

The ultimate pullout capacity of anchors in frictional soils

R.S. Merifield and S.W. Sloan

Abstract: During the last 30 years various researchers have proposed approximate techniques to estimate the uplift capacity of soil anchors. As the majority of past research has been experimentally based, much current design practice is based on empiricism. Somewhat surprisingly, very few numerical analyses have been performed to determine the ultimate pullout loads of anchors. This paper presents the results of a rigorous numerical study to estimate the ultimate pullout load for vertical and horizontal plate anchors in frictional soils. Rigorous bounds have been obtained using two numerical procedures that are based on finite element formulations of the upper and lower bound theorems of limit analysis. For comparison purposes, numerical estimates of the break-out factor have also been obtained using the more conventional displacement finite element method. Results are presented in the familiar form of break-out factors based on various soil strength profiles and geometries and are compared with existing numerical and empirical solutions.

Key words: anchor, pullout capacity, finite elements, limit analysis, lower bound, sand.

Résumé : Au cours des trente dernières années, divers chercheurs ont proposé des techniques approximatives pour estimer la capacité d'arrachement des ancrages dans le sol. Comme la majorité des recherches antérieures ont été expérimentales, une bonne part de la pratique courante de conception est basée sur l'empirisme. Il est quelque peu surprenant que très peu d'analyses numériques ont été réalisées pour déterminer les charges ultimes d'arrachement des ancrages. Cet article présente les résultats d'une étude numérique rigoureuse pour estimer la charge ultime d'arrachement de plaques d'ancrage verticales et horizontales dans des sols pulvérulents. Des limites rigoureuses ont été obtenues au moyen de deux procédures numériques qui sont basées sur des formulations en éléments finis des théorèmes de limites supérieure et inférieure de l'analyse limite. Pour fins de comparaison, de nombreuses estimations numériques du coefficient de rupture ont été obtenues au moyen de la méthode plus conventionnelle de déplacement par éléments finis. Les résultats sont présentés sous la forme familière de coefficients de rupture sur divers profils de résistance au cisaillement, et ils sont comparés avec les solutions numériques et empiriques existantes.

Mots clés : ancrage, capacité d'arrachement, éléments finis, analyse limite, limite inférieure, sable.

[Traduit par la Rédaction]

Introduction

The design of many engineering structures requires that foundation systems resist vertical uplift or horizontal pullout forces. In such cases, an attractive and economic design solution may be achieved through the use of tension members. These members, which are referred to as soil anchors, are typically fixed to the structure and embedded in the ground to sufficient depth so that they can resist pullout forces with safety. Soil or "ground" anchors are a lightweight foundation system designed and constructed specifically to resist any

uplifting force or overturning moment placed on a structure. Plate anchors can be installed by excavating the ground to the required depth, placing the anchor, and then backfilling with soil. For example, when used as a support for retaining structures, anchors are installed in excavated trenches and connected to tie rods that may be driven or placed through augered holes. This type of anchor is the subject of interest in this paper.

The application of soil anchors in supporting transmission towers appears to be the driving force behind much of the initial research into anchor behaviour (Balla 1961). Initially, these towers were supported by large-mass concrete blocks where the required uplift capacity was supplied entirely by the self-weight of the concrete. This simple design came at a considerable cost and, as a result, research was undertaken to find a more economical design solution. The result was what is known as belled piers or mushroom foundations. As the range of applications for anchors expanded to include the support of more elaborate and substantially larger structures, a more concerted research effort has meant soil anchors today have evolved to the point where they now provide an economical and competitive alternative to these mass foundations.

It is clear that the majority of past research has been experimentally based and, as a result, current design practices are largely based on empiricism. In contrast, very few thor-

Received 11 April 2005. Accepted 28 April 2006. Published on the NRC Research Press Web site at <http://cgj.nrc.ca> on 4 August 2006.

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ough numerical analyses have been performed to determine the ultimate pullout loads of anchors. Of the numerical studies that have been presented in the literature, few can be considered as rigorous.

Problem definition

Soil anchors may be positioned either vertically or horizontally depending on the load orientation or type of structure requiring support. A general layout of the problem to be analysed is shown in Fig. 1.

For numerical convenience, the ultimate anchor capacity, q_u , is presented in a form analogous to Terzaghi's equation, which is used to analyse surface footings:

$$[1] \quad q_u = \gamma H N_\gamma$$

where γ is the unit weight of soil, H is the depth of the anchor, and N_γ is referred to as the anchor break-out factor.

Previous studies

To provide a satisfactory background to subsequent discussions, a summary of research into plate anchor behaviour is presented. A comprehensive overview on the topic of anchors is given by Das (1990).

Research into the behaviour of soil anchors can take one of two forms, namely experimental-based or numerical-theoretical-based studies. The brief summary of existing research herein has been separated based on this distinction. No attempt is made to present a complete bibliography of all research, but rather a more selective overall summary of research with greatest relevance to the current paper is presented. In addition, contributions made to the behaviour of circular, multiple underreamed, or multihelix anchors have not been reviewed.

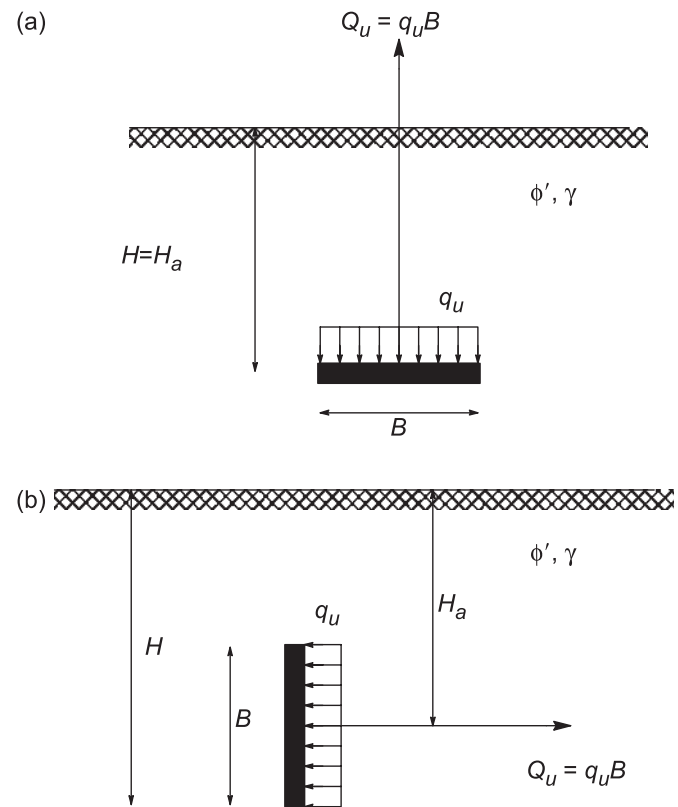
Previous experimental investigations

Although there are no entirely adequate substitutes for full-scale field testing, tests at the laboratory scale have the advantage of allowing close control of at least some of the variables encountered in practice. In this way, trends and behaviour patterns observed in the laboratory can be of value in developing an understanding of performance at larger scales. In addition, observations made in laboratory testing can be used in conjunction with mathematical analyses to develop semi-empirical theories. These theories can then be applied to solve a wider range of problems.

Experimental investigations into plate anchor behaviour have generally adopted one of two approaches, namely, conventional methods under "normal gravity" conditions or centrifuge systems. Of course, both methods have advantages and disadvantages, and these must be borne in mind when interpreting the results from experimental studies of anchor behaviour.

Numerous investigators have performed model tests in an attempt to develop semi-empirical relationships that can be used to estimate the capacity of anchors in cohesionless soil. This is evidenced by the number of studies shown in Tables 1 and 2. More specific details of these studies are provided in later sections of the paper when several previous laboratory studies are compared with the new theoretical predictions obtained in the current study.

Fig. 1. Problem notation. The distribution of pressure q_u is unlikely to be uniform as shown. (a) Horizontal plate anchors. (b) Vertical plate anchors.



The works prior to 1970 have not been presented in Tables 1 and 2. This includes the field and (or) model testing of horizontal circular anchors or belled piles by Mors (1959), Giffels et al. (1960), Balla (1961), Turner (1962), Ireland (1963), Sutherland (1965), Mariupolskii (1965), Kananyan (1966), Baker and Kondner (1966), and Adams and Hayes (1967). A number of these studies were primarily concerned with testing foundations for transmission towers (Mors 1959; Balla 1961; Turner 1962; Ireland 1963). In the majority of these earlier studies, a failure mechanism was assumed and the uplift capacity was then determined by considering the equilibrium of the soil mass above the anchor and contained by the assumed failure surface. Subsequent variations on these early theories have been proposed by Balla, Baker and Kondner, Sutherland, and Kananyan.

Previous theoretical investigations

Although there are a variety of experimental results in the literature, very few rigorous numerical analyses have been performed to determine the pullout capacity of anchors in sand. Although it is essential to verify theoretical solutions with experimental studies wherever possible, results obtained from laboratory testing alone are typically problem specific. Since the cost of performing laboratory tests on each and every field problem combination is prohibitive, it is necessary to be able to model soil uplift resistance numerically for the purposes of design.

Table 1. Laboratory model tests on horizontal anchors in cohesionless soil.

Source	Type of testing	Anchor shape	Anchor size (mm)	Friction angle (°)	Anchor roughness (°)	H/B or H/D
Das and Seeley 1975	Chamber	Square; rectangular; $L/B = 1-5$	51	31	?	1-5
Rowe 1978	Chamber	Square; rectangular	51	32	16.7	1-8
Ovesen 1981	Centrifuge; field	Circular; square	20	29.5-37.7	?	1-3.39
Murray and Geddes 1987	Chamber	Circular; rectangular; $L/B = 1-10$	50.8	44, dense; 36, medium dense	11, smooth; 42, rough	1-10
Frydman and Shamam 1989	Field chamber (summary)	Strip; rectangular	19; 200	30, loose; 45, dense	?	2.5-9.35
Dickin 1988	Centrifuge chamber	Square; rectangular; $L/B = 1-8$	25; 50	38-41, ^a loose; 48-51, ^a dense	?	1-8
Tagaya et al. 1988	Centrifuge	Circular; rectangular	15	42	?	3-7.02
Murray and Geddes 1989	Chamber	Square; rectangular; $L/B = 1-10$	50.8	43.6, dense; 36, medium dense	10.6	1-8

Note: D , anchor diameter; L , anchor length.

^aPlane strain friction angle.

Table 2. Laboratory model tests on vertical anchors in cohesionless soil.

Source	Type of testing	Anchor shape	Anchor size (mm)	Friction angle (°)	Anchor roughness (°)	H/B
Neely et al. 1973	Chamber	Square; rectangular	50.8	38.5	21	1-5
Das 1975	Chamber	Square; circular	38-76	34	?	1-5
Akinmusuru 1978	Chamber	Strip; rectangular; square; circular; $L/B = 2, 10$	50	24; 35	?	1-10
Ovesen 1981	Centrifuge; field	Square	20	29.5-37.7	?	1-3.39
Dickin and Leung 1983, 1985	Centrifuge chamber	Square; rectangular; strip	25; 50	41 ^a	Polished, 29	1-8; 1-13
Hoshiya and Mandal 1984	Sand chamber	Square; rectangular; $L/B = 2, 4, 6$	25.4	29.5	?	1-6
Murray and Geddes 1989	Sand chamber	Square; rectangular; $L/B = 1-10$	50.8	43.6, dense	10.6	1-8

^aMobilized plane strain friction angle, ϕ'_{mp} .

A summary of previous theoretical studies post-1968 for horizontal and vertical anchors is provided in Tables 3 and 4, respectively.

Previous numerical studies of anchors in sand have typically utilized simple analytical approaches such as limiting equilibrium, cavity expansion, and limit analysis. These can be found in the works of Meyerhof and Adams (1968), Meyerhof (1973), Neely et al. (1973), Murray and Geddes (1987, 1989), Hanna et al. (1988), Basudhar and Singh (1994), and Smith (1998).

In the limit equilibrium method, a failure surface is assumed along with a distribution of stress along that surface. Equilibrium conditions are then considered for the failing soil mass, and an estimate of the collapse load is obtained. In the case of horizontal anchors, the failure mechanism is generally assumed to be a logarithmic spiral in shape (Murray and Geddes 1987), and the distribution of stress is obtained either by using Kotter's equation (Balla 1961) or by making an assumption regarding the orientation of the resultant force acting on the failure plane.

Neely et al. (1973) used both a trial failure surface approach and the method of characteristics to analyse a vertical strip anchor in a cohesionless material. In the first method, a

trial surface was adopted which consisted of a combination of straight lines and logarithmic spirals. It was assumed initially that the soil above the level of the top of the anchor would act as a simple surcharge. This was defined as the "surcharge method of analysis". Since this approach ignores the shearing resistance of the soil above the anchor, however, the approach was modified by incorporating the strength of the soil above the anchor through what was termed an *equivalent free surface*. This method was defined as the "equivalent free surface" method.

Limit analysis techniques have been used to estimate the capacity of horizontal and vertical strip anchors in sand. Basudhar and Singh (1994) obtained estimates using a generalized lower bound procedure based on finite elements and nonlinear programming similar to that of Sloan (1988). The solutions of Murray and Geddes (1987, 1989) were obtained by manually constructing kinematically admissible failure mechanisms (upper bound), and Smith (1998) presented a novel rigorous limiting stress field (lower bound) solution for the trap-door problem.

More rigorous studies have been performed by Rowe and Davis (1982), Tagaya et al. (1983, 1988), Vermeer and Sutjiadi (1985), and Koutsabeloulis and Griffiths (1989), using the dis-

Table 3. Theoretical studies on horizontal anchors in cohesionless soil.

Source	Analysis method	Anchor shape	Anchor roughness	Friction angle (°)	H/B or H/D
Meyerhof and Adams 1968	Limit equilibrium – semi-analytical	Strip; square; circular	?	—	—
Rowe and Davis 1982	Elastoplastic finite element	Strip	Smooth	0–45	1–8
Vermeer and Sutjiadi 1985	Elastoplastic finite element – upper bound	Strip	?	All	1–8
Tagaya et al. 1988	Elastoplastic finite element	Circular; rectangular; $L/B = 2$?	31.6; 35.1	0–30
Tagaya et al. 1983	Elastoplastic finite element	Circular; rectangular; $L/B = 2$?	42	0–30
Murray and Geddes 1987	Limit analysis and limit equilibrium	Strip; rectangular; circular	?	All	All
Koutsabeloulis and Griffiths 1989	Finite element – initial stress method	Strip; circular	?	20; 30; 40	1–8
Basudhar and Singh 1994	Limit analysis – lower bound	Strip	Rough; smooth	32	1–8
Kanakapura et al. 1994	Method of characteristics	Strip	Smooth	5–50	2–10
Smith 1998	Limit analysis – lower bound	Strip	Rough?	25–50	1–28

Table 4. Theoretical studies on vertical anchors in cohesionless soil.

Source	Analysis method	Anchor shape	Anchor roughness	Friction angle (°)	H/B
Biarez et al. 1965	Limit equilibrium	Strip	?	All	All
Meyerhof 1973	Limit equilibrium – semi-analytical	Strip	?	All	All
Neely et al. 1973	Limit equilibrium and method of characteristics	Strip	Rough, $\phi/2^a$	30–45	1–5.5
Rowe and Davis 1982	Elastoplastic finite element	Strip	Smooth	0–45	1–8
Hanna et al. 1988	Limiting equilibrium	Strip; inclined	?	All	All
Murray and Geddes 1989	Limit analysis – upper bound	Strip; inclined	Smooth; rough	43.6	1–8
Basudhar and Singh 1994	Limit analysis – lower bound	Strip	Rough; smooth	32; 35; 38	1–5

^a ϕ , soil friction angle.

placement finite element technique. Unfortunately, only limited results were presented in the studies of Tagaya et al. and Vermeer and Sutjiadi. The most complete numerical study appears to be that by Rowe and Davis. In their investigation, results were presented for both horizontal and vertical strip anchors embedded in sand. An elastoplastic finite element analysis was used and modelled the soil–structure interaction at the soil–anchor boundary. Koutsabeloulis and Griffiths investigated the trap-door problem using the initial stress finite element method. Both plane strain and axisymmetric studies were conducted.

Analysis of anchors

At present, there are a number of techniques available for use by geotechnical practitioners and researchers when analysing geotechnical problems. These include the limit equilibrium method, slip line method – method of characteristics, limit analysis theorems, and displacement finite element method (DFEM).

The displacement finite element technique is now widely used for predicting the load–deformation response, and hence collapse, of geotechnical structures. This technique can deal with complicated loadings, excavation and deposition sequences, geometries of arbitrary shape, anisotropy, layered deposits, and complex stress–strain relationships. Although the ability to incorporate all of these variables can be a distinct advantage when compared to other methods of analysis, it can also be perceived as the greatest disadvantage of the method. In practice, great care must be exercised when finite element analysis is employed to predict collapse loads. In general, such an analysis needs to be performed by qualified and experienced personnel. Even for quite simple problems, experience has indicated that results from the displacement finite element method tend to overestimate the true limit load and, in some instances, fail to provide a clear indication of collapse altogether. The ability of this technique to accurately predict incipient collapse has been studied in Toh and Sloan (1980) and Sloan and Randolph (1982).

Another approach for analysing the stability of geotechnical structures is to use the upper and lower bound

limit theorems developed by Drucker et al. (1952). These theorems can be used to bracket the exact ultimate load from above and below and are based, respectively, on the notions of a kinematically admissible velocity field and a statically admissible stress field. A kinematically admissible velocity field is simply a failure mechanism in which the velocities satisfy both the flow rule and the velocity boundary conditions; a statically admissible stress field is one where the stresses satisfy equilibrium, the stress boundary conditions, and the yield criterion.

Three different numerical methods have been used in this paper to determine the ultimate capacity of anchors. These are the upper and lower bound theorems of limit analysis and the more conventional DFEM.

Numerical limit analysis

Rigorous bounds on the ultimate pullout load presented in this paper are obtained by using two numerical procedures that are based on finite element formulations of the upper and lower bound theorems of limit analysis. These formulations assume a perfectly plastic soil model with an associated flow rule and generate large linear programming problems. For drained loading in purely frictional soils, the soil is assumed to obey a Mohr–Coulomb failure criterion in terms of the soil friction angle ϕ . A detailed overview of the numerical procedures used here can be found in Sloan (1988) and Sloan and Kleeman (1995). The following paragraphs provide a brief summary of the limit theorems.

The lower bound solution is obtained by modelling a statically admissible stress field using finite elements where stress discontinuities can occur at the interface between adjacent elements. Application of the stress boundary conditions, equilibrium equations, and yield criterion leads to an expression of the collapse load, which is maximized subject to a set of constraints on the stresses. The lower bound theorem states that if an equilibrium distribution of stress covering the whole body can be found that balances a set of external loads on the stress boundary and nowhere exceeds the yield criterion of the material, the external loads are not higher than the true collapse load. By examining different admissible stress states, the best (highest) lower bound value on the external loads can be found.

An upper bound on the exact pullout capacity can be obtained by modelling a kinematically admissible velocity field with finite elements. To be kinematically admissible, a velocity field must satisfy the set of constraints imposed by compatibility, the velocity boundary conditions, and the flow rule. After prescribing a set of velocities along a specified boundary segment, we can equate the power dissipated internally (caused by plastic yielding within the soil mass and sliding of the velocity discontinuities) to the power dissipated by the external loads to yield a strict upper bound on the true limit load.

Mesh details

Previous numerical studies using the formulations of Sloan (1988) and Sloan and Kleeman (1995) (Merifield et al. 2001) have provided several important guidelines for mesh generation. These, and a number of other studies, indicated that successful mesh generation typically involves ensuring (i) the overall mesh dimensions are adequate to

contain the computed stress field (lower bound) or velocity–plastic field (upper bound), and (ii) there is an adequate concentration of elements within critical regions.

The overall mesh dimensions for a lower bound problem can be checked by comparing the result obtained using a mesh with extension elements with that obtained using a mesh without extension elements. The use of the rectangular and triangular extension elements enables the stress field to be extended indefinitely in the half-plane without violating equilibrium, the stress boundary conditions, or the yield criterion, and the solution is thus a rigorous lower bound. It is then reasonable to conclude that if the two results differ significantly, then the overall mesh dimensions are inadequate and should be increased. For an upper bound problem, a visual representation of the plastic regions will provide adequate information for determining the overall mesh size. In this case the mesh need only be large enough to contain the zones of plastic shearing.

When generating upper and lower bound meshes there are several points worth considering. First, a greater concentration of elements should be provided in areas where high stress gradients (lower bound) or high velocity gradients (upper bound) are likely to occur. For the problem of a soil anchor, these regions exist at the anchor edges. Second, in areas where there is a significant change in principal stress direction (lower bound), appropriate shaped meshes should be used. Such principal stress rotations are, for example, observed at the edges of footings and anchors in cohesionless soil. The best solution in this case is generally obtained by adopting a fan-type mesh. Lastly, where possible, elements with severely distorted geometries should be avoided. This is particularly the case in upper bound analyses, where such elements can have a significant effect on the observed mechanism and collapse load.

In accordance with the previous discussion, the final finite element mesh arrangements (both upper and lower bound) were selected only after considerable refinements had been made. The process of mesh optimization followed an iterative procedure, and the final selected mesh characteristics were those which were found to either minimize the upper bound solution or maximize the lower bound solution. This will have the desirable effect of reducing the error bounds between both solutions, hence bracketing the actual collapse load more closely.

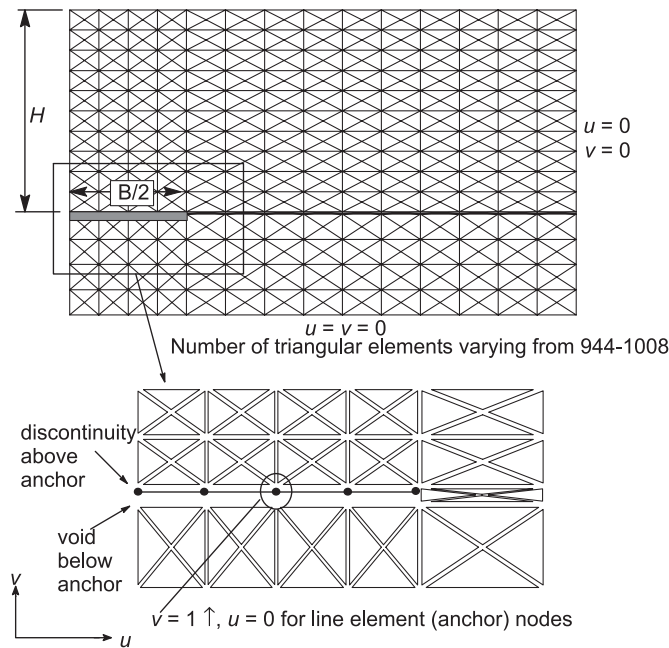
A typical upper bound mesh for the problem of a horizontal and vertical anchor, along with the applied velocity boundary conditions and mesh dimensions, is shown in Figs. 2 and 3, respectively. The symmetrical nature of the horizontal anchor problem was utilized, and only half the anchor was incorporated in the finite element mesh (Fig. 2).

In all cases, the effect of mesh variation on the results for different embedment ratios was avoided by maintaining fairly constant element dimensions.

Limit analysis collapse load determination

An upper bound solution is obtained by prescribing a certain set of velocity boundary conditions, which depend on the type of problem being analysed. As an example, for the vertical anchor problem (Fig. 3) a unit horizontal velocity ($u = 1$) is prescribed for the nodes along the line element that represents the anchor, with the additional constraint that

Fig. 2. Upper bound finite element mesh geometry for horizontal anchors. u , horizontal velocity; v , vertical velocity.



it cannot move vertically (vertical velocity $v = 0$). After the corresponding optimization problem is solved for the imposed boundary conditions, the collapse load is found by equating the internal dissipated power to the power expended by the external forces.

A lower bound solution for the anchor problem is obtained by maximizing the integral of the compressive stress along the soil–anchor interface. The task is to find a statically admissible stress field that maximizes the collapse load over the area of the anchor.

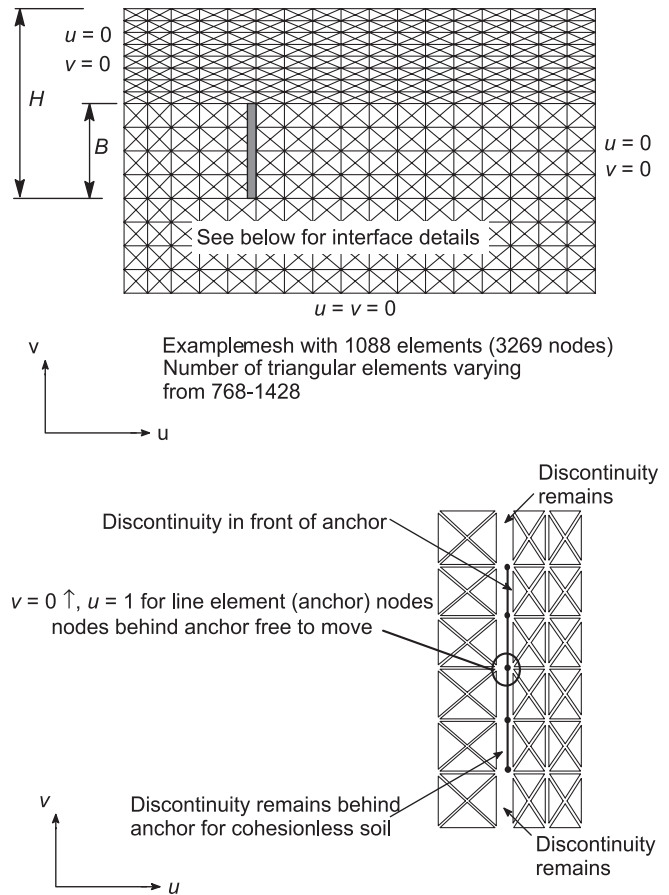
Displacement finite element analysis (DFEA)

The use of the finite element method is now widespread among researchers and practitioners, but it has not been used widely when estimating the capacity of soil anchors.

The finite element formulation used by the authors is that presented by Abbo (1997) and Abbo and Sloan (1998). This formulation, called the solid nonlinear analysis code (SNAC), was developed with the aim of reducing the complexity of elastoplastic analysis by using advanced solution algorithms with automatic error control. The resulting formulation greatly enhances the ability of the finite element technique to predict collapse loads accurately and avoids many of the inherent problems discussed by Toh and Sloan (1980) and Sloan and Randolph (1982).

The soil was modelled as an isotropic elastic – perfectly plastic continuum with yielding described by the Mohr–Coulomb yield criterion. A 15-noded triangular element was used in the analyses. It is anticipated that such a high-order element will provide a good estimate of the anchor collapse load. Again, for drained loading in purely frictional soils, the soil is assumed to obey a Mohr–Coulomb failure criterion in terms of the soil friction angle ϕ . The elastic behaviour was described by a Poisson’s ratio of $\nu = 0.3$ and a dimensionless

Fig. 3. Upper bound finite element mesh geometry for vertical anchors. u , horizontal velocity; v , vertical velocity.



stiffness ratio of $E/\gamma B = 500$, where E is the Young’s modulus of the soil, and B is the width of the anchor.

To determine the collapse load of an anchor, displacement-defined analyses were performed where the anchor was considered as being perfectly rigid. That is, a uniform prescribed displacement was applied to those nodes representing the anchor. The total displacement was applied over a number of substeps, and the nodal forces along the anchor were summed to compute the equivalent force.

Results and discussion

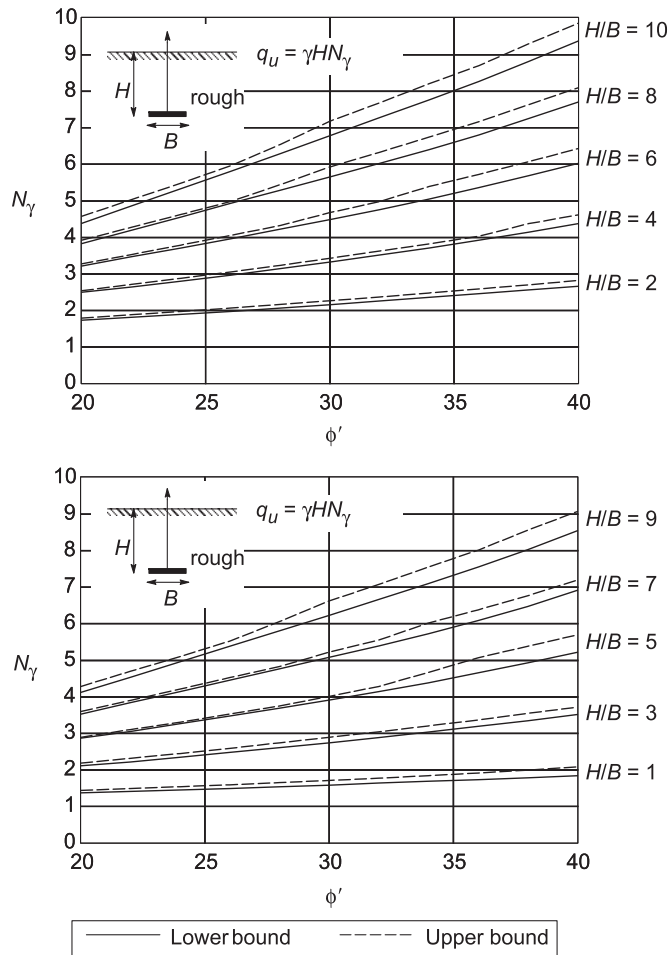
Horizontal anchors

Numerical limit analysis results

The computed lower and upper bound estimates of the anchor break-out factor N_γ are shown graphically in Fig. 4 for embedment ratios of 1–10. It was found that for a given embedment ratio (H/B), the break-out factor increases almost linearly with an increase in the soil friction angle. This linear behaviour is more apparent in the lower bound results than in the upper bound results.

The results shown in Fig. 4 indicate that the limit analysis is very successful, with only very small error bounds observed between the upper and lower bound solutions. This has enabled the true value of the anchor break-out factor to be bracketed to within $\pm 5\%$ in all cases. The error bounds

Fig. 4. Break-out factors N_γ for horizontal anchors in sand.



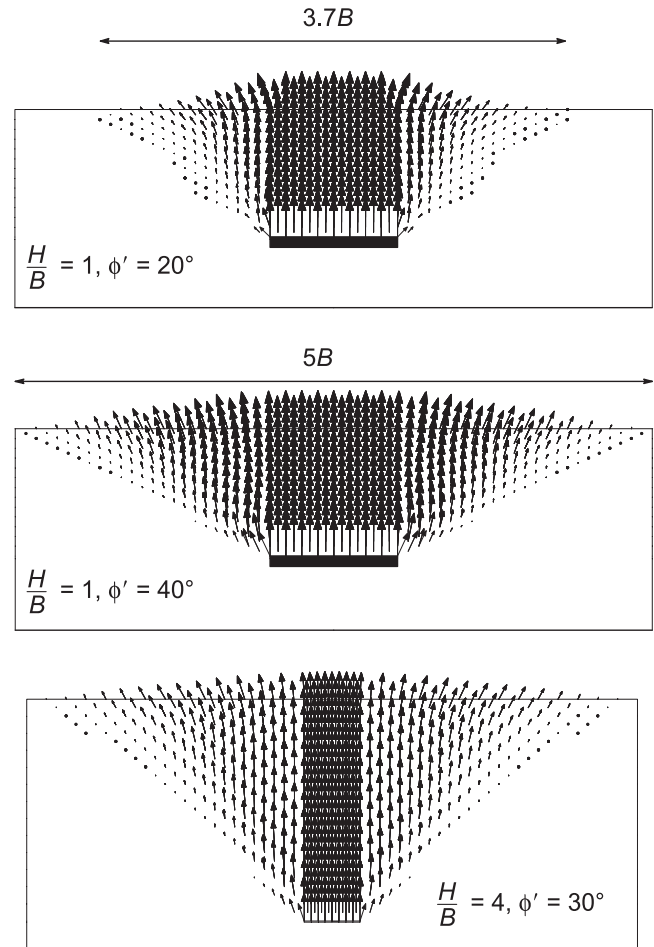
were found to increase with an increase in the friction angle, being less than $\pm 3\%$ for friction angles less than 34° .

To date, a great deal of uncertainty has surrounded the likely modes of failure for horizontal anchors in sand. The observed upper bound velocity diagrams at collapse for anchors at several embedment depths are shown in Fig. 5.

In general, it was found that failure consists of the upward movement of a rigid column of soil immediately above the anchor, accompanied by lateral deformation extending out and upwards from the anchor edge. As the anchor is pulled vertically upwards, the material above the anchor tends to lock up as it attempts to dilate during deformation. As a consequence, to accommodate the rigid soil column movement, the observed plastic zone is forced to extend a large distance laterally outwards into the soil mass. This soil locking phenomenon was noticed by Rowe (1978), who found that soil dilatancy had a significant effect on the observed plastic region at failure, which in turn resulted in substantial variations in the predicted anchor break-out factor.

For relatively shallow anchors ($H/B \leq 4$), the bulk of the soil column above the anchor remains nonplastic, whereas the lateral extent of plastic shearing is extensive. For $\phi' = 20^\circ$, the vertical extent of the nonplastic zone extends to the soil surface. The vertical extent of the nonplastic zone decreases with an increase in the friction angle, however, and no longer reaches the soil surface for friction angles greater

Fig. 5. Observed velocity plots from upper bound analyses.



than 20° . This is also the case for relatively deep anchors ($H/B \geq 5$).

For relatively deep anchors ($10 \geq H/B \geq 5$) where ϕ' is around 20° , the soil column above the anchor no longer “locks up”, and the material changes from an elastic state to a plastic state at ultimate collapse. When $\phi' = 20^\circ$, a localized zone of compression and lateral deformation forms above the anchor, which leads to a reduction in the magnitude of the deformations observed throughout the soil mass and at the surface.

Local compression of the soil adjacent to the anchor also produces a velocity jump along the soil–anchor interface. It appears logical to anticipate that when such a velocity jump at the anchor interface exists, the effect of interface roughness will be most significant.

The extent of lateral shearing was found to increase with an increase in the friction angle at a given embedment depth. For example, referring to Fig. 5, the lateral extent of vertical deformation observed at the surface for $H/B = 1$ increases from approximately $3.7B$ for $\phi' = 20^\circ$ to around $5B$ for $\phi' = 40^\circ$. This is also the case for larger embedment depths, where the lateral extent of vertical deformation observed at the surface was found to increase from $2.5B$ to $4.5B$ and from $3.5B$ to $6.0B$ for $H/B = 2$ and 4 , respectively.

Unlike the case for a horizontal anchor in purely cohesive soil (Merifield et al. 2001), no plastic shearing or flow was

observed below the anchor for all cases. This is in contrast to the observations of Rowe (1978), who observed quite extensive plastic failure above and below the anchor for $\phi = 15^\circ$. This plastic region was found to shrink, and plastic flow below the anchor was nonexistent for $\phi = 45^\circ$.

Displacement finite element results

The displacement finite element (SNAC) estimates of the break-out factor N_γ are shown graphically in Fig. 6. As to be expected, the SNAC results plot above the lower bound estimates and close to the upper bound solutions ($\pm 4\%$).

Vertical anchors

Numerical limit analysis results

The upper and lower bound estimates of the break-out factor N_γ are shown graphically in Fig. 7. The limit analysis procedure was again successful, and the true break-out factor has been estimated to within $\pm 10\%$.

There are two significant features that emerge from the results shown in Fig. 7. First, for a given embedment ratio H/B , the break-out factor was found to increase in a nonlinear manner with an increase in soil friction angle ϕ . This is in contrast with the results for horizontal anchors (see Fig. 4), where the break-out factor was found to increase almost linearly with an increase in the soil friction angle. Second, the increase in break-out factor with an increase in the friction angle is far more significant for vertical anchors compared with horizontal anchors. For example, for $H/B = 5$, the increase in break-out factor between $\phi = 20^\circ$ and 40° is approximately 400% for vertical anchors and only 80% for horizontal anchors. The rate of increase in the break-out factor for vertical anchors tends to be greatest for medium-dense to dense sands where $\phi \geq 30^\circ$.

The complex nature of vertical anchor behaviour, due to the lack of symmetry, has led to a great deal of uncertainty regarding the likely failure patterns at collapse. As a result, it is difficult to predict the capacity of vertical anchors using existing approaches that require assumptions regarding the shape of the failure surface. Such approaches include the limit equilibrium method, the method of characteristics, and any analytical upper bound method.

A distinct advantage of the numerical upper bound formulation used is that the form of the failure mechanism can be obtained automatically without any assumptions being made in advance. The observed modes of failure for several vertical anchor cases are shown in Fig. 8.

In general, it was found that failure could be characterized by the development of an active failure zone immediately behind the anchor, and an extensive passive failure zone in front of and above the anchor. The size of the active failure zone behind the anchor decreases with an increase in the friction angle and is most extensive for shallow anchors ($H/B \leq 2$) in loose sand ($\phi \leq 20^\circ$).

Displacement finite element results

The displacement finite element (SNAC) estimates of the break-out factor N_γ are shown graphically in Fig. 9. The results again compare favourably with the limit analysis estimates and are typically within $\pm 5\%$ of the upper bound.

Fig. 6. Comparison of lower bound limit analysis and solid non-linear analysis code (SNAC) break-out factors for horizontal anchors in cohesionless soil.

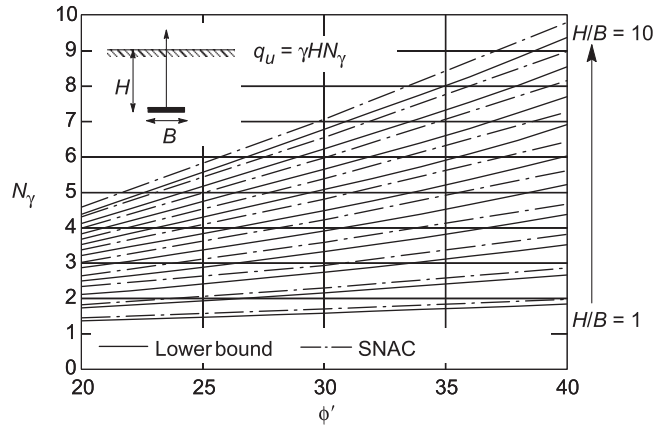
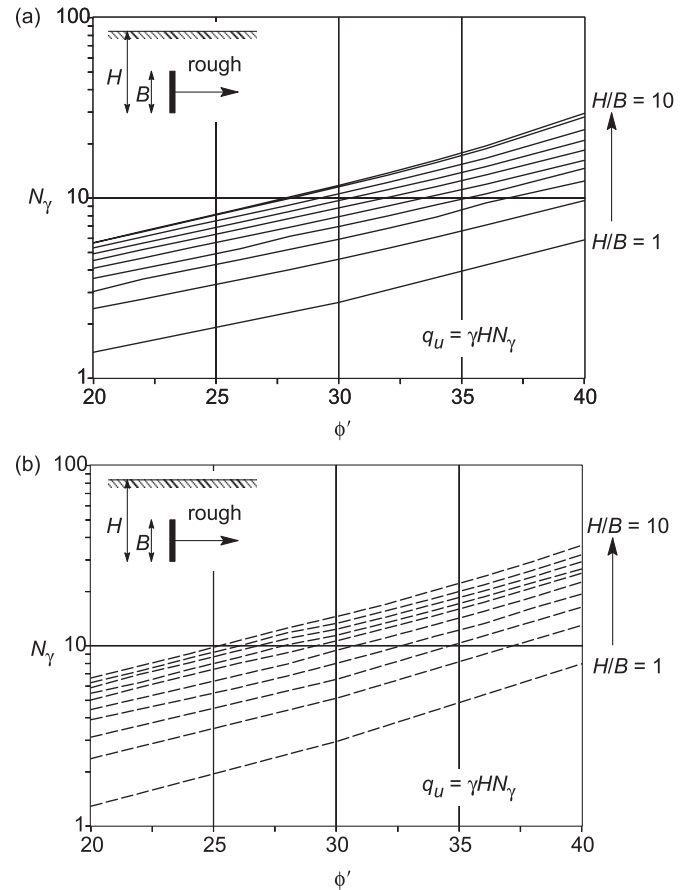


Fig. 7. Break-out factors for vertical anchors in sand: (a) lower bound; (b) upper bound.



The extent of the elastoplastic zone at failure confirms the formation of an active zone of collapse behind the anchor, as predicted by the limit analysis results. This is illustrated in Fig. 10. It was found that by modifying the boundary conditions behind the anchor to either allow or deny active collapse will have an effect on the collapse load for anchors with $H/B \leq 2$ and $\phi \leq 20^\circ$. For these cases, any analysis that ignores the zone of soil directly behind the anchor will

Fig. 8. Velocity plots from upper bound analyses.

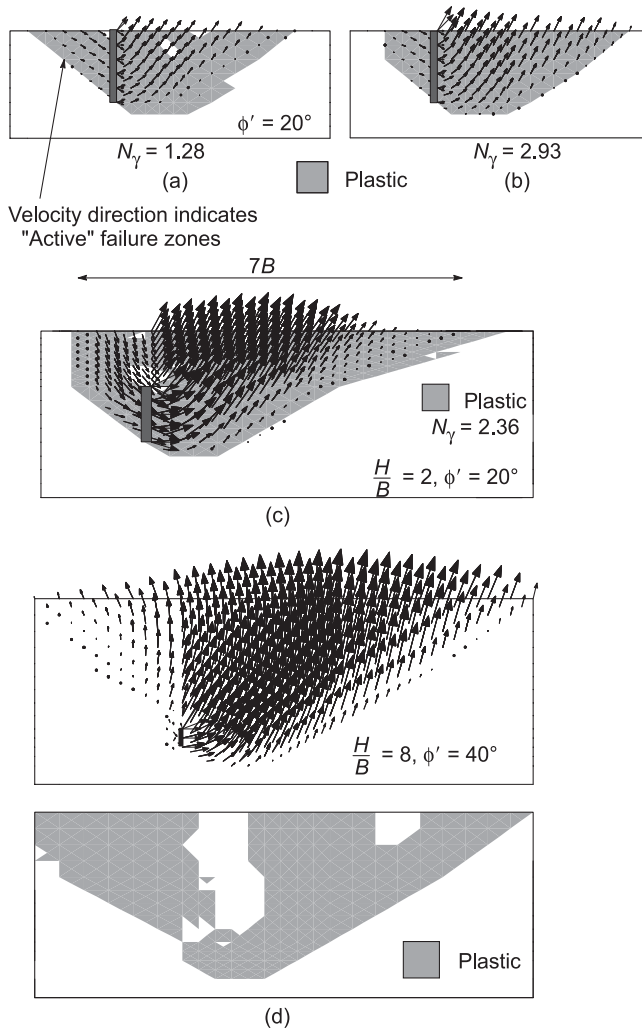


Fig. 9. Break-out factors for vertical anchors in sand.

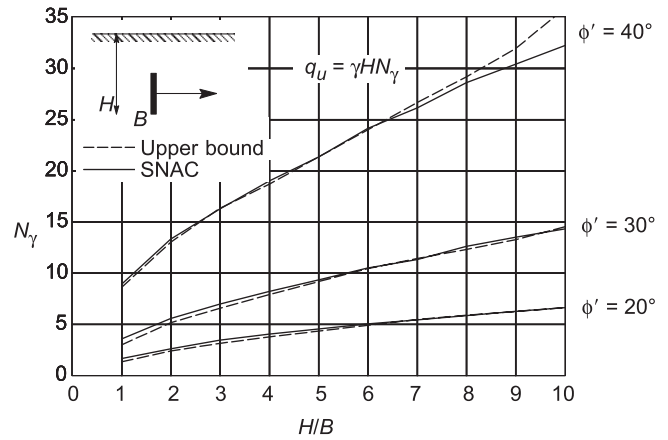
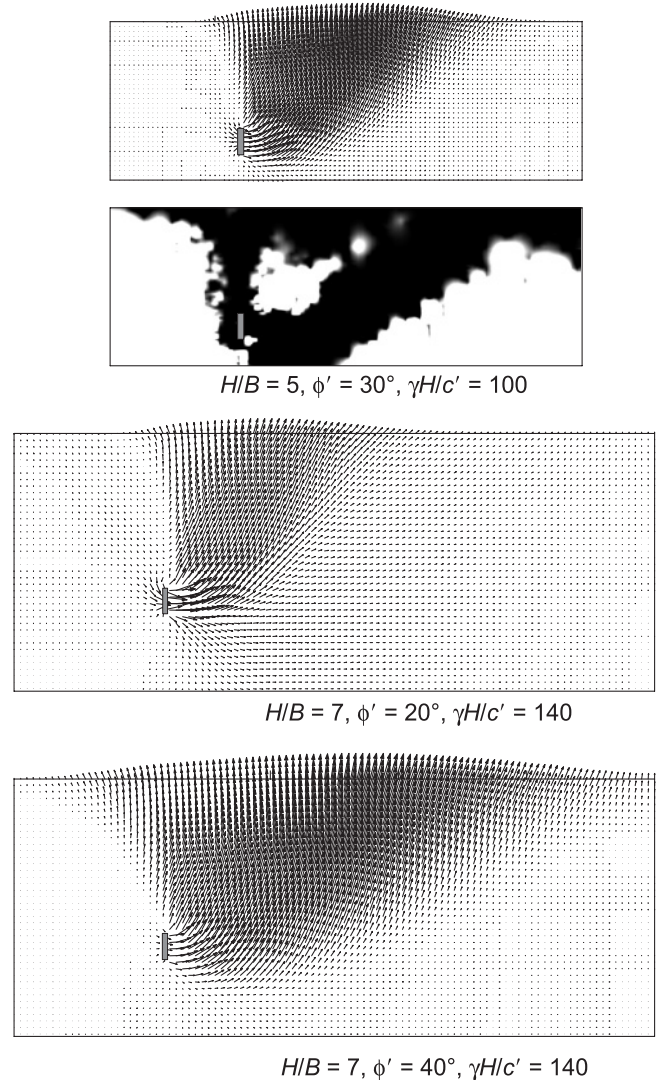


Fig. 10. Failure modes and zones of plastic yielding for rough vertical anchors in cohesionless soils as predicted by SNAC. c' , effective cohesion intercept.



overpredict the collapse load by up to 18%. For $H/B > 2$, any active soil pressure behind the anchor has little influence on the ultimate collapse load.

Effect of anchor roughness

The results discussed in the preceding sections were based on the assumption that the soil–anchor interface could be considered as being perfectly rough. For this case, the angle of friction between the anchor plate and the surrounding soil δ is taken equal to the soil friction angle ϕ . In reality, the true interface friction angle will vary depending on the type of anchor and surrounding material.

To determine the effect interface friction has on the anchor break-out capacity, the specific case of a perfectly smooth interface has been analysed using the limit analysis formulations. For the upper bound analysis the shear strength of the soil–anchor interface is assumed to be zero, whereas for the lower bound analysis the shear stresses at the soil–anchor interface are assumed to be zero. Such analyses will provide an indication as to the likely maximum effect of interface friction.

To quantify the effect of anchor roughness, a correction factor is introduced as

$$[2] \quad F_{\delta} = q_{u,\text{rough}}/q_{u,\text{smooth}} = N_{\gamma,\text{rough}}/N_{\gamma,\text{smooth}}$$

which simply equates the ratio of the pullout capacity for a rough anchor to that of a smooth anchor.

For horizontal anchors, the interface roughness was found to have little or no effect (<4%) on the calculated pullout capacity for all embedment depths and friction angles analysed. For an anchor with $H/B = 3$ and $\phi' = 30^\circ$, for example, changing the roughness from perfectly rough to perfectly smooth reduces the break-out factor N_{γ} by just 1%. The effect of anchor roughness is greatest for relatively deep anchors ($10 \geq H/B \geq 5$) where ϕ' is around 20° .

Earlier studies on vertical anchors in sand have revealed that anchor interface roughness significantly affects the ultimate anchor capacity (Rowe and Davis 1982).

The finite element limit analysis results confirm the significance of interface roughness when estimating the capacity of vertical anchors. The roughness correction factor for $\phi' = 30^\circ$ is shown as a function of embedment ratio in Fig. 11. The limit analysis, displacement finite element (SNAC) results, and a line of best fit have been included in Fig. 11. Changing the interface roughness from perfectly rough to perfectly smooth can lead to a reduction in the anchor capacity of as much as 67%. The effect of anchor roughness was found to decrease with an increase in embedment depth for all friction angles. The greatest rate of change in the roughness correction factor occurs between embedment ratios of 1 and 4. For $\phi' = 30^\circ$, for example, the anchor correction factor F_{δ} increases from approximately 1.15 for $H/B = 10$ to around 1.90 for $H/B = 1$. Two thirds of the total variation in fact occurs between $H/B = 4$ and 1, however.

The reason interface roughness can significantly alter the estimated pullout capacity is because the movement of the soil in front of the anchor is upwards at an angle to the soil-anchor interface. In the case of a rough anchor, significant shear stresses will develop at the anchor interface in response to this upward movement. These shear stresses are resisted by the interface and contribute to the anchor capacity. The shearing resistance provided along a rough anchor interface tends to force the failure zone downwards below the anchor level.

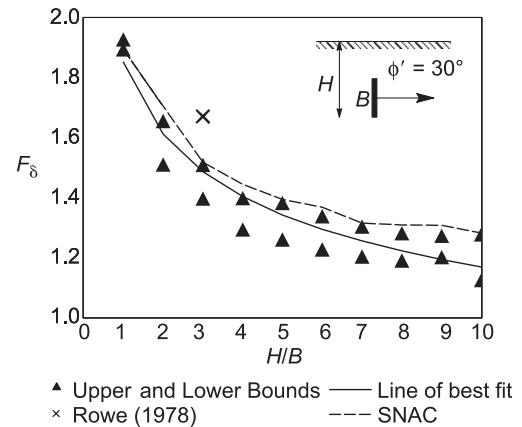
Effect of soil dilation

The effect of soil dilation ψ' on the anchor capacity was investigated using the displacement finite element technique. Two separate cases were investigated, namely half dilation where $\psi' = \phi'/2$ and zero dilation where $\psi' = 0^\circ$.

For a horizontal anchor in a dense soil that deforms plastically with zero volume change ($\phi' = 40^\circ$, $\psi' = 0^\circ$), soil dilation was found to reduce the anchor capacity by up to 40%. The occurrence of localization, however, makes it difficult to predict the anchor capacity reliably for this case with the finite element method. The effect was found to be less significant ($\leq 15\%$) for friction angles less than 40° and for cases where $\psi' = \phi'/2$.

Soil dilatancy was found to have a significant effect on the response of a vertical anchor. In the extreme case of an anchor in a dense nondilatant soil ($\phi' = 40^\circ$, $\psi' = 0^\circ$), the ultimate capacity was estimated to be approximately half of that

Fig. 11. Effect of anchor roughness.



for the same anchor in an associated soil ($\phi' = 40^\circ$, $\psi' = 40^\circ$). Due to the effect of localization, however, it is suggested that these results be treated with caution. Further investigations are required to accurately determine the true effect of dilation. In particular, further work is also needed to better understand the reasons for the numerical instabilities observed in nondilatant soil.

Comparison with existing numerical and laboratory results

Horizontal anchors

There have been numerous approximate or semi-empirical methods proposed for estimating the ultimate pullout capacity of anchors in sand. A number of these previous theoretical results are reproduced in the following sections so that they can be compared with the finite element limit analysis and SNAC results.

The limit equilibrium solution of Murray and Geddes (1987) shown in Fig. 12 provides a poor estimate of the break-out factor for all friction angles and is unconservative by as much as 30%. Clearly the assumptions implicit in this solution, regarding the shape of the failure surface and the average orientation of the frictional resistance, lead to an overestimate of the anchor capacity.

The lower bound solutions of Smith (1998), shown in Fig. 13a, compare well with the finite element lower bound solutions. In fact, the results of Smith are hard to distinguish from those of the current study, and lie within $\pm 1\%$ of them. In contrast, the lower bound break-out factors presented by Basudhar and Singh (1994) do not compare well with the finite element lower bounds (Fig. 13b). Indeed, their predictions for rough anchors are at least 30% above the finite element lower bounds and are also greater than the finite element upper bound results. This inconsistency suggests that the results of Basudhar and Singh are not true lower bounds. Their predictions for smooth anchors, although better than those for the rough case, are still unacceptably high. The likely explanation for these overestimates is that the lower bounds of Basudhar and Singh are based on stress fields that are not properly extended throughout the soil domain. Extending the stress field can significantly reduce the lower

Fig. 12. Comparison of theoretical break-out factors for rough horizontal anchors in cohesionless soil based on the limit equilibrium solution of Murray and Geddes (1987).

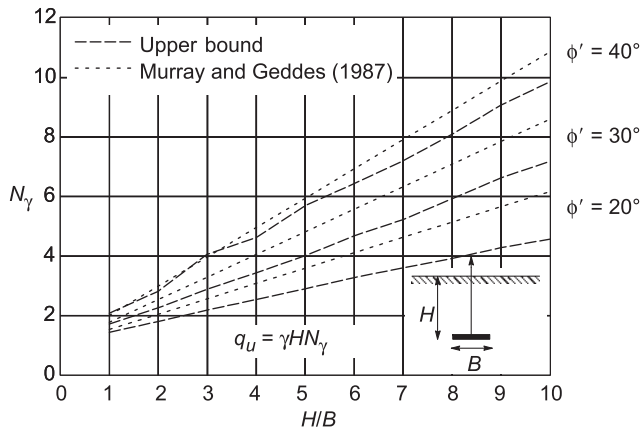
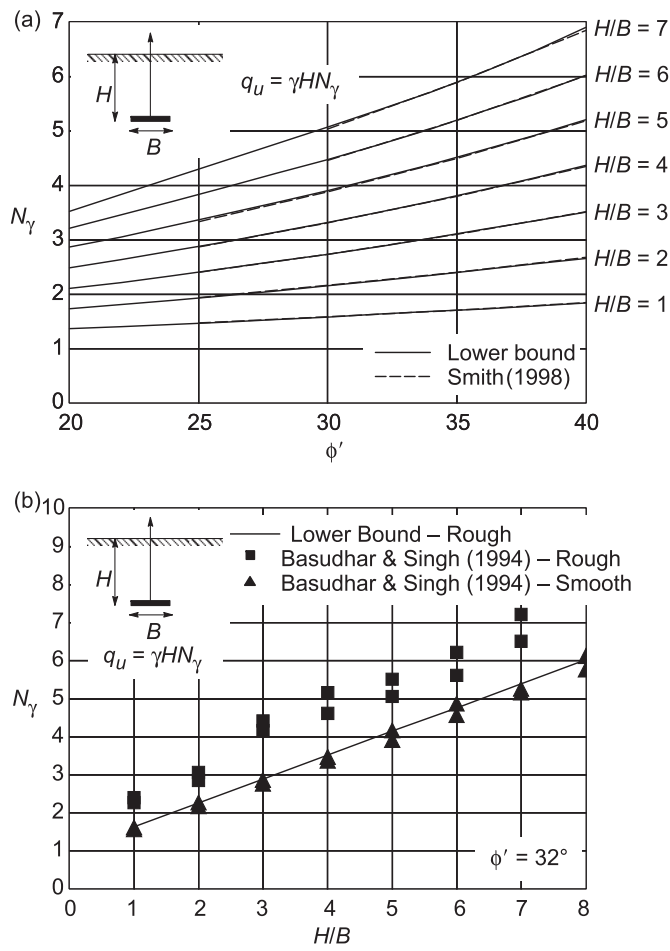


Fig. 13. Comparison of theoretical break-out factors for rough horizontal anchors in cohesionless soil: (a) lower bound solutions of Smith (1998) compared with finite element lower bounds; (b) lower bound solutions of Basudhar and Singh (1994) compared with finite element lower bounds.



bound, especially if the mesh is too coarse or too small in dimension.

The semi-empirical theory presented by Meyerhof and Adams (1968) is compared against the finite element lower

bounds in Fig. 14a. Meyerhof and Adams suggest that, for large H/B ratios, the compressibility and deformation of the soil mass above the anchor will eventually prevent the failure surface from reaching the ground surface. To account for this in their theory, a limit is placed on the vertical extent of the failure surface leading to the definition of a critical embedment ratio. The magnitude of the critical embedment ratio can only be determined from the failure surface observed in laboratory testing, and once it is exceeded, the anchor break-out factor reaches a limiting value. For embedment ratios below this critical ratio, the predictions of Meyerhof and Adams compare favourably with the finite element lower bounds, the difference being less than 5%. Because no critical embedment ratio was observed in the current study, the results of Meyerhof and Adams are very conservative once this ratio is exceeded.

Kanakapura et al. (1994) produced approximate estimates of the break-out factor using theory based on the method of characteristics. Their results, shown in Fig. 14b, are conservative for all friction angles and embedment depths. The conservative nature of these predictions was acknowledged by Kanakapura et al. in their paper.

The displacement finite element method results by Koutsabeloulis and Griffiths (1989) are shown in Fig. 15, which shows that the break-out factor proposed by Koutsabeloulis and Griffiths overpredicts the effect of dilation on the anchor capacity. This is particularly so for soils with a friction angle of $\phi' = 20^\circ$, where the break-out factor of Koutsabeloulis and Griffiths is up to 70% above the finite element estimate obtained in the current study.

Results obtained from the comprehensive displacement finite element study of Rowe (1978) are shown in Fig. 16. Allowing for the possible combination of effects of dilation, anchor roughness, and the definition of failure adopted by Rowe, the discrepancy among the results in Fig. 16 does not seem unreasonable.

Comparisons between experiment and theory are complicated by the wide range of friction angles for which results have been obtained and the uncertainty regarding the soil dilatancy and anchor roughness. Furthermore, as model tests are typically conducted at low stress levels, where Mohr's failure envelope may be curved, difficulty arises in selecting the correct soil friction angle for comparison. Despite these difficulties, it is possible to compare the theoretical anchor break-out factors with those from several published experimental studies.

Chamber testing programs have been performed by Murray and Geddes (1987, 1989), who performed pullout tests on horizontal strip, circular, and rectangular anchors in dense and medium-dense sand with $\phi' = 43.6^\circ$ and 36.0° , respectively. Their anchors were typically 50.8 mm in width (diameter) and were tested at aspect ratios (L/B) of 1, 2, 5, and 10. Murray and Geddes concluded that the uplift capacity of rectangular anchors in very dense sand increases with an increase in the embedment ratio and with a decrease in the aspect ratio L/B . In addition, their experimental results suggest that an anchor with an aspect ratio of $L/B = 10$ behaves like a strip and does not differ much from an anchor with $L/B = 5$.

A comparison between the finite element lower bound and the model anchor tests of Murray and Geddes (1989) is

Fig. 14. Comparison of theoretical break-out factors for rough horizontal anchors in cohesionless soil: (a) semi-empirical theory presented by Meyerhof and Adams (1968) compared with finite element lower bounds; (b) estimates of Kanakapura et al. (1994) compared with finite element lower bounds.

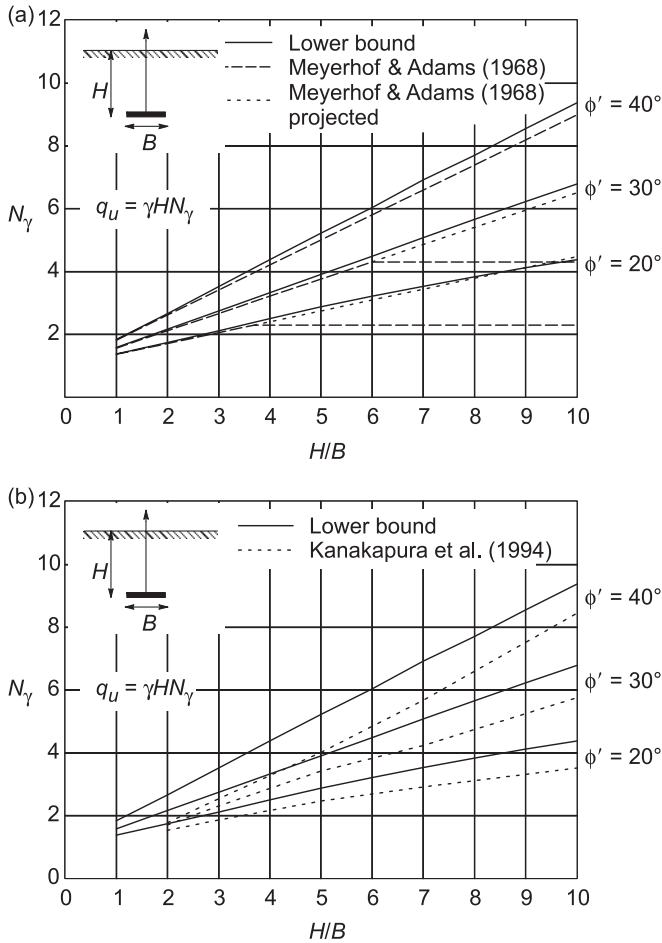
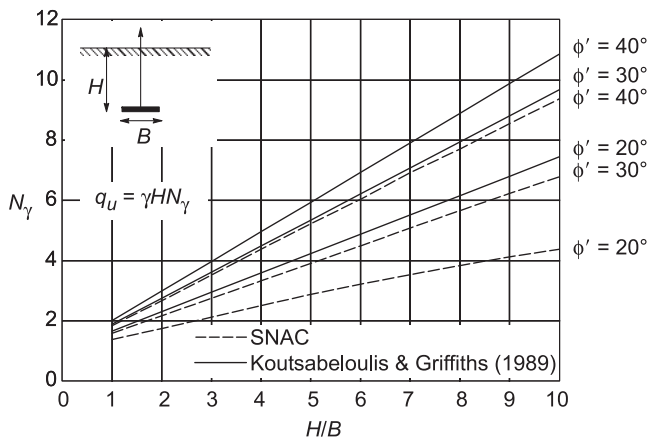
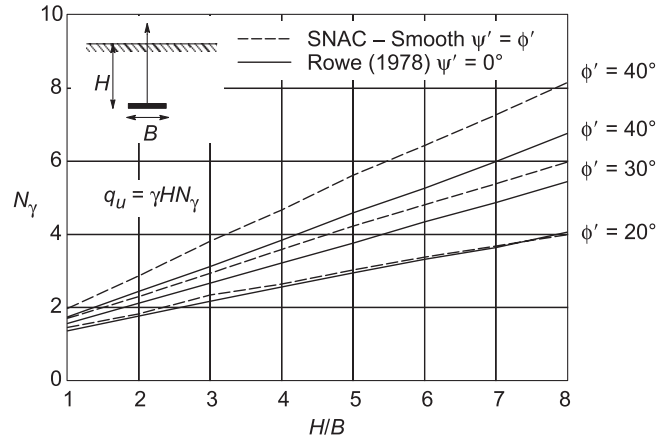


Fig. 15. Comparison of theoretical break-out factors for rough horizontal anchors in cohesionless soil as predicted by SNAC and Koutsabeloulis and Griffiths (1989).



shown in Fig. 17a. The lower bounds compare reasonably well with the greatest variation occurring at shallow embedment depths of $H/B \leq 2$. Note that the results of

Fig. 16. Comparison of finite element break-out factors for smooth horizontal anchors in cohesionless soil as predicted by SNAC and Rowe (1978).



Murray and Geddes are for the case where the anchor-soil friction angle was measured to be around 11° , which is close to smooth.

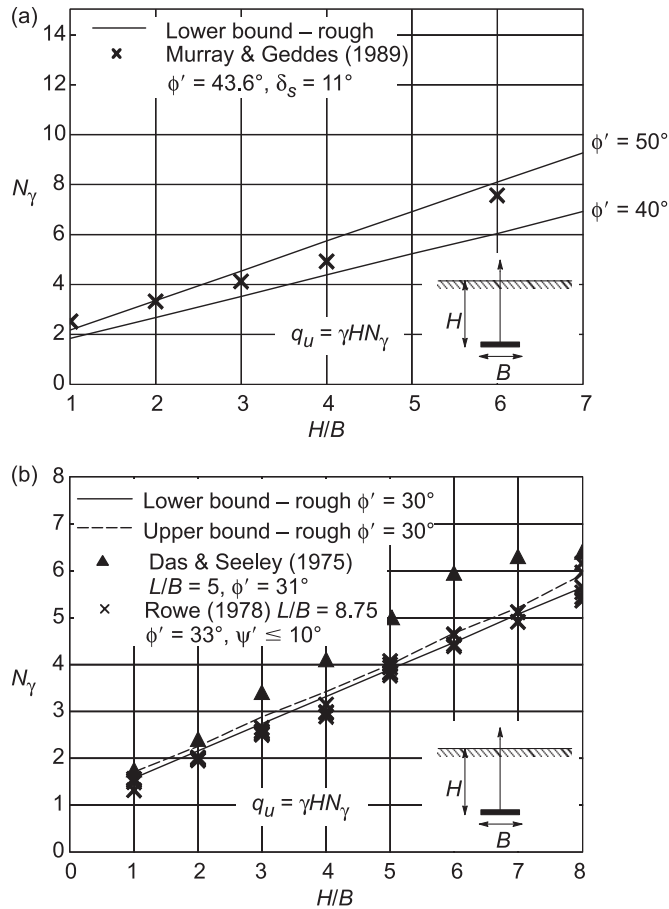
Das and Seeley (1975) performed pullout tests for rectangular anchors in dry sand with $\phi = 31^\circ$. A similar investigation was conducted by Rowe (1978) in dry sand with friction angles of $\phi = 31^\circ-33^\circ$ and dilation angles of $\psi = 4^\circ-10^\circ$. Polished steel plates were used for the anchors, and the interface roughness was measured as $\delta = 16.7^\circ$. Most tests were performed on anchors with an aspect ratio L/B of 8.75. The results of Das and Seeley for $L/B = 5$ are summarized in Fig. 17b together with the measurements of Rowe for $\phi = 33^\circ$.

The anchor capacities of Das and Seeley (1975) (for $\phi = 31^\circ$) are higher than the numerical bound predictions (for $\phi = 30^\circ$). This may be partly due to the shape of the anchor. The laboratory findings of Rowe (1978) suggest the capacity of anchors with an aspect ratio of $L/B = 5$ exceeds those for strip anchors where $L/B = \infty$. Allowing for the effects of dilatancy and shape, the discrepancy between the experimental results of Das and Seeley and the numerical bound predictions for a strip anchor do not seem unreasonable.

The numerical bound predictions compare reasonably well with the experimental results of Rowe (1978) but are a little too high. This may, in part, be attributed to the lower dilation angle of 10° observed by Rowe.

Dickin (1988) performed tests on anchor plates with aspect ratios of $L/B = 1, 2, 5$, and 8 at embedment ratios H/B of up to 8 in both loose and dense sand. A number of conventional gravity tests were also performed and compared with the centrifuge results. The results of laboratory tests performed by Dickin are shown in Fig. 18. With the exception of the centrifuge tests in dense sand ($\phi = 48^\circ-51^\circ$), which compare reasonably well, the results of Dickin compare rather poorly with the numerical lower bound solutions for the anchor break-out factor. Dickin observed a significant disparity in the capacity of model anchors tested under simple gravity and centrifuge loading, with the former yielding much higher capacities. This is highlighted in Fig. 18. Consequently, Dickin concluded that the extrapolation of results from small-scale gravity testing to field scale will produce overoptimistic predictions. A similar conclusion was

Fig. 17. Comparison of theoretical and experimental break-out factors for horizontal anchors in cohesionless soil: (a) finite element lower bounds compared with model anchor tests of Murray and Geddes (1989); (b) numerical bound predictions compared with the results of Das and Seeley (1975) and Rowe (1978). δ_s , soil-anchor interface roughness.



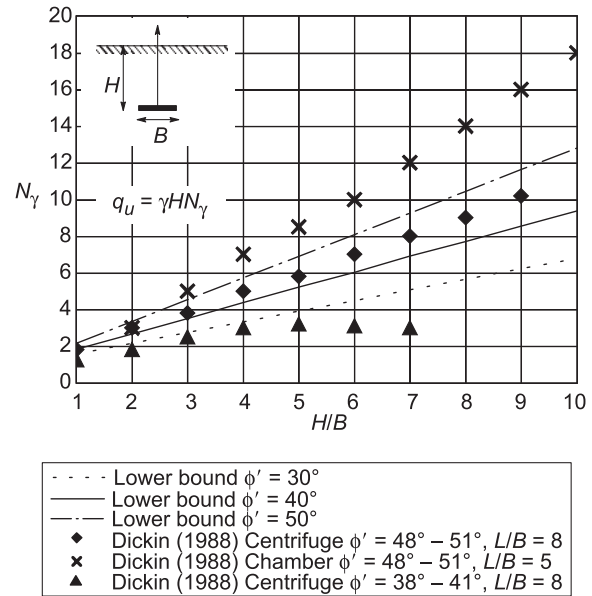
made by Ovesen (1981) in relation to circular anchors in dense sand (for loose sand the discrepancy between conventional gravity and centrifuge pullout capacities was not significant). Unfortunately, very few centrifuge testing programs have been undertaken to determine anchor capacity in cohesionless soils, and the conclusions of Dickin and Ovesen effectively remain unconfirmed. There is clearly scope for further experimental work to identify whether scale effects alone are the cause of these discrepancies.

Vertical anchors

The results obtained from a selection of existing theoretical and laboratory studies are reproduced in this section. These are compared with the new predictions and a brief discussion is presented.

Break-out factors estimated by Basudhar and Singh (1994), who used a lower bound procedure based on finite elements and nonlinear programming, are compared with the finite element lower bound estimates obtained from the current study in Fig. 19. For the case of a rough vertical anchor, the predictions of Basudhar and Singh are close to the current lower bound predictions, even though the former are not based on a properly extended stress field. For a smooth an-

Fig. 18. Comparison of theoretical and experimental break-out factors for rough horizontal anchors in cohesionless soil based on the results of laboratory tests performed by Dickin (1988).



chor, the results of Basudhar and Singh underestimate the anchor break-out factor by up to 25%. This discrepancy is greatest for higher friction angles and could be due to the very simple finite element mesh they adopted.

The results obtained by both the “surcharge” and “equivalent free surface” methods of analysis proposed by Neely et al. (1973) are shown in Fig. 20a. For relatively shallow anchors ($H/B \leq 5$), both of these methods give predictions that are generally conservative. The equivalent free surface is more accurate than the surcharge method, however, which tends to be overly conservative. Over the range of embedment ratios shown, the break-out factors given by the equivalent free surface method are generally within 15% of the numerical lower bound.

The semi-empirical theory presented by Meyerhof (1973) is compared with the finite element upper bounds in Fig. 20b. Unfortunately, no information was provided by Meyerhof regarding the interface roughness ratio (δ/ϕ) and how it was considered when selecting the relevant earth pressure coefficients from the charts produced by Caquot and Kerisel (1949). Consequently it is difficult to determine whether the results of Meyerhof are for rough or smooth anchors. Assuming they are applicable to the latter, Meyerhof’s predictions compare favourably with the finite element upper bounds for $H/B \leq 6$ and $\phi = 20^\circ - 30^\circ$. Above this embedment ratio with $\phi = 20^\circ - 30^\circ$, Meyerhof’s theory tends to overpredict the break-out factor by 5%–30%. For a denser soil with $\phi = 40^\circ$, the solution of Meyerhof is conservative over the range of embedment ratios considered, plotting 5%–25% below the upper bound solution.

The displacement finite element results obtained by Rowe (1978) for smooth anchors in a nonassociated cohesionless material ($\psi' = 0^\circ$) are reproduced in Fig. 21 along with the SNAC results from the current study for an associated material ($\psi' = \phi$). As expected, the break-out factors from the current study are greater than those obtained by Rowe be-

Fig. 19. Comparison of theoretical break-out factors for rough (a) and smooth (b) vertical anchors in cohesionless soil as estimated by Basudhar and Singh (1994).

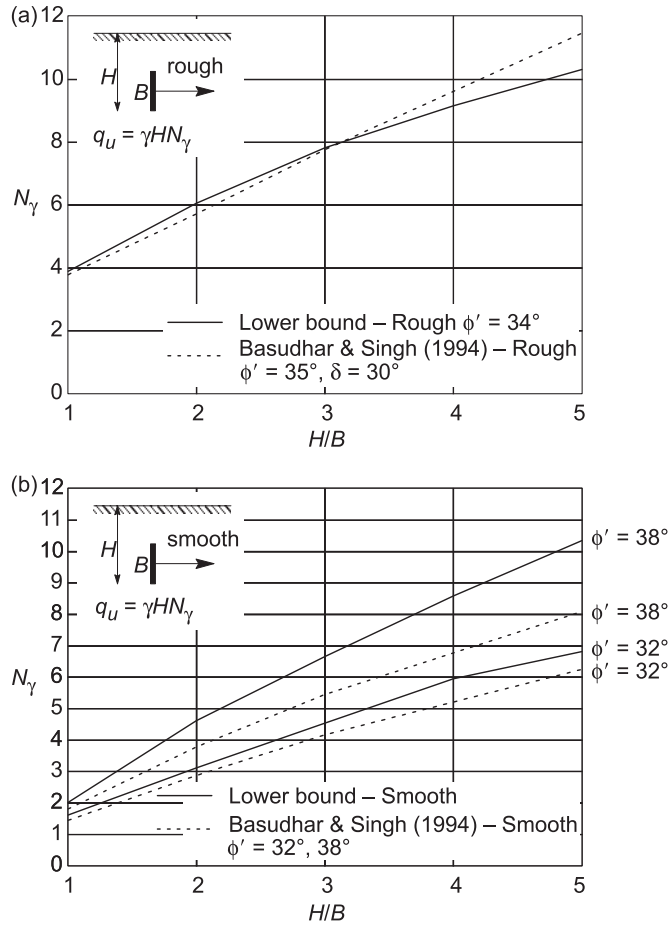
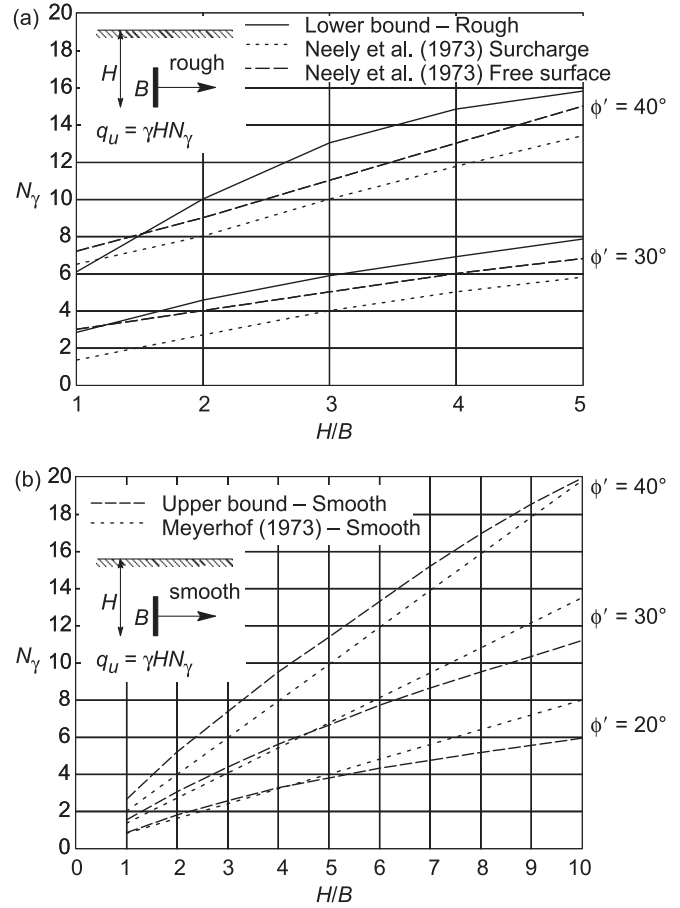


Fig. 20. Comparison of theoretical break-out factors for vertical anchors in cohesionless soil: (a) lower bounds compared with “surcharge” and “equivalent free surface” methods of analysis proposed by Neely et al. (1973); (b) upper bounds compared with semi-empirical theory presented by Meyerhof (1973).

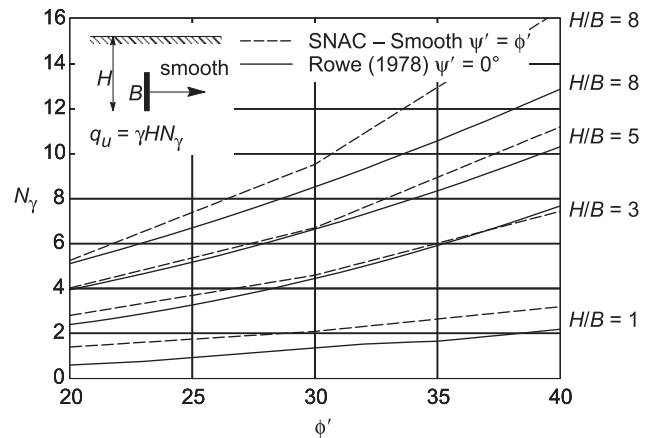


cause of the effect of soil dilatancy. Although it is difficult to compare the results of Rowe with those from the current study owing to the different definitions of failure that are adopted (particularly for break-out factors at larger embedment ratios), the discrepancy between the two sets of predictions is not unreasonable.

Laboratory testing of vertical anchors in cohesionless soil has attracted limited attention. This is highlighted by the lack of studies summarized in Table 2. The results obtained from several of these studies are now compared and discussed.

In addition to performing laboratory tests on horizontal anchors, Dickin and Leung (1983, 1985) also conducted small-scale conventional chamber and centrifuge tests on polished vertical anchors in sand. The results of these laboratory tests are reproduced in Fig. 22a. Dickin and Leung concluded that considerable scale errors are incurred by direct extrapolation of small-scale chamber tests to field scale, as evidenced by the data shown in Fig. 22a. They also argue that results obtained from centrifuge tests provide a more reliable basis for full-scale anchor design. This point is debatable, however, as centrifuge testing of granular materials is also prone to significant scale effects, especially at high gravity. A greater number of centrifuge tests on anchors are

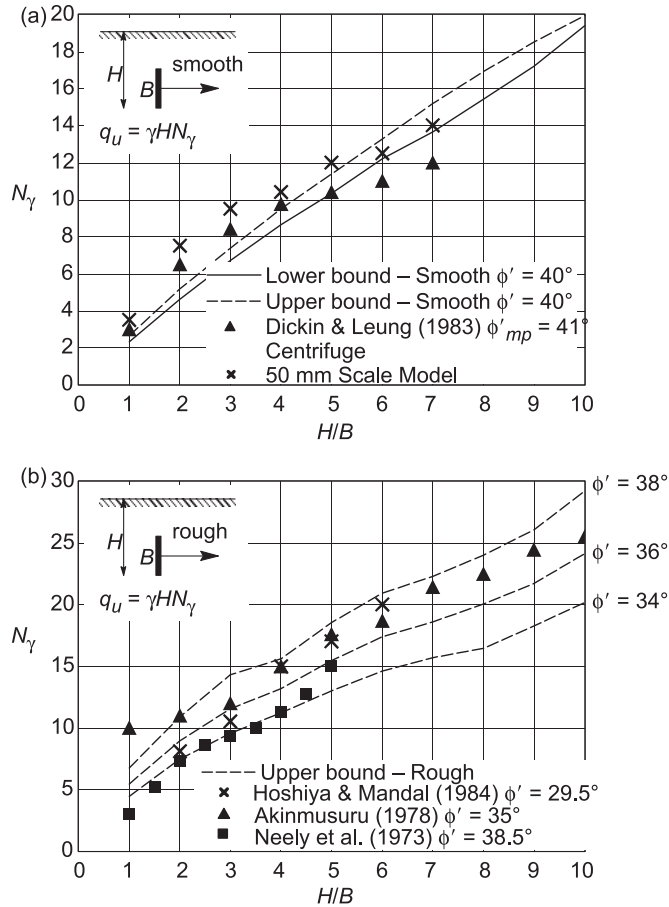
Fig. 21. Comparison of theoretical break-out factors obtained by Rowe (1978) with the SNAC results from the current study for smooth vertical anchors in cohesionless soil.



needed before reaching any conclusions regarding the accuracy of conventional small-scale gravity tests.

Unfortunately, Dickin and Leung (1983, 1985) do not give any data for the friction and dilatancy angle of their sand.

Fig. 22. Comparison of theoretical and experimental break-out factors for vertical anchors in cohesionless soil: (a) upper and lower bound predictions from this study compared with the results of Dickin and Leung (1983); (b) results from several conventional sand chamber tests compared with finite element bound predictions from this study.



They do, however, state that they used a mobilized friction angle of $\phi = 41^\circ$ in conjunction with limit state analyses when making comparisons between their work and existing theories. Therefore, it seems reasonable to compare the laboratory results of Dickin and Leung with the numerical solutions obtained in the current study for a friction angle of $\phi = 40^\circ$. The upper and lower bound predictions in Fig. 22a underestimate the centrifuge results of Dickin and Leung by around 15% for shallow anchors ($H/B \leq 3$) but overestimate the break-out factor above $H/B \geq 6$. For embedment ratios between these limits, the finite element bounds predict the break-out factor accurately.

The capacity of vertical anchors was reported by Akinmusuru (1978). Square, circular, and rectangular anchors ($L/B = 1$ and 10) were tested at embedment ratios ranging from 1 to 10. In a novel attempt to better observe the failure mechanism for anchors at $L/B = 10$, the soil was simulated by steel pins (76 mm length) placed to give a friction angle of 24° . The movement of the pins during each test was photographed with the aid of long-exposure film. Hoshiya and Mandal (1984) also investigated the capacity of square and rectangular anchors but focused on their behav-

our in loose sand. A small sand box was used for the anchor pull-out testing.

Neely et al. (1973) reported results for small-scale testing on vertical square and rectangular anchors in sand with a friction angle of 38.5° . Anchors at aspect ratios of 1 (square), 2, and 5 were used and embedded up to $H/B = 5$. Very large displacements were observed for square anchors when the embedment ratio was greater than 2. In fact, it appears that the load-displacement curves were still increasing when the test was terminated, and an alternative criterion was used to define the ultimate load.

The results from several of these conventional sand chamber tests along with some finite element bound predictions are shown in Fig. 22b. The comparison is again complicated by the uncertainty regarding soil properties and the anchor roughness in these laboratory studies. Although there is some scatter evident in Fig. 22b, the overall agreement is encouraging.

Mention should be made of the results of Hoshiya and Mandal (1984), who conducted pullout tests on small-scale anchors in a small sand chamber (300 mm \times 400 mm). Based on the observations in the current study, this chamber is of insufficient size to contain the collapse mechanism, and thus the results will be affected by the boundary conditions. This may, in part, explain the larger than expected break-out factors shown in Fig. 22b, particularly at embedment ratios of $H/B = 5$ and 6.

Conclusions

Rigorous lower and upper bound solutions have been presented for the ultimate capacity of horizontal and vertical strip anchors in sand. These have been compared with anchor break-out factor estimates obtained using an advanced displacement finite element formulation. Consideration has been given to the effect of soil friction angle, soil dilation, anchor embedment depth, and anchor roughness. The results obtained have been presented in terms of familiar break-out factors in graphical form to facilitate their use in solving practical design problems.

The following main conclusions can be drawn from the results presented in this paper:

- (1) The upper bound, lower bound, and displacement finite element estimates for the anchor break-out factors compare favourably over the range of embedment depths (H/B) and soil friction angles (ϕ) studied.
- (2) In general, it was found that the failure mode for horizontal anchors consists of the upward movement of a rigid column of soil immediately above the anchor, accompanied by lateral deformation extending out and upwards from the anchor edge. As the anchor is pulled vertically upwards, the material above the anchor tends to lock up as it attempts to dilate during deformation.
- (3) The failure mode for vertical anchors indicates active zones of failure can form behind the anchor when $H/B \leq 2$ and $\phi \leq 20^\circ$. For these cases, any analysis that ignores the zone of soil directly behind the anchor will over-predict the collapse load by up to 18%. For $H/B > 2$, any active soil pressure behind the anchor has little influence on the ultimate collapse load.

- (4) The effect of anchor interface roughness was found to have little or no effect on the calculated pullout capacity for horizontal anchors at all embedment depths and friction angles analysed. In contrast, anchor roughness was found to have a significant effect on the capacity of vertical anchors. Indeed, changing the interface roughness from perfectly rough to perfectly smooth can lead to a reduction in the anchor capacity by as much as 67%. The effect of anchor roughness was found to decrease with an increase in the embedment ratio and is most significant for dense soils with high friction angles ($\phi \geq 40^\circ$).
- (5) Soil dilation was found to have a significant effect on the anchor capacity. This is particularly the case for vertical anchors. In the extreme case of a vertical anchor in a dense non-dilatant soil, the ultimate capacity was estimated to be approximately half of that for the same anchor in an associated soil. Because of the effect of localization, however, it is suggested that these results be treated with caution.
- (6) The results obtained from a selection of existing theoretical and laboratory studies have been compared with the new theoretical predictions obtained in the current study. The comparisons between the new and existing theoretical results show encouraging agreement. The comparisons with the laboratory results were complicated by the uncertainty of the soil properties and the anchor roughness. Despite these effects, the discrepancy between the theoretical predictions and the laboratory results does not seem unreasonable and suggests that, for the cases considered, the theoretical results presented by this study will provide a reasonable prediction of anchor capacity for design purposes.

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