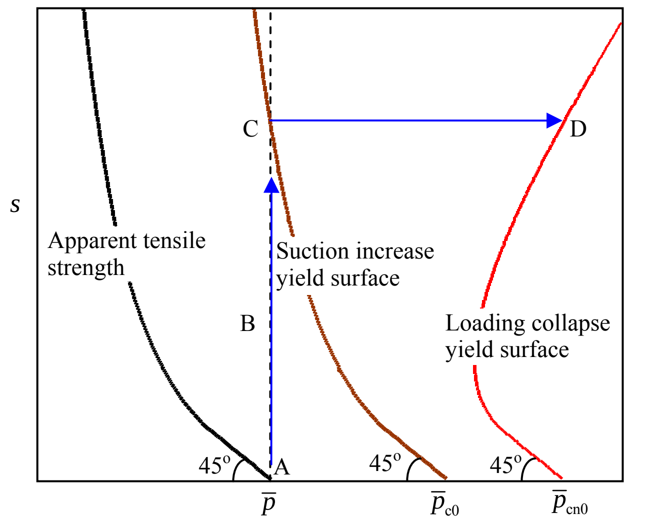


Figure 10. Yield stress variation with suction in the SFG model (Sheng *et al*. 2008a).

to describe the volume change, the loading-collapse yield surface will take the following form:

$$\overline{p}_c = \begin{cases} \overline{p}_{c0} - s, & s \le s_{sa} \\ \overline{p}_r \left(\dfrac{\overline{p}_{c0} - s_{sa}}{\overline{p}_r} \right)^{\frac{\lambda_{vp}(0)-\kappa}{\lambda_{vp}(s)-\kappa}}, & s > s_{sa} \end{cases} \tag{18}$$

where $\overline{p}_{c0}$ is the yield stress at zero suction, $\overline{p}_r$ a reference stress, and κ the elastic compression index. The specific shape of this function depends on the variation of the compressibility with suction, i.e. the function $\lambda_{vp}(s)$. The suction-increase yield surface can also be derived from equation (5). If the shrinkability (λ_{vs}) is assumed to be independent of stress, the suction-increase yield surface is simply:

$$s = s_0 \tag{19}$$

where s_0 is the yield suction. Equation (19) represents a horizontal line in the stress – suction space. Because equation (5) is not defined at zero suction and zero mean stress, the apparent tensile strength surface can not be derived from (18). A separate function is usually introduced:

$$\overline{p}_0 = -k\, s \tag{20}$$

where k was assumed to be a constant in Alonso *et al.* (1990), but as a function of suction in Georgiadis *et al*. (2005).

If Approach B, i.e. equation (7), is adopted to describe the volume change, the loading-collapse surface can then be written as:

$$p'_c = \begin{cases} p'_{c0}, & s \le s_{sa} \\ p'_r \left(\dfrac{p'_{c0}}{p_r} \right)^{\frac{\lambda(0)-\kappa}{\lambda(s)-\kappa}}, & s > s_{sa} \end{cases} \tag{21}$$

The suction-increase surface is the same as equation (19). The apparent tensile strength surface is recovered from (21) by setting $\overline{p}_{c0} = 0$:

$$p'_0 = 0 \tag{22}$$

which represents the vertical line going through the origin of the stress space.

If the SFG model, i.e. equations (13) and (15), is adopted to describe the volume change, the yield stress takes the following form:

$$\overline{p}_c = \begin{cases} \overline{p}_{c0} - s, & s \le s_{sa} \\ \overline{p}_{c0} - s_{sa} - s_{sa} \ln \dfrac{s}{s_{sa}}, & s > s_{sa} \end{cases} \tag{23}$$

The function defines the suction-increase surface and is independent of the specific function $f(s)$ used in (13). The apparent tensile strength function is also defined in (23) by setting $\overline{p}_{c0} = 0$. The loading-collapse surface takes a very similar form:

$$\overline{p}_c = \begin{cases} \overline{p}_{cn0} - s, & s \le s_{sa} \\ \dfrac{\overline{p}_{cn0}}{\overline{p}_{c0}} \left(\overline{p}_{c0} + f(s) - s_{sa} - s_{sa} \ln \dfrac{s}{s_{sa}} \right) - f(s), & s > s_{sa} \end{cases} \tag{24}$$

where $\overline{p}_{cn0}$ is the new yield stress at zero suction (Figure 10). This function is however dependent on the specific forms of $f(s)$ used in (13).

The functions defined in (18) to (24) are all continuous over the entire ranges of possible suction or pore pressure values. However, functions (18) and

(21) may not be smooth, dependent on functions $\lambda_{vp}(s)$ and $\lambda(s)$, respectively. On the other hand, functions (23) and (24) are continuous and smooth. All these functions can be incorporated into existing constitutive models for saturated soils. For example, if the modified Cam clay model is used for saturated soil behaviour, the yield function can be generalised to unsaturated states along the suction axis:

$$f = q^2 - M^2(\bar{p}-\bar{p}_0)(\bar{p}_c - \bar{p}) \equiv 0 \tag{25}$$

where f is the yield function in the stress space, q is the deviator stress, M is the slope of the critical state line in $q-\bar{p}$ space, and $\bar{p}_0$ and $\bar{p}_c$ are defined above. Again, equation (25) is valid for all pore pressure and suction values.

5 SHEAR STRENGTH

The change of shear strength with suction or saturation is one of the main reasons behind rainfall-induced landslides. It is related to the volume change equation (Sheng *et al*. 2008c). However, this relationship has been overlooked in most existing models for unsaturated soils. If the slope of the critical state line is assumed to be independent of suction, such as supported by experimental data of Toll & Ong (2003), Ng & Chiu (2001) and Thu *et al.* (2007b), the shear strength – suction relationship can indeed be derived from the volume change equation. If the slope of the critical state line depends on suction, as supported by data of Toll (1990) and Merchán *et al.* (2008), two equations are needed to define the shear strength – suction relationship, namely the volume change equation, and the $M(s)$ function, with M being the slope of the critical state line in the deviator – mean stress space.

Bishop & Blight (1963) first proposed an effective stress definition to interpret the shear strength of unsaturated soils:

$$\begin{aligned}\tau &= c' + \sigma_n' \tan\phi' = c' + (\bar{\sigma}_n + \chi s)\tan\phi' \\ &= \bar{c} + \bar{\sigma}_n \tan\phi'\end{aligned} \tag{26}$$

where τ is the shear strength, c' is the effective cohesion for saturated states and is usually assumed to be zero, σ_n' and $\bar{\sigma}_n$ are respectively the effective and net normal stress on the failure plane, ϕ' is the effective friction angle of the soil, χ is the well-known Bishop's effective stress parameter, and $\bar{c}$ is the apparent cohesion which includes the friction term due to suction, i.e. $\bar{c} = c' + \chi s \tan\phi'$.

Fredlund *et al.* (1978) proposed the following relationship which conveniently separates the shear strength due to stress from that due to suction:

$$\begin{aligned}\tau &= \left[c' + (\sigma_n - u_a)\tan\phi'\right] + \left[(u_a - u_w)\tan\phi^b\right] \\ &= \bar{c} + (\sigma_n - u_a)\tan\phi'\end{aligned} \tag{27}$$

where τ is the shear strength, c' the effective cohesion for saturated states and is usually assumed to be zero, σ_n the normal stress on the failure plane, ϕ' the effective friction angle of the soil, and ϕ^b the frictional angle due to suction. Obviously, if ϕ^b is set to ϕ' in equation (27), the Coulomb friction criterion in terms of effective stress for saturated soils is recovered.

The shear strength of an unsaturated soil is fully defined if the apparent cohesion ($\bar{c}$) or the friction angle due to suction (ϕ^b), and the friction angle due to stress ϕ' in equation (27) are known. A number of alternative equations have been used in the literature to define $\bar{c}$ or ϕ^b (e.g., Fredlund *et al.* 1996; Öberg & Sällfors 1997; Vanapalli *et al.* 1996; Khalili & Khabbaz 1998; Toll & Ong 2003; Miao *et*

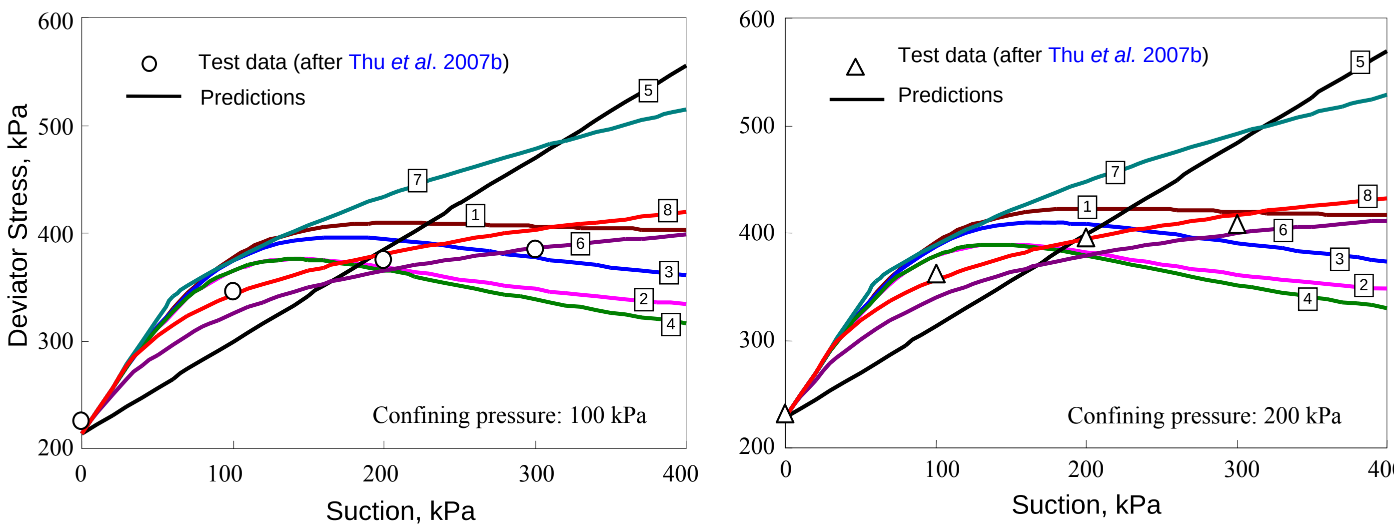


Figure 11. Predictions of triaxial test data on compacted kaolin clay by various shear strength equations (data by Thu *et al.* 2007b).

al. 2007). Most of these equations are empirically based and are defined independently from the volume change equation, which, if incorporated into a constitutive model, may lead to inconsistence with the yield surface as discussed in the Section above. Rojas (2008) presented an analytical expression for Bishop's parameter χ in terms of the saturated fraction and S_r of the unsaturated fraction of the soil and used it to interpret shear strength. One issue with this approach is that χ can not be easily determined either experimentally or theoretically. A similar approach was proposed by Pereira and Alonso (2009) where χ is expressed in terms of the effective degree of saturation that depends on the micro- and macro-saturation.

Table 1. Shear strength equations and parameters used for compacted kaolin clay.

Equations	$\tan\phi^b / \tan\phi'$	*Parameters*
1. Öberg & Sällfors (1997)	S_r	SWCC
2. Fredlund *et al.* (1996)	$(S_r)^{\kappa}$	κ=1.3, θ_s=0.659, SWCC
3. Vanapalli *et al.* (1996)	$\left(\dfrac{\theta-\theta_r}{\theta_s-\theta_r}\right)$	θ_s=0.659, SWCC
4. Toll & Ong (2003)	$\left(\dfrac{S_r-S_{r2}}{S_{r1}-S_{r2}}\right)^k$	k=1.2, SWCC
5. Alonso *et al.* (1990)	α	α=0.4
6. Sun *et al.* (2000)	$\dfrac{a}{s+a}$	a = 110 kPa
7. Khalili & Khabbaz (1998)	$(s_{ae}/s)^r$	s_{ae} = 60kPa, r =0.55
8. Sheng *et al.* (2008a)	$\left(\dfrac{s_{sa}}{s}+\left(\dfrac{s_{sa}}{s}\right)\ln\dfrac{s}{s_{sa}}\right)$	s_{sa} = 25kPa

Sheng *et al.* (2010) recently compared various shear strength equations for unsaturated soils against a large number of data sets. The equations include those empirically based and those embedded in constitutive models. The equations studied by Sheng *et al.* (2010) are listed in Table 1. Figure 11 presents one example of such comparisons. The parameters used in the shear strength equations are given in Table 1. The comparative study by Sheng *et al.* (2010) reveals that:

1. If the friction angle of the soil is assumed to be independent of suction, all shear strength equations in the literature can be formulated either in form of equation (26) or (27). In this case, there is little difference in formulating shear strength in one single stress variable or in two independent stress variables. The real challenge is to find an effective stress when the friction angle depends on suction.
2. The performance of the shear strength equations in predicting experimental data depends on the careful determination of material parameters and also on the specific data set. A shear strength equation may predict one data set better than other data sets.
3. The shear strength equations that incorporate the soil-water characteristic curve (SWCC) generally require more parameters. This group of equations seem to provide reasonable prediction of shear strength for unsaturated soils (e.g. Fredlund *et al.* 1996; Vanapalli *et al.* 1996; Toll & Ong 2003). However, some equations are sensitive to the residual suction (Vanapalli *et al.* 1996; Toll & Ong 2003), which can be difficult to determine accurately from the SWCC.

Simpler shear strength equations that are embedded in constitutive models appear to provide reasonable predictions of shear strength (Khalili & Khabbaz 1999; Sun *et al.* 2000; Sheng *et al.* 2008a). These equations contain typically one or two parameters. However, these equations usually do not predict any peak value of the shear strength attained at an intermediate suction.

6 WATER RETENTION BEHAVIOUR AND HYDRO-MECHANICAL COUPLING

The issue of interaction between the mechanical and hydraulic behaviour was perhaps first raised by Wheeler (1996) and then by Dangla *et al.* (1997). The first complete model that deals with coupled hydro-mechanical behaviour of unsaturated soils was perhaps due to Vaunat *et al.* (2000). A number of coupled models soon followed (e.g. Wheeler *et al.* 2003; Sheng *et al.* 2004). With respect to hydraulic behaviour of unsaturated soils, many models (van Genuchten 1980; Fredlund & Xing 1994) take advantage of the fact that the influence of suction on degree of saturation is more significant than the influence of deformation. The dependency of degree of saturation on suction is described by a soil-water characteristic curve (also called soil-water retention curve, SWRC). Only until recently, the effects of deformation on SWCCs have been considered (e.g., Gallipoli *et al.* 2003b; Wheeler *et al.* 2003; Sun *et al.* 2007b; Miller *et al.* 2008; Zhou 2009).

As pointed out by Wheeler *et al.* (2003), the mechanical behaviour of an unsaturated soil depends on degree of saturation even if the suction, net stress and specific volume are kept the same for the soil. Separate treatment of mechanical and hydraulic components in modelling unsaturated soil behaviour has certain limitations in reproducing experimental observation. It would be difficult to consider the saturation dependency in a mechanical model that is independent of the hydraulic behaviour. Similarly, a hydraulic model that is independent of mechanical behaviour can not easily take into account the effects of soil density on the SWCC. Experimental data generally demonstrate the following points:

1. A SWCC obtained under a higher net mean stress tends to shift towards the higher suction (Matyas & Radhakrishna 1968; Ng & Pang 2000; Gallipoli *et al.* 2003b; Lee *et al.* 2005; Tarantino & Tombolato 2005). This means that the incremental relationship between degree of saturation (S_r) and suction (s) depends on net mean stress ($\bar{p}$) or soil density.
2. When the suction is kept constant, isotropic loading or unloading can also change the degree of saturation of an unsaturated soil (Wheeler *et al.* 2003). This implies that the degree of saturation is related to stress or soil density when the suction is kept constant.

One of the early models that fully couple the hydraulic and mechanical components of unsaturated soil behaviour is that by Wheeler *et al.* (2003). Models that appeared before or soon after Wheeler *et al.* (2003) tend to accentuate the influences of the hydraulic component on the mechanical component, not vice versa (e.g., Vaunat *et al.* 2000; Sheng *et al.* 2004; Nuth & Laloui 2008b). The interaction between the mechanical and hydraulic components in the model by Wheeler *et al.* (2003) was realised through the use of the average soil skeleton stress (effective stress), the modified suction and the coupling between the loading-collapse and suction-increase and suction-decrease surfaces. The average soil skeleton stress (σ_{ij}^{*}) is an amalgam of stress, suction and degree of saturation. The modified suction (s^{*}) is a combination of suction and porosity. Therefore, the influence of hydraulic behaviour on the stress-strain relationship is considered via the definition of the average stress. The influence of porosity on the saturation-suction relationship is considered via the definition of the modified suction. The model by Wheeler *et al.* (2003) is one of the few models that are qualitatively tenable in terms of coupling mechanical behaviour with hydraulic behaviour for unsaturated soils. However, the use of the modified suction and the soil skeleton stress, which is one of the advantages that makes the model rigorously consistent in thermodynamics, can become a disadvantage as well, particularly in terms of quantitative prediction and the application of the model. For example, one of the difficulties in using this model is to quantify the synchronised movement between the loading-collapse (LC) surface and the suction-increase (SI) and suction-decrease (SD) surfaces. This synchronicity can not easily be calibrated by laboratory experiments (Raveendiraraj, 2009) or defined theoretically.

In more recent models, the influences of mechanical properties on the hydraulic behaviour are usually modelled via the dependency of the SWCC on soil volume (Gallipoli *et al.* 2003b; Tarantino 2009), soil density (Sun *et al.* 2007b; Mašín 2010), or volumetric strain (Nuth & Laloui 2008a). Gallipoli *et al.* (2003b) suggested including a function of specific volume (v) in the SWCC equation of van Genuchten (1980). Tarantino (2009) showed there is a unique relationship between the water ratio (product of S_r and e) and the matric suction and used this relationship to modify van Genuchten's equation. The modified van Genuchten's equation takes a similar form as Gallipoli *et al.* (2003b). It is also common to express the SWCC equation in incremental forms. For example, Sun *et al.* (2007b) proposed a hydraulic model in the following form:

$$\mathrm{d}S_r = \lambda_{se}\mathrm{d}e - \lambda_{ss}\,\mathrm{d}s/s \tag{28}$$

where λ_{ss} is the slope of main drying or wetting curve, and λ_{se} the slope of degree of saturation versus void ratio curve under constant suction. In theory, λ_{ss} in the equation above can only be determined from constant-volume tests ($\mathrm{d}e$=0). However, such tests are not common. Mašín (2010) used a similar equation as (28). In his model both air entry value (s_{ae}) and the slope of main drying curve (λ_{ss}) vary with void ratio. Nuth & Laloui (2008a) provided an alternative approach of modelling SWCC for a deforming soil. They assume there is an intrinsic SWCC for a non-deforming soil and deformation of the soil can shift this intrinsic SWCC along the suction axis. The shift is governed by an air entry value that depends on the volumetric strain.

The models by Sun *et al.* (2007b), Nuth & Laloui (2008a), Mašín (2010) and many others (e.g. Sheng *et al.* 2004, 2008a; Nuth & Laloui 2008b; Khalili *et al.* 2008; Zhou 2009) essentially all adopt a water retention equation in the following form:

$$\mathrm{d}S_r = (\ldots)\mathrm{d}s + (\ldots)\mathrm{d}\varepsilon_v \tag{29}$$

This equation is not wrong, but the embedded $S_r - s$ relationship is for constant volume ($\mathrm{d}\varepsilon_v = 0$). Therefore, it does not recover the conventional SWCC equations, which are obtained under constant stress. The volume change along a conventional SWCC can be significant. It is not common to obtain an $S_r - s$ relationship for constant soil volume. The vast available data on SWCCs that were obtained under constant stresses would be of limited use in these models. In addition, neglecting the volume change along

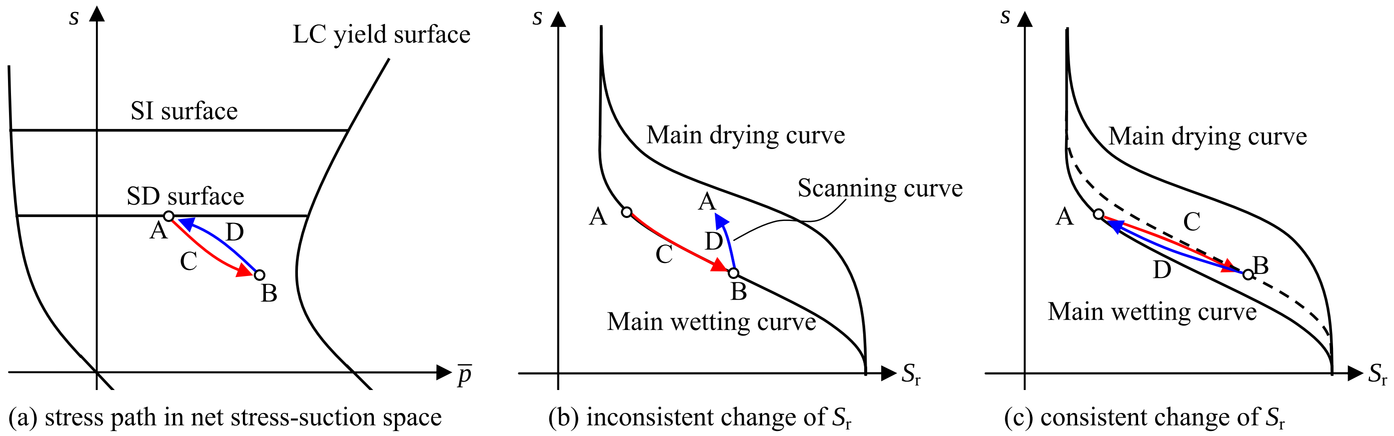


Figure 12. Qualitative analysis of isotropic compression under undrained condition.

SWCC can lead to inconsistent prediction of the degree of saturation during undrained compression, an issue raised by Zhang & Lytton (2008).

Sheng & Zhou (2010) proposed a new method for coupling hydraulic with mechanical behaviour. This new method is based on the fact that SWCCs are obtained under constant stress. In Sheng & Zhou (2010), the volume change behaviour and the water retention behaviour under isotropic stress states are represented by the following incremental equations, respectively:

$$d\varepsilon_v = A d\overline{p} + B ds \tag{30}$$

$$\begin{aligned} dS_r &= E\,ds + \frac{S_r}{n}(1-S_r)^m A d\overline{p} \\ &= \left(E - B\frac{S_r}{n}(1-S_r)^m\right)ds - \frac{S_r}{e}(1-S_r)^m de \end{aligned} \tag{31}$$

where parameters A and B are related to the specific volume change equation as discussed in Section 3, parameter E is the gradient of the conventional SWCC, e is the void ratio, n is the porosity, and m is a fitting parameter. If the SFG model is used to describe the volume change, parameters A and B would then take the form:

$$A = \frac{\lambda_{vp}}{\overline{p} + f(s)}, \qquad B = \frac{\lambda_{vs}}{\overline{p} + f(s)} \tag{32}$$

The $S_r - s$ relationship is defined for constant stress ($d\overline{p}$=0) in equation (31) and hence parameter E refers to the gradient of the SWCC:

$$E = \frac{d\left(S_r^{SWCC}(s)\right)}{ds} \tag{33}$$

where S_r^{SWCC} represents the conventional SWCC equation. The void ratio in equation (31) refers to the value at the current stress and suction.

It is clear from equation (31) that the $S_r - s$ relationship for constant volume (de=0) is more complex than the conventional SWCC equation ($dS_r = E\,ds$). Equation (31) was proposed based on experimental observation as well as the intrinsic phase relationship for undrained condition:

$$dS_r = \frac{S_r}{n} d\varepsilon_v, \qquad dw=0 \tag{34}$$

where w is the gravimetric water content. Equation (34) actually imposes a constraint on suction change under undrained compression. This constraint is obtained by substituting equation (34) into equation (31):

$$\left(S_r - S_r(1-S_r)^m\right)A d\overline{p} = (nE - B)ds, \quad dw=0 \tag{35}$$

Zhang & Lytton (2008) recently noted that some models fall short in predicting undrained behaviour of unsaturated soils. The specific issue raised is illustrated in Figure 12. Assume the initial state of a soil is inside the elastic zone, but on the main wetting curve, i.e. point A in Figure 12a. Compressing the soil under undrained condition will lead to some suction decrease (Sun *et al.* 2008; Tang *et al.* 2008), say to point B. Assume B is still inside the elastic zone. Unloading from B to A will recover the initial volume of the soil and hence the initial degree of saturation should be recovered as well. However, if the volume change along the SWCC is neglected, the change of S_r along path ACB would follow the main wetting curve and is hence 'elastoplastic', whereas the change of S_r along path BDA would follow the scanning curve and is hence 'elastic', leading to inconsistent change of S_r (Figure 12b). It would then seem unlikely that a model where the irreversible volume change is not synchronised with the irreversible saturation change could lead to a consistent prediction of saturation change over the closed path ABA (Zhang & Lytton 2009a). This inconsistency is actually due to the assumption that the main wetting curve is defined for constant volume.

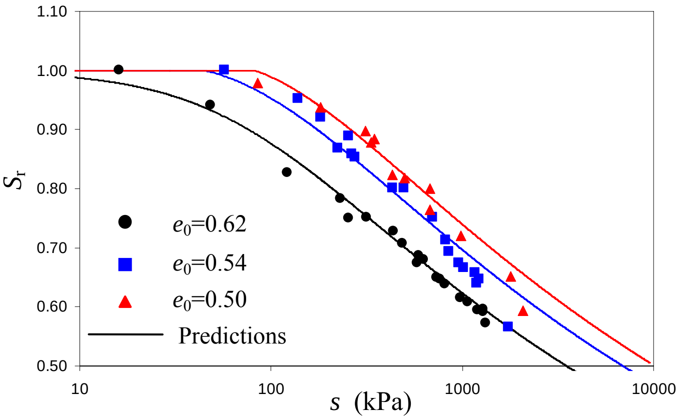


Figure 13. SWRCs of reconstituted Barcelona silt with different initial voids ratios (data after Tarantino 2009).

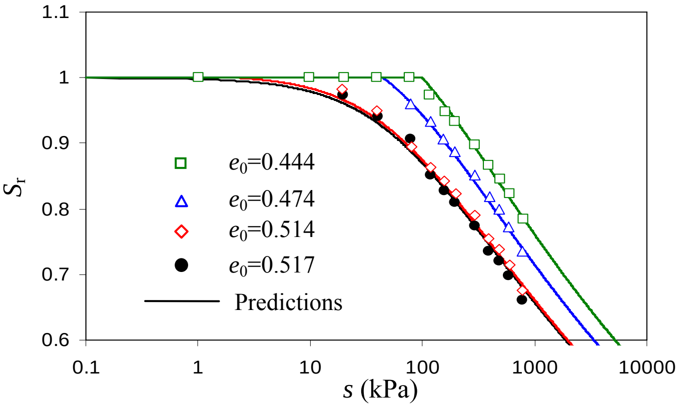


Figure 14. SWCCs for specimens compacted at optimum water content (data after Vanapalli *et al.* 1999).

If equation (31) is used, the inconsistence in Figure 12b will be avoided. Because of equation (34), the model will predict no change of S_r as long as $\varepsilon_v = 0$, irrespective of evolution of the suction-decrease surface. Equation (34) is satisfied as long as the suction changes according to (35). It is easy to understand that the loading path ACB is not on the initial main wetting curve and the unloading path BDA is not on the scanning curve, because the mean stress is not constant along those stress paths. The main wetting curve at point A also shifts to that at point B, as the mean stress changes (Figure 12c). The synchronised evolution of the LC, SI and SD surfaces (as in Wheeler *et al.* 2003) is not necessary for the consistent prediction in Figure 12c. Indeed, the suction path can be 'elastoplastic' albeit the elastic stress path.

The following equation can be derived from equation (31):

$$\frac{\partial S_\mathrm{r}}{\partial e} = -\frac{S_\mathrm{r}(1-S_\mathrm{r})^m}{e} \qquad (36)$$

The void ratio (e) in equation (36) refers to the initial void ratio at the current stress. It can also be interpreted as the initial void ratio at the start of the SWCC tests. Equation (36) shows that the SWCC for a soil shifts with its initial void ratio. This is similar to the approach by Gallipoli *et al.* (2003b) where the van Genuchten equation was modified to include the initial void ratio. Indeed, their SWCC equation can be re-written as:

$$\frac{\partial S_\mathrm{r}}{\partial e} = -mn\psi \frac{S_\mathrm{r}(1-S_\mathrm{r}^{1/m})}{e} \qquad (37)$$

where m and n are two fitting parameters in the original van Genuchten equation, and ψ is another parameter introduced by Gallipoli *et al.* (2003b). If the product ($mn\psi$) was set to 1, equation (37) would be equivalent to (36). Sheng & Zhou (2010) showed that the intrinsic phase relationship requires:

$$\frac{1-S_\mathrm{r}}{e} \geq \frac{\partial S_\mathrm{r}}{\partial e} \geq \frac{-S_\mathrm{r}}{e} \qquad (38)$$

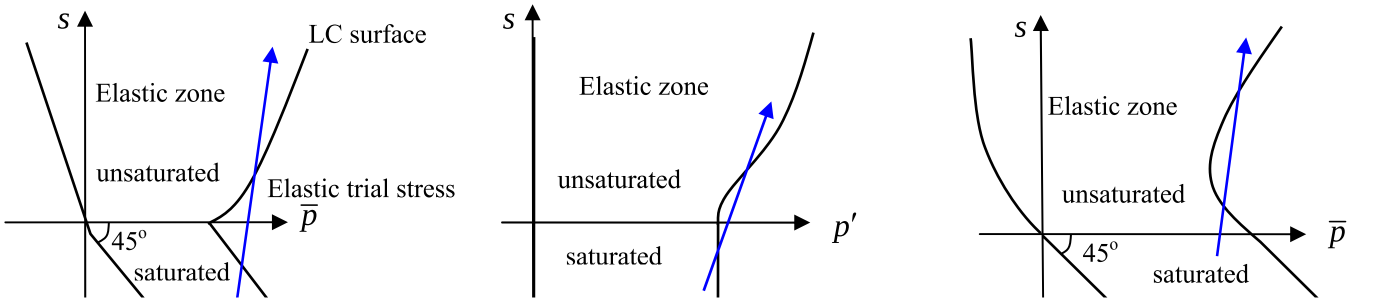
The above constraint is satisfied if $mn\psi$=1 in equation (37). It is also interesting to note that all the numerical examples in Gallipoli *et al.* (2003b) used a value of 1.1 for $mn\psi$.

Equation (36) can be integrated either analytically for certain m values or numerically in more general cases. Because equation (36) is in an incremental form, integration of the equation requires one specific SWCC equation that corresponds to a reference initial void ratio. In other words, the conventional SWCC equation is only used for the reference initial void ratio and the new SWCC for a new initial void ratio is obtained by integration of (36).

The model by Sheng & Zhou (2010) is validated against a variety of data sets. In Figure 13, the predicted SWCCs are compared with the data set by Tarantino (2009) on reconstituted Barcelona silt. In the prediction, the SWCC for e_0=0.62 was fitted by van Genuchten equation. The other SWCCs were predicted with $m = 0.2$. In Figure 14, the data by Vanapalli *et al.* (1999) on a compacted till were used. The SWCC for e_0=0.517 was fitted by the van Genuchten equation, while the other SWCCs are predicted by equation (36) with m=0.03. Figure 14 shows that both the slope and the air entry value of the SWCCs change with the initial void ratio. In the two cases studied, the model by Sheng & Zhou (2010) seems to be able to capture the effect of initial void ratio on the soil water retention behaviour.

7 FINITE ELEMENT IMPLEMENTATION

One of the ultimate goals of constitutive modelling is to implement the model in a numerical method to solve boundary value problems. A constitutive model can generally be formulated in the following incremental form (Sheng *et al.* 2004, 2008c):



(a) net stress (Alonso *et al.*, 1990) (b) effective stress (Sheng *et al.*, 2004) (c) net stress (Sheng *et al.*, 2008a)

Figure 15. Non-convexity of yield surfaces in unsaturated soil models in the suction-stress space.

$$\begin{pmatrix} \mathrm{d}\bar{\boldsymbol{\sigma}} \\ \mathrm{d}\theta \end{pmatrix} = \begin{pmatrix} \mathbf{D}^{\mathrm{ep}} & \mathbf{W}^{\mathrm{ep}} \\ \mathbf{T} & H \end{pmatrix} \begin{pmatrix} \mathrm{d}\boldsymbol{\varepsilon} \\ \mathrm{d}s \end{pmatrix} \tag{39}$$

where $\bar{\boldsymbol{\sigma}}$ is the stress vector, $\boldsymbol{\varepsilon}$ is the vector of soil skeleton strain, θ is the volumetric water content, $\mathbf{D}^{\mathrm{ep}}$, $\mathbf{W}^{\mathrm{ep}}$, $\mathbf{T}$ and H are constitutive matrices (Sheng *et al.* 2008c).

In the displacement finite element method, the nodal displacements and pore pressures are first solved from the equilibrium and continuity equations, based on the current stress states and the current volumetric water content. The strain and suction increments are then derived from the displacements and pore pressures. For given strain and suction increments, the current stress vector, the volumetric water content and the internal variables must be updated according to equation (39). This update is generally carried out by numerical integration.

One of the main challenges in integrating equation (39) arises from the non-convexity of the yield surface around the transition between saturated and unsaturated states (Wheeler *et al.* 2002, Sheng *et al.* 2003b; Sheng 2003). The non-convexity seems to persist irrespective of the stress variables and is demonstrated in Figure 15 (Sheng *et al.* 2008d).

Both implicit and explicit schemes have been used to integrate unsaturated soils models. In implicit schemes, all gradients and functions are estimated at advanced unknown stress states and the solution is achieved by iteration. This group of stress integration schemes include Vaunat *et al.* (2000), Borja (2004), Hoyos & Arduino (2008), Zhang & Zhou (2008), and Tamagnini & De Gennaro (2008). They usually do not deal with the non-convex problem by avoiding the transition between saturated and unsaturated states. Borja (2004) was perhaps the only one in this group who noticed the problem. He suggested keeping the step size sufficiently small to avoid overshooting the plastic zone by the elastic trial stress (Figure 15). Otherwise, there is currently no general method used to tackle the non-convexity problem in implicit schemes.

On the other hand, explicit schemes estimate the gradients and functions at the current known stress states and proceed in an incremental fashion. These methods are theoretically more appropriate for non-convex models (Pedroso *et al.* 2008). González & Gens (2010) compared both an implicit and an explicit scheme for integrating the BBM and found that the latter is more robust and efficient. However, explicit schemes usually require to determine the intersection between the current yield surface and the elastic trial stress path and some substepping methods to control the integration error (Sheng *et al.* 2003a, 2003b; Sheng *et al.* 2008c, 2008d; Sánchez *et al.* 2008; Sołowski & Gallipoli 2009a, 2009b).

A key issue in integrating the incremental stress-strain relationships using an explicit scheme is thus to find the intersection between the elastic trial stress path and the current yield surface. The most complex situation occurs when the yield surface is crossed more than once, such as shown in Figure 15a. However, it is not possible to know *a priori* how many times the yield surface is crossed. Therefore, for non-convex yield surfaces, the key task is to find the very first intersection for any possible stress and hydraulic path.

Finding the intersection between the elastic trial stress increment and the current yield surface can be cast as a problem of finding the multiple roots of a nonlinear equation:

$$f(\alpha) = f(\bar{\boldsymbol{\sigma}}_\alpha, s_\alpha, \mathbf{z}_\alpha) = 0 \tag{40}$$

where $0 \le \alpha \le 1$, $f(\bar{\boldsymbol{\sigma}}, s, \mathbf{z})$ is the yield function, $\mathbf{z}$ is a set of internal variables, and subscript α indicates the quantity is estimated at strain increment $\alpha\Delta\varepsilon$ and suction increment $\alpha\Delta s$. Pedroso *et al.* (2008) proposed a novel method to bracket the roots (α). The method is illustrated in Figure 16. For a given increment (α=1), the number of root of $f(\alpha)$ is first computed. If there are more than one roots, the increment is divided into two equal sub-increments. The number of roots of each sub-increment is then

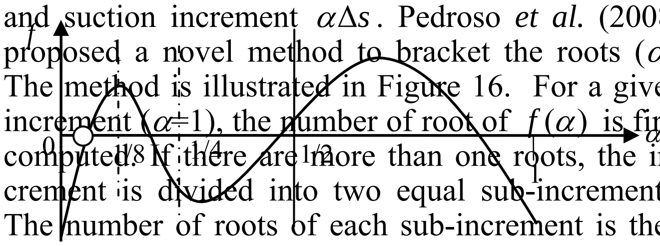


Figure 16. Bracketing the roots for nonlinear function according to Pedroso *et al.* (2008).

increments. This process is repeated until the first sub-increment contains at most one root (Figure 16). Once the roots are bracketed, the solution of the first root can be found by using numerical methods such as the Pegasus method (Sloan *et al*. 2001).

Sheng *et al.* (2008d) applied the method by Pedroso *et al.* (2008) to integrate the SFG model and found that the method can indeed provide an accurate solution of the intersection problem. However, this method is found to be computationally expensive. The formula used to find the number of roots of a nonlinear function requires both the first and second orders of gradients of the function and requires numerical integration. Furthermore, the root-finding procedure must be applied for all suction increments near the non-convexity. Further research is required to improve its efficiency.

8 CONCLUSIONS

Some conclusions can be drawn from this review of constitutive modelling of unsaturated soils:

1. Partial saturation is only a state of soil. Constitutive models for soils should represent the soil behaviour over entire ranges of possible stress, pore pressure and suction values. This requires a coherent merge of fundamental soil mechanics principles for saturated and unsaturated states.
2. The volume change behaviour is one of the most fundamental properties of soils. For soils in unsaturated states, the volume change equation also underpins the yield stress – suction and shear strength – suction relationships. It also affects the soil water retention behaviour.
3. Three groups of volumetric models are compared and it is shown that each has advantages and disadvantages. There is certainly room for improvement of all these models. One observation here is that it seems difficult to describe the volume change of unsaturated soils in terms of one single stress variable.
4. The methods used to define the loading-collapse, suction-increase and apparent tensile strength surfaces should be consistent with the volume change equation. In addition, these surfaces and the shear strength function are all related to each other and hence should be defined consistently with each other.
5. If the friction angle of the soil is assumed to be independent of suction, all shear strength equations in the literature can be formulated either in terms of one single stress variable or in terms of two independent stress variables. The real challenge is to find a single effective stress when the friction angle depends on suction.
6. The performance of the shear strength equations in predicting experimental data depends on the careful determination of material parameters and also on the specific data set. A shear strength equation may predict one data set better than other data sets.
7. When coupling the hydraulic component with the mechanical component in a constitutive model, it is recommended to take into account the volume change along soil-water characteristic curves. Neglecting this volume change can lead to inconsistent prediction of volume and saturation changes.
8. Unsaturated soil models are characterised by non-convex yield surfaces at the transition between saturated and unsaturated states. This non-convexity, if handled rigorously, can significantly complicate the implementation of these models into finite element codes. In this case, an explicit stress integration scheme incorporating an efficient root search algorithm is preferred.

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