Residual Strains in Calcareous Sand due to Irregular Cyclic Loading

By W. S. Kaggwa,[1] J. R. Booker,[2] and J. P. Carter[3]

Abstract: This paper describes a procedure for predicting the residual strains developed in calcareous sand under irregular cyclic triaxial loading conditions. The rates of accumulation of volumetric and shear strains are treated separately, with the volumetric strain response being used as the basis for characterizing the effects of cyclic loading. The residual volumetric strain developed as a result of repeated loading is related to the mean and the amplitude of the cyclic deviator stress. Thus, by using empirical correlations between the applied deviator stresses, the number of cycles, and the measured residual strains, the effect of cyclic loading under one set of cyclic loading conditions can be expressed in terms of an equivalent number of cycles under a different set of cyclic loading conditions. By using this equivalent number of cycles for the cyclic loading stresses, the residual strains developed as a result of further loading are then determined, so that the residual strains developed at any stage during irregular cyclic loading can be evaluated.

Introduction

Earthquakes and wave storms induce in the underlying soil layers cyclic stresses of different magnitudes over a range of stress cycles, the pattern of which is difficult to reproduce under laboratory conditions. Thus, when analyzing field problems, data obtained from laboratory tests in which the amplitude of the cyclic deviator stress is constant have to be synthesized into irregular applied stress-time histories similar to those observed in the field. This is commonly achieved by using the concept of equivalent uniform stress cycles.

It is shown in this paper that because of the nonlinear residual strain responses under fully drained conditions, the pattern of loading affects the response of the soil. Here, the analyses are restricted to drained loading conditions only, so that effective stresses are used throughout. The problem of analyzing undrained loading behavior can be accomplished by linking the residual volumetric strain response to the generation of excess pore pressures, which is currently under investigation by the writers. Thus, before any procedure for converting an irregular stress-time history into an equivalent uniform stress-time history can be used with confidence, it is necessary to take into consideration the likelihood of partially drained conditions under the field conditions.

Equivalent Uniform Cycle Concept

The practice of equating the effects of uniform stress cycles with irregular stress-time histories for the soil has been used widely in laboratory studies

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of the liquefaction phenomenon. The irregular time history is replaced by a number of uniform cycles at a proportion of the maximum stress level observed in the field, such that the damage, e.g., the generated excess pore pressure, is the same as that produced by the irregular cyclic stresses observed in the field. The most widely used procedure for determining the equivalent number of uniform stress cycles, n_e, is that based on the hypothesis by Miner (1945) for metals, which states that

$$\frac{n_e}{N_e} = \sum_{i=1}^{m} \frac{n_i}{N_i} \quad (1)$$

where n_i = number of cycles at stress level i; N_i = number of cycles at stress level i required to cause a specified failure condition, e.g., liquefaction; n_e = number of equivalent cycles at the stress level of interest that produce the same damage as that produced by an irregular stress-time history; N_e = number of cycles at the stress level of interest required to produce the same specified failure condition; and m = number of stress levels in the irregular stress-time history.

Thus, in the case of liquefaction, Eq. 1 can be written as

$$\left(\frac{n_{eq}}{N_{le}}\right) = \left(\frac{n_1}{N_{l1}}\right) + \left(\frac{n_2}{N_{l2}}\right) + \left(\frac{n_3}{N_{l3}}\right), \ldots, \left(\frac{n_m}{N_{lm}}\right) \quad (2)$$

where N_{l1} = the number of cycles required to cause liquefaction at the corresponding stress level i; and N_{le} = the number of cycles required to cause liquefaction at the stress level of interest.

This hypothesis has been tested in studies by Lee and Chan (1972) and Seed et al. (1975). However, the use of the ratio n_i/N_{li} implies that the damaged caused by n_i cycles is irrespective of prior cyclic loading, i.e., that there is a linear relationship between the number of cycles and the excess pore pressure generated by cyclic loading in undrained tests. Thus, of the many models available for expressing the generation of excess pore pressure during cyclic loading, only the β-parameter model by Bjerrum (1973) can be used with this hypothesis.

The results of drained cyclic triaxial tests on cohesionless soils, e.g., Martin et al. (1975), Marr et al. (1982), and Kaggwa (1988), show that the accumulation of residual strains with the number of stress cycles is nonlinear. Clearly, attention must be focused on the stress history of the soil. A more appropriate expression for the contribution of a parcel of cyclic loading i would be determined by using an equation that is similar to the variation of residual strain with the number of stress cycles. Thus, where the relationship between the residual volumetric strain ϵ and the number of stress cycles can be expressed by a power function, e.g., Marr et al. (1982) and Kaggwa (1988), then the following equation would be used:

$$\epsilon = \left(\frac{n_i}{N_i}\right)^{xi} \quad (3)$$

where x_i = exponent that depends on the loading condition for level i and prior cyclic loading; $n_i = n_{ei} + dn_i$; N_i = number of cycles required to develop failure or reference strain; n_{ei} = number of cycles at stress level i that produce the same strain as all the preceding irregular cycles; and dn_i = number of cycles at stress level i.

A procedure for determining n_{ei} at any stage during irregular cyclic loading is described in this paper. First, empirical relationships that express the fully drained response of calcareous sand under uniform cyclic stress conditions are presented.

The analysis of the accumulation of residual strains in cohesionless soils due to cyclic loading has been reported in many previous works, and has been expressed by empirical relationships, analytical equations, and numerical constitutive models. Examples of laboratory test results include Timmerman and Wu (1969), Martin et al. (1975), Hedberg (1977), Marr et al. (1982), and Bouckovalas et al. (1984). Empirical models such as Bouckovalas et al. (1984) and analytical methods such as Chang and Whitman (1988) express the relationships between residual volumetric strains and shear strains produced by cyclic loading, and the cyclic stresses that produce them. The numerical constitutive models, e.g., the critical state model by Carter et al. (1982) and the viscoplastic model by Mroz and Norris (1982), involve the determination of a number of parameters; it is uncertain whether the accuracy by which the parameters are determined will ever match the sophistication involved in these models.

Residual Strain Response of Calcareous Sand under Uniform Cyclic Loading

The calcareous sand used in the present study, as well as in an earlier study by Kaggwa and Booker (1990), was recovered from the seabed at the North Rankin A platform off the northwest coast of Australia. The grains of the sediments varied widely with a wide range of bioclastic particles present, although foraminifera were predominant. Larger particles composed of shells were screened out and only the finer fraction used. A summary of the properties of the calcareous sand is given in Table 1.

The cyclic drained tests were carried out in a stiff loading frame using a standard triaxial cell capable of accommodating 62-mm diameter samples. Top and bottom drainage connections were used. The cell pressure and the back pressure were controlled by two GDS pressure controllers. The axial

TABLE 1. Properties of Calcareous Sand Used in Two Studies

Property (1)	Calcareous Sand: Kaggwa and Booker (1990) study (2)	Calcareous Sand: Present study (3)
Specific gravity	2.725	2.725
D_{10} (mm)	0.005	0.005
C_u (D_{60}/D_{10})	4.3	3.9
Minimum dry density (kN/m^3)	10.3	10.3
Maximum dry density (kN/m^3)	13.4	13.5
Carbonate content (%)	91.4	94.0
Initial voids ratio	1.04–1.15	1.05–1.09
Friction angle φ (degrees)	44	44
Critical stress ratio η_{crit} (q/p') at no volume change	1.722	1.722

TABLE 2. Mean and Cyclic Deviator Stress Conditions during Each Test (Kaggwa and Booker 1990)

Test number (1)	Mean deviator stress, q_m (kPa) (2)	Cyclic deviator stress, Δq (kPa) (3)	Number of cycles (4)
(a) Isotropically Consolidated Samples			
1	0	100	160
2	0	200	160
3	0	280	160
(b) Anisotropically Consolidated Samples			
4	50	200	160
5	100	200	160
6	150	200	160
7	200	200	160
8	200	100	160
9	200	400	160

load was applied using an MTS servo-controlled hydraulic actuator, using the Microprofiler™ to apply the sinsusoidal wave form. A back pressure of 100 kPa was used in all the tests.

The samples were formed in the triaxial cell following the standard procedure of preparing cohesionless soils. Before forming the samples, a predetermined weight of oven-dried sand was placed in a 1-L beaker, distilled water was added to the sand, and the wet soil was then placed in a vacuum chamber to remove all air bubbles trapped between the soil particles. With the rubber membrane, sample preparation mold, and porous stone in place, the sand was carefully spooned into the mold. Care was taken to ensure that the soil was covered with water during soil placement. No vibration of the sample was allowed. This way, each soil sample was not subjected to any cyclic loading prior to the cyclic drained test.

In the cyclic tests, the samples were isotropically consolidated at 200 kPa effective pressure. In tests where the cyclic deviator stresses were applied about a nonzero value, the deviator stress was increased slowly up to the predetermined value. This deviator stress is referred to as the mean deviator stress. The sample was allowed to reach equilibrium conditions by maintaining this stress condition for a standard period of 60 min. Thereafter, cyclic deviator stresses were applied about the mean deviator stress following a sinusoidal waveform with periods of 60 sec. In this relatively free-draining soil, this cycle period was sufficient to ensure that no excess pore pressures were developed during cycling. The mean and amplitude of the cyclic deviator stresses used in each test are summarized in Table 2.

The residual strains developed in calcareous sand under drained cyclic triaxial test conditions have been analyzed by Kaggwa and Booker (1990). A summary of the stress conditions are given in Table 2. The following empirical relationships have been developed to express the volumetric and shear strain responses, with the stressess acting on the soil defined using the p'-q stress space, where $p' = (\sigma_1' + \sigma_2' + \sigma_3')/3$; and $q = (\sigma_1' - \sigma_3')$. In the triaxial test, it is assumed that $\sigma_2' = \sigma_3'$, so that $p' = (\sigma_1' + 2\sigma_3')/3$.

Representation of Residual Strain Response under Uniform Cyclic Loading

The residual volumetric strain $\epsilon = (\epsilon_1 + \epsilon_2 + \epsilon_3)$ and the residual shear strain $\gamma = (\epsilon_1 - \epsilon_3)$ developed are expressed as follows:

$$\epsilon = \epsilon_i + \epsilon_c \quad \text{(4a)}$$

$$\gamma = \gamma_i + \gamma_c \quad \text{(4b)}$$

where $\epsilon = \epsilon_a + 2\epsilon_r$; $\gamma = \epsilon_a - \epsilon_r$; ϵ_a = axial strain in triaxial test; and ϵ_r = radial strain in triaxial test. ϵ_i and γ_i = the volumetric and shear strains due to initial (or virgin) loading, respectively. These are the strains produced by the first cycle of loading. ϵ_c and γ_c = the volumetric and shear strains due to repeated loading at stresses less than the maximum virgin stresses, respectively. The sign convention used in the analysis is compression negative.

The residual strain components ϵ_c, γ_c, developed due to N cycles of repeated loading, are shown in Fig. 1 and can be expressed by the following relationships:

$$1 + \epsilon_c = N^F \quad \text{(4c)}$$

$$1 + \gamma_c = N^D \quad \text{(4d)}$$

where N = the number of stress cycles of constant amplitude; and F and D = exponents expressing the rate of accumulation of residual volumetric and shear strain, respectively. The exponents depend on type of soil, initial voids ratio, and the mean and amplitude of the applied cyclic stresses. F and D have been related to the amplitude of shear strains that the soil undergoes during each cycle of loading using the following relationships:

$$F = X\gamma_{cy}^a \quad \text{(4e)}$$

$$D = Y\gamma_{cy}^b \quad \text{(4f)}$$

where γ_{cy} = amplitude of cyclic shear strain in the soil which equals $\gamma_{max} - \gamma_{min}$; $\gamma = (\epsilon_a - \epsilon_r)$ for triaxial test conditions; ϵ_a, ϵ_r = axial and radial residual strains, respectively; X, Y = coefficients determined from experimental data; a, b = exponents determined from experimental data; and the subscripts max and min = the maximum and minimum values during a stress cycle, respectively.

The amplitude of shear strain γ_{cy} has been related to the amplitude of shear stress and the proximity of the failure envelope using a stress-dependent cyclic shear modulus G defined by

$$G = \frac{\Delta q}{\gamma_{cy}} \quad \text{(4g)}$$

where Δq = the amplitude of shear stress, defined by $\Delta q = q_{max} - q_{min}$; and the subscripts max and min = the maximum and minimum values during a stress cycle, respectively. These measures of the applied shear stress are shown in Fig. 2.

In order to relate the cyclic shear modulus to the amplitude of the shear stresses and the proximity of the failure envelope, the following empirical relationship was found to fit the results of drained cyclic triaxial tests on calcareous sand:

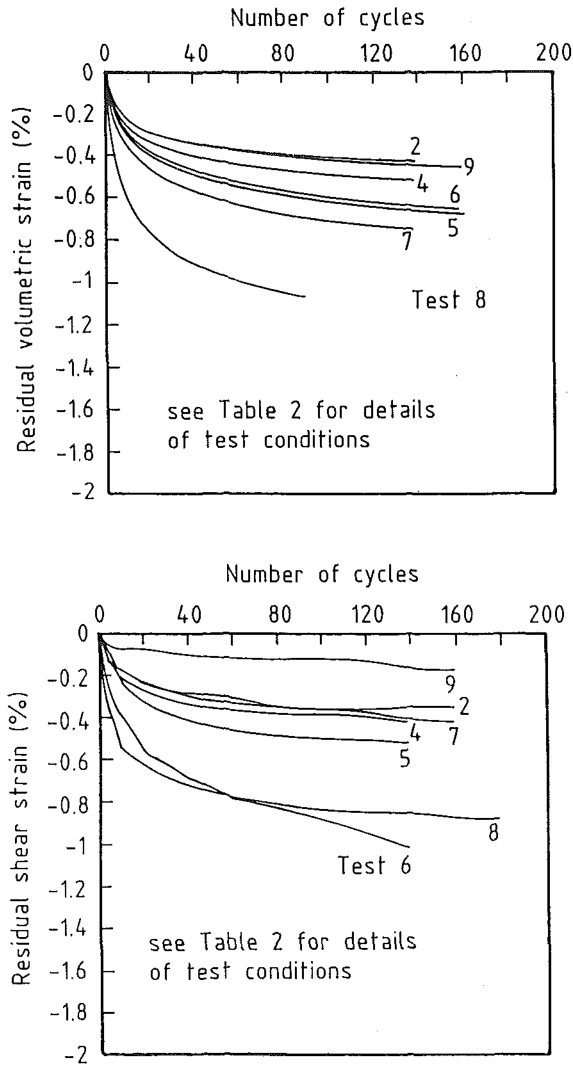

FIG. 1. Residual Volumetric and Shear Strain Responses of Calcareous Sand

$$G = G_0\left(1 - \alpha \frac{\Delta q}{q_{mx}}\right) \quad \text{(4h)}$$

where α = a coefficient close to unity that is used as a correction for loss of shear stiffness ($G = 0$) at stress conditions slightly lower than the failure stresses. The ratio $\Delta q/q_{mx}$ = a measure of the proximity of the maximum shear stress to the failure envelope; and the shear stress q_{mx} = the difference between the shear stress at the failure envelope and the minimum shear stress during a stress cycle, as shown in Fig. 2, i.e.

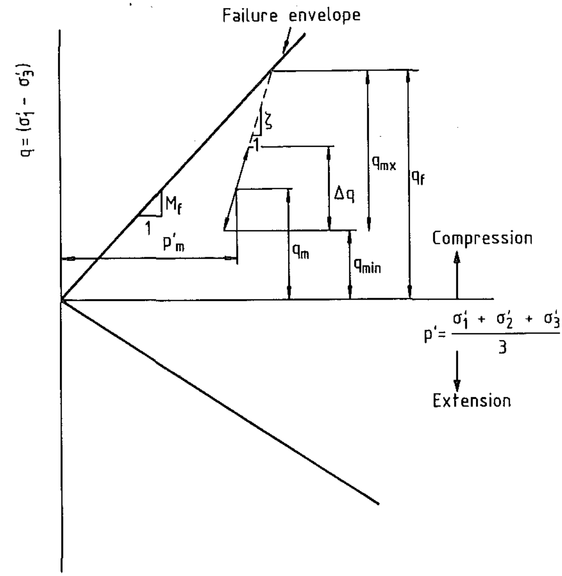

FIG. 2. Relationship between Maximum Shear Stress and Stresses Applied to Soil Sample

$$q_{mx} = q_f - q_{\min} \qquad (4i)$$

where q_f = the shear stress at the failure envelope, i.e.

$$q_f = M_f \cdot \left(\frac{\zeta - \eta_m}{\zeta - M_f}\right) p'_m \qquad (4j)$$

and η_m = ratio of average stresses about which the cyclic stresses are applied, which equals q_m/p'_m; ζ = average slope of the stress path during a stress cycle, $(q_{\max} - q_{\min})/(p'_{\max} - p'_{\min})$; M_f = slope of failure envelope; p' = $(\sigma'_1 + \sigma'_2 + \sigma'_3)/3$; $q = (\sigma'_1 - \sigma'_3)$; and the subscripts max, min, and m, = maximum, minimum, and mean values, respectively. The values of the coefficients determined from results of drained cyclic triaxial tests on calcareous sand are summarized in Table 3.

Analysis of Residual Strain Response

The following steps may be used in order to determine the residual strain response of calcareous sand under uniform cyclic triaxial loading conditions. It is assumed that the average and cyclic shear stresses acting on the soil sample are known or can be determined beforehand:

1. For the known average and cyclic shear stresses, calculate the stress ratios ζ, η_m, and M_f in Eq. 4*j*. Calculate the maximum shear stress q_{mx} using Eq. 4*i*.
2. Calculate the cyclic shear modulus G using Eq. 4*h* and the cyclic shear strain from Eq. 4*g*.
3. Calculate the exponents F and D using Eqs. 4*e* and 4*f*, respectively.

TABLE 3. Summary of Exponents Determined from Analysis of Cyclic Triaxial Test Results for Calcareous Sand

Coefficient (1)	Cyclic compression (2)	Cyclic compression and extension (3)	Equation (4)
(a) Residual Volumetric Strain Exponent F			
X	−0.124	−6.297	4e
a	0.554	1.162	
(b) Residual Shear Strain Exponent D			
Y	−1.515	−384.6	4f
b	0.938	1.582	
(c) Cyclic Shear Strain γ_{cy}			
G_0 (MPa)	800	—	4h
α	0.99	—	

4. Tabulate the cyclic component of the residual strains versus the number of cycles.

5. Calculate the residual strains due to the first cycle of loading (termed initial loading) using a suitable model (e.g., nonlinear elastic, elastoplastic). Obtain the total residual strain response using Eqs. 4a and 4b.

6. Plot the variation of total residual strains ϵ and γ versus the number of stress cycles. In this way, the residual strain response of the soil due to uniform cyclic loading can be determined.

Procedure for Analyzing Effects of Irregular Cyclic Loading

As an alternative to analyzing cycle by cycle the effects of irregular cyclic loading, it is necessary to divide the irregular stress time history into a series of "parcels" of cyclic loading. Each parcel is chosen such that the cyclic stresses are of the same mean and amplitude. The resulting pattern of loading is termed a multistage cyclic loading pattern. The term *multistage cyclic loading* is used here to represent cyclic loading conditions in which several parcels of uniform cyclic loading are applied to the soil. Each parcel is assumed to consist of a number of stress cycles with the same mean and amplitude of cyclic shear stresses.

Consider the following loading sequence. Several parcels of cyclic loading (each with a different mean or amplitude of deviator stress acting for specified numbers of cycles), e.g., m parcels with corresponding cycles ΔN_1, ΔN_2, ΔN_3, $\ldots$, ΔN_m, are applied to the soil. We can associate a parcel of loading i with a corresponding exponent for the rate of accumulation of residual volumetric strain, F_i, parcels 1, 2, 3, $\ldots$, m with corresponding exponents F_1, F_2, F_3, $\ldots$, F_m, respectively.

Now, because of the nonlinear accumulation of residual strain during cyclic loading (see Eq. 4c), the increment in residual volumetric strain produced by ΔN_i cycles will depend on the current voids ratio of the soil (and thus on the current residual volumetric strain). This can be seen from the following equation:

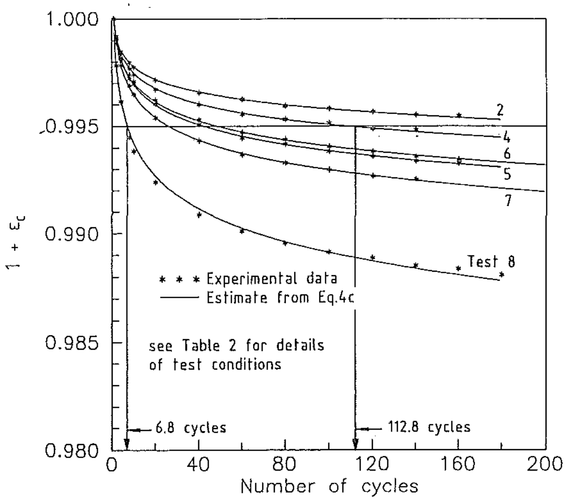

FIG. 3. Equivalent Number of Cycles at Different Cyclic Stress Conditions

$$\Delta\epsilon = (1 + \epsilon_j) - (1 + \epsilon_i) = (N_j)^F - (N_i)^F = (N_i)^F\left[\left(\frac{1 + \Delta N_i}{N_i}\right)^F - 1\right] \quad \text{.... (5)}$$

where $\Delta\epsilon$ = increase in residual volumetric strain due to ΔN_i cycles; $N_j = N_i + \Delta N_i$; and $j = i + 1$. Eq. 5 shows that as the ratio $\Delta N_i/N_i$ decreases (due to a higher N_i for a given cycle increment ΔN_i) so does the term in brackets.

Now, at the end of parcel j, the residual volumetric strain due to all cyclic loading to date ϵ_{cj} is given by

$$1 + \epsilon_{cj} = (N_{ej} + \Delta N_j)^{Fj} \quad \text{.......... (6a)}$$

$$1 + \epsilon_{cj} = (N_{ej})^{Fj}\left(\frac{1 + \Delta N_j}{N_{ej}}\right)^{Fj} \quad \text{.......... (6b)}$$

where N_{ej} = number of cycles at loading level j that would produce the same residual strain ϵ_{ci} as that produced by all prior cyclic loading; ΔN_j = number of cycles in parcel j; and F_j = exponent expressing the rate of accumulation of residual volumetric strain associated with the mean and cyclic stresses for parcel j.

The term $(N_{ej})^F$ is a direct measure of the residual volumetric strain or change in voids ratio (refer to Eq. 4c) caused by prior cyclic loading and provides an easy way of relating the accumulated volumetric strain to the equivalent number of cycles under any loading conditions that would produce the same residual volumetric strain. Fig. 3 shows an example of the equivalent number of cycles at different stress conditions corresponding to a residual volumetric strain $\epsilon_c = -0.5\%$ or $1 + \epsilon_c = 0.995$. For example, 6.8 cycles at Δq = 400 kPa and mean stresses p'_m = 267 kPa and q_m = 200 kPa (see the curve labeled Test 8 in Fig. 3) produce the same residual volumetric strain as 112.8 cycles at Δq = 200 and mean stresses p'_m = 233.3

kPa and q_m = 50 kPa (see the curve labeled Test 4 in Fig. 3). We can therefore express $(N_{ej})^F$ in terms of the preceding cyclic loading conditions using the following relationship:

$$(N_{ej})^{F_j} = (N_{ei} + \Delta N_i)^{F_i} = (N_{ei})^{F_i}\left(\frac{1 + \Delta N_i}{N_{ei}}\right)^{F_i} \qquad (7)$$

where $i = j = 1$.

We can now write the following equations:

1. At the start of the first parcel of cyclic loading, the residual volumetric strain due to cyclic loading $\epsilon_c = 0$. Therefore

$$(N_{e1})^{F_1} = 1 \qquad (8a)$$

2. For parcel 2, the effect of the previous parcel 1 can be expressed by

$$(N_{e2})^{F_2} = (\Delta N_1)^{F_1} = Y_1 \qquad (8b)$$

where $Y_1 = (\Delta N_1)^{F_1}$.

3. For parcel 3, the combined effect of parcel 1 and parcel 2 is expressed by

$$(N_{e3})^{F_3} = Y_1\left[\frac{1 + \Delta N_2}{(Y_1)^{1/F_2}}\right]^{F_2} \qquad (8c)$$

which can be written as

$$(N_{e3})^{F_3} = Y_1 Y_2 \qquad (8d)$$

where

$$Y_2 = \left[\frac{1 + \Delta N_2}{(Y_1)^{1/F_2}}\right]^{F_2} \qquad (8e)$$

4. Thus, for parcel j, the effect of $j - 1$ prior parcels can be expressed by

$$(N_{ej})^{F_j} = Y_1 Y_2 Y_3, \ldots, Y_{j-1} \qquad (8f)$$

where

$$Y_j = \left[\frac{1 + \Delta N_i}{(Y_1 Y_2 Y_3, \ldots, Y_i)^{1/F_j}}\right]^{F_j}, \qquad i = j - 1 \text{ and } j > 2 \qquad (8g)$$

Examples of Analyses of Irregular Cyclic Loading

The following examples are used to demonstrate the influence of a sequence of cyclic loading. The values of the exponents shown in Table 3 are used in the theoretical analyses. Examples of laboratory tests in which three parcels of cyclic loading are applied to a calcareous sand recovered from the same site, but with a higher fines content, are presented. These experimental results are compared with the results of analyses in which the values of the exponents F and D are determined from the cyclic response to the first parcel of cyclic loading of each test. A procedure for estimating the effects of prior cyclic loading on the basis of voids ratio and precompression pressure is also presented.

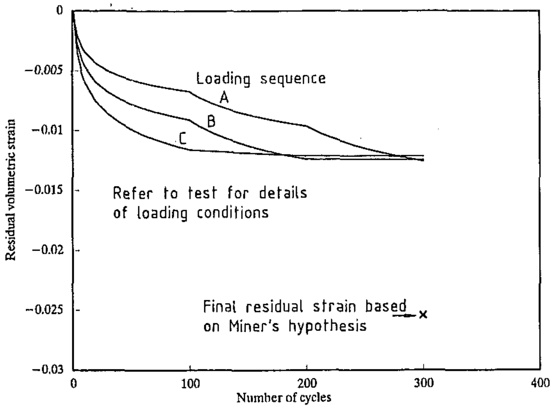

FIG. 4. Predicted Residual Volumetric Strain Response

Influence of Pattern of Loading

Predictions in which three parcels of cyclic loading (each 100 stress cycles) are applied to a calcareous sand under fully drained triaxial test conditions are presented to demonstrate the influence of the sequence of cyclic loading. The following stress conditions have been used: effective lateral pressure $\sigma_r' = 200$ kPa and mean deviator stress $q_m = 200$ kPa. The cyclic stresses were $\Delta q = 200$ kPa for parcel 1, 300 kPa for parcel 2, and 400 kPa for parcel 3. Three loading sequences have been analyzed. The order of application of the parcels was as follows: (1) Loading sequence A: parcel 1 + parcel 2 + parcel 3; (2) loading sequence B: parcel 2 + parcel 3 + parcel 1; and (3) loading sequence C: parcel 3 + parcel 2 + parcel 1. Thus overall, the same number of stress cycles was applied to the calcareous sand, with the only difference being the sequence of application of the parcels. High cyclic stresses have been used in order to show clearly the differences in the residual strain responses.

The residual volumetric and shear strains predicted by the analysis are shown in Fig. 4 and Fig. 5. In both figures, the intermediate residual strain responses are dependent on the sequence of application of the parcels of cyclic loading, although the residual strains at the end of cycling are essentially the same. Furthermore, the change in residual volumetric or shear strain due to a given parcel of loading depends on the extent of prior cyclic loading.

It should be pointed out that if the equivalent uniform cycle concept is used in the given predictions of the residual strain responses, as well as with Miner's hypothesis, a linear relationship results. The residual strain at the end of the loading stage simply becomes the sum of the residual strains contributed by each parcel of loading, and the final values are shown on the figures. It can be seen from Figs. 4 and 5 that the residual strains obtained using Miner's hypothesis are higher than those obtained using the present method. The dependence of the response on the loading history highlights one of the shortcomings of using Miner's hypothesis to determine the equivalent number of uniform stress cycles for evaluating residual strains.

Thus, when analyzing problems such as wave loading where the sequence

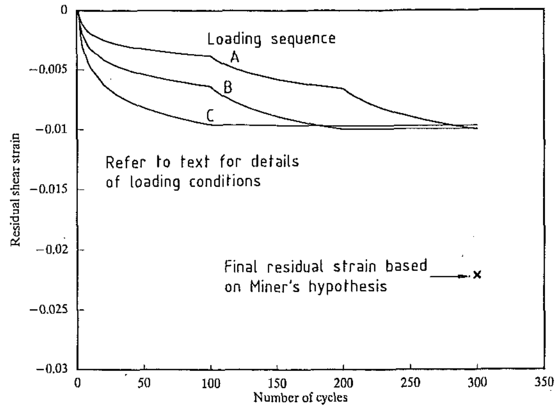

FIG. 5. Predicted Residual Shear Strain Response

of cyclic loading can be represented by a series of parcels of cyclic loading, the present procedure would provide more realistic results than the use of a time series of equivalent uniform stress cycles.

In order to check the validity of the present procedure for analyzing multistage cyclic loading, three laboratory tests were performed in which three parcels of cyclic stresses (each of 50 stress cycles) were applied to a calcareous sand with a higher percentage of particles sizes falling in the range of fine sand than the calcareous sand used by Kaggwa and Booker (1990), as can be seen from Table 1. The stress conditions used in each test, as well as the sequence of application of the parcels of cyclic loading, are summarized in Table 4.

Fig. 6 and Fig. 7 show the results of the three triaxial tests. Again, the experimental results show that the residual volumetric and shear strains developed during any parcel of loading depend on the extent of prior cyclic loading. The residual volumetric and shear strains recorded at the end of each parcel of cyclic loading are shown in Table 5. By using the first parcel

TABLE 4. Stress Conditions during Multistage Cyclic Testing

Test number (1)	Parcel number (2)	Number of cycles (3)	Mean deviator stress, q_m (kPa) (4)	Cyclic deviator stress, Δq (kPa) (5)
1	1	50	200	200
	2	50	200	300
	3	50	200	400
2	1	50	200	300
	2	50	200	400
	3	50	200	200
3	1	50	200	400
	2	50	200	300
	3	50	200	200

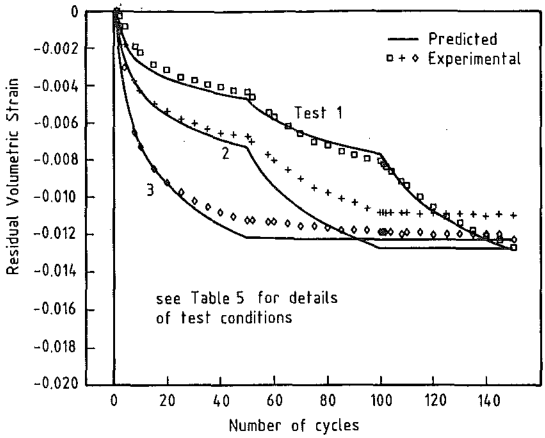

FIG. 6. Results of Residual Volumetric Strain Response under Multistage Cyclic Loading

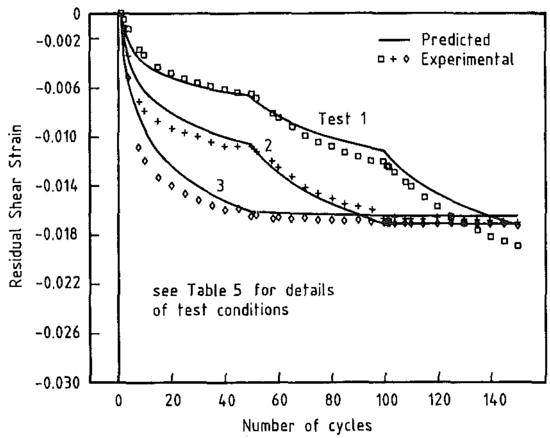

FIG. 7. Results of Residual Shear Strain Response under Multistage Cyclic Loading

of cyclic loading in each of the three tests to determine the residual strain accumulation exponents F and D (see Eqs. 4c and 4d), the residual strain responses due to the subsequent parcels were then determined using Eq. 6. The values of the exponents F and D obtained from the analyses are shown in Table 5, and the predicted responses obtained from these analyses are shown as the solid lines in Figs. 6 and 7. There is good agreement between the experimental and predicted responses.

Application to Field Conditions

It does not appear possible to determine whether a soil has been subjected to prior cyclic loading in situ on the basis of commonly measured engi-

TABLE 5. Summary of Results of Multistage Cyclic Triaxial Tests on Calcareous Sand

Results (1)	Test 1 (2)	Test 2 (3)	Test 3 (4)
(*a*) Initial Voids Ratio			
—	1.091	1.053	1.073
(*b*) Residual Volumetric Strain ϵ_c			
At 50 cycles	−0.00437	−0.00673	−0.01125
At 100 cycles	−0.00831	−0.01081	−0.01189
At 150 cycles	−0.01270	−0.01102	−0.01221
(*c*) Residual Shear Strain γ_c			
At 50 cycles	−0.00708	−0.01081	−0.01645
At 100 cycles	−0.01271	−0.01666	−0.01708
At 150 cycles	−0.01930	−0.01705	−0.01723
(*d*) Residual Strain Accumulation Exponents			
Volumetric strain, F	−0.001206	−0.001871	−0.003128
Shear strain, D	−0.001722	−0.002721	−0.004172
(*e*) Cyclic Shear Strain Coefficients			
X	−0.5994		
a	0.9041		
Y	−0.5751		
b	0.8442		

neering properties such as density or voids ratio, permeability coefficient, or compressibility coefficient. Thus, it is necessary to perform laboratory tests on "undisturbed" soil samples in order to determine the values of the exponents F and D, which express the rates of accumulation of the residual volumetric and shear strains, respectively. Of the two residual strains, i.e., volumetric and shear strains, only the volumetric strain can be directly related to the voids ratio and the in situ stresses. A procedure that can be used to estimate the effects of prior field stresses (static as well as cyclic stresses) for cohesionless soils using the current in situ stresses in conjunction with the results of laboratory tests on reconstituted soil samples is as follows:

1. Using results of triaxial tests on reconstituted samples, prepare graphs of voids ratio versus effective mean normal pressure p' for different ratios of ζ ($\Delta q/\Delta p'$) as shown schematically in Fig. 8. In Fig. 8, it is assumed that as Δq increases, so does the volumetric strain. Accordingly, as the value of the ratio ζ increases, the proportion of the volumetric strain due to Δq becomes higher, whereas that due to $\Delta p'$ becomes less.

2. Determine the value of the mean stress p_0' corresponding to the preconsolidation pressure of the "undisturbed" soil from the oedometer test, as well as the value of K_0 ($\sigma_{h0}'/\sigma_{v0}'$), where σ_{h0}' = the horizontal in situ stress; and σ_{v0}' = the vertical in situ stress.

3. Determine the overconsolidation ratio OCR (the ratio of the vertical preconsolidation pressure to the in situ vertical stress). If OCR > 1, then use the precompression pressure p_{oc}' and $(q/p')_{oc}$ to determine the precompression voids

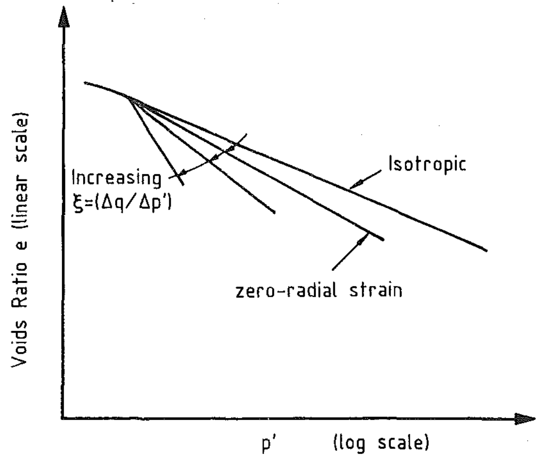

FIG. 8. Relationship between Voids Ratio and Consolidation Stresses

ratio $e_0 = e_{oc}$, using Fig. 8. If OCR = 1, then the in situ stresses are used to calculate the corresponding precompression voids ratio $e_0 = e_{nc}$.

4. If the in situ voids ratio e_{is} is less than e_{nc} for a normally consolidated soil, this indicates that the soil has been subjected to prior field stresses greater than the current in situ stresses. If the in situ voids ratio e_{is} is less than e_{oc}, which corresponds to the precompression pressure, then the overconsolidated soil is considered to have experienced prior cyclic loading.

5. The difference between the in situ voids ratio e_{is} and the precompression voids ratio e_0 can be expressed in terms of the equivalent number of cycles at any given cyclic loading level that are required to cause the same change in voids ratio. This is achieved by using laboratory cyclic test results on reconstituted soil samples with an initial voids ratio e_0, subjected to the average stresses p'_m, q_m equal to the precompression stresses p'_0, q_0.

Eq. 4*c* can be expressed in terms of changes in voids ratio, since

$$\epsilon = \frac{\Delta e}{(1 + e_0)} \quad (9)$$

Thus, the equivalent number of cycles N_e can be determined using the following relationship:

$$(N_e)^F = 1 + \frac{\Delta e_c}{1} + e_0 \quad (10)$$

where e_0 = voids ratio of reconstituted soil sample when subjected to preconsolidation stresses p'_0 and q_0; Δe_c = change in voids ratio due to prior cyclic loading in the field; $\Delta e_c = e - e_i$; e = current in situ voids ratio; e_i = voids ratio of reconstituted soil after the first stress cycle; and F = exponent expressing the rate of increase of residual volumetric strain at the cyclic stress conditions of interest.

The following example illustrates the procedure for calculating the equivalent number of cycles. Consider a normally consolidated, uncemented calcareous sediment with in situ voids ratio $e = 1.05$, and in situ stresses $\sigma_v' = 60$ kPa and $\sigma_h' = 24$ kPa. Laboratory tests show that $e_0 = 1.08$, when $p_0' = 36$ kPa and $q_0 = 36$ kPa. If for the cyclic conditions given by $\zeta = 3$ and $\Delta q = 20$ kPa, $e_i = 1.06$ and the residual volumetric strain accumulation exponent $F = -0.0008$, then the equivalent number of stress cycles at these stresses is calculated as follows. The voids ratio is related to the volumetric strain by

$$\epsilon_c = \frac{\Delta e_c}{(1 + e_0)} = \frac{(1.05 - 1.06)}{2.08} = -0.0048 \qquad (11)$$

From Eq. 4*c*, $(N_e)^F = 0.9952$. Thus, substituting $F = -0.0008$, the equivalent number of cycles is calculated as $N_e = 409.3$ cycles. If the soil is subjected to a further 200 cycles, the sample will undergo further residual volumetric strains that can be calculated since

$$\Delta e_c = (N_e = 200)^F - (N_e)^F = (609.3)^{-0.0008} - 0.0952 = -0.03\% \qquad (12)$$

The volumetric strain ϵ_c, neglecting any effects of prior cyclic loading, would be $\epsilon_c = 200^F = 0.42\%$. It can be seen that the residual strain for the field sample is only about 7% of that for the reconstituted laboratory sample.

Conclusions

A procedure for analyzing the behavior of calcareous sand under irregular cyclic loading has been presented. It has been pointed out that the use of existing procedures for converting irregular stress time histories into equivalent uniform stress series may lead to erroneous predictions of the effects of the irregular cyclic loading. Accordingly, a procedure has been presented that divides the irregular stress time history into a sequence of parcels of uniform loading which are then analyzed parcel by parcel. Examples have been presented for determining the residual strains developed in multistage cyclic triaxial tests. These examples demonstrate that the sequence of cyclic loading affects the response of the sand, and therefore the sequence of loading should be taken into consideration in analyses that require that an irregular cyclic stress time history be replaced by an equivalent uniform cyclic stress series.

The procedure presented in this paper can be extended to other cohesionless soils provided that the appropriate empirical equations are developed in order to express the relationships between accumulation of residual strains and the applied cyclic stresses.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

a = exponent;
b = exponent;
D = residual shear strain exponent;
e = current voids ratio;
e_i = voids ratio at end of first cycle;
e_{is} = in situ voids ratio;
e_0 = initial voids ratio;

F_i = residual volumetric strain exponent for loading parcel i;
G^* = cyclic shear modulus;
G_0 = cyclic shear modulus at small strain;
M_f = slope of failure envelope;
m = number of stress levels in irregular stress-time history;
N_e = number of cycles at level of interest that are required to produce same failure conditions;
N_i = number of cycles at stress level i that are required to cause specified failure condition;
n_e = equivalent number of cycles at stress level of interest;
n_{ei} = equivalent number of cycles at stress level i which produce same strain as all preceding irregular cycles;
n_i = number of stress cycles at stress level i;
p' = effective mean normal stress;
p'_m = average effective mean normal stress during cyclic loading;
q = deviator shear stress;
q_f = deviator shear stress at failure;
q_m = average deviator shear stress during cyclic loading;
$q_{\max}$ = maximum deviator shear stress;
$q_{\min}$ = minimum deviator shear stress;
q_{mx} = maximum cyclic shear stress;
X = coefficient;
x_i = exponent;
Y = coefficient;
α = coefficient;
β = coefficient;
γ = total residual shear strain;
γ_c = residual shear strain due to cyclic loading;
γ_i = residual shear strain at end of first cycle;
Δe_c = change in voids ratio due to cyclic loading;
Δq = amplitude of cyclic deviator shear stress;
ϵ = total residual volumetric strain;
ϵ_a = axial strain in triaxial test;
ϵ_c = residual volumetric strain due to cyclic loading;
ϵ_i = residual volumetric strain at end of first cycle;
ϵ_r = radial strain in triaxial test;
ζ = average slope of stress path during cyclic loading;
η = ratio of stresses in p'-q stress space;
η_m = value of ratio of average stresses (q_m/p'_m);
σ'_i = effective normal stress, subscript i = 1, 2, 3 represents principal stresses;
σ'_{h0} = effective in situ horizontal stress; and
σ'_{v0} = effective in situ vertical stress.