Experimental Techniques for the Investigation of the Elasto-Plastic Transition Zone of Foamed Materials

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The classical assumption in solid materials, i.e. that the plastic behaviour is incompressible, does no longer hold in the case of cellular materials. The plastic behaviour is pressure-sensitive due to the cellular structure even when the pure base material is independent of the hydrostatic pressure in the plastic range. Therefore, the yield criterion needs to incorporate the hydrostatic pressure. In many cases, the yield criterion can be simplified to an additive form where an arbitrary scalar functions weights the influence of the hydrostatic stress. The yield stress can be obtained from uniaxial tests but the determination of the weighting function for the hydrostatic stress requires the realisation of multi-axial stress states. This work presents two experimental procedures, i.e. an experiment under plane strain conditions and the axial compression, for the determination of the parameters of the yield criterion in the elasto-plastic transition zone. Furthermore, both experiments aim to determine a second elastic constant if for example Young's modulus is known from uniaxial compression tests. The proposed procedures are numerically applied to a material obeying the Deshpande-Fleck yield criterion.[1,2]

Keywords: *plasticity, yield surface, elasto-plastic transition zone, experimental procedure, material properties*

1. Introduction

The application of low density foams in lightweight constructions is a promising approach in modern industry (e.g. aerospace, automotive or navigation). Foamed materials are characterised by good heat insulation and damping as well as high energy absorption and a high specific stiffness (e.g. [3]). Combinations of these properties allow for new future-oriented applications. In addition to the well-established utilisation of polymer foams, new porous materials with metallic matrix are continuously gaining importance. One often quoted example is the utilisation of foamed cores for crash elements and recent studies underline the potential of metallic foams.[4-7] These metal foams are characterised by a higher specific stiffness as well as higher chemical and thermal resistance compared to their polymer based equivalents and enable the design of a new group of advanced structural lightweight components. In order to fully exploit these new materials, an accurate knowledge of their material parameters is required. Consequently, a successful industrial application depends on efficient procedures for material testing which enable the accurate determination of material parameters for design and quality control.

The plastic behaviour can be described e.g. by the yield criterion of Deshpande-Fleck.[1,2] This yield criterion requires the determination of the plasticity parameter α, weighing the influence of the hydrostatic stress. Classical uniaxial approaches for the determination of this parameter, requiring the direct measurement of the strain (for a general overview on classical test methods see e.g. [8]), cannot be applied in the case of cellular materials since the application of strain gauges is impossible on the rough surface of cellular or porous materials. In order to overcome this problem, two new experimental procedures which additionally allow for the determination of a second elastic constant, e.g. Poisson's ratio, will be presented.

2. Continuum Mechanics

2.1 General description of yield criteria

One of the commonly used constitutive equations for foamed materials was developed by Deshpande and Fleck based on experimental investigations of metallic[1] and polymer foams[2]. The mathematical formulation is given by:

$$F = \frac{1}{\left[1+\left(\frac{\alpha}{3}\right)^2\right]} \cdot \left[\sigma_{\mathrm{e}}^2 + \alpha^2 \sigma_{\mathrm{m}}^2\right] - k_{\mathrm{t}}^{\,2} = 0, \quad (1)$$

where $\sigma_{\mathrm{m}} = \frac{1}{3}\sigma_{ii}$ is the mean stress and $\sigma_{\mathrm{e}} = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$ is the von Mises effective stress. The plasticity parameter α and the uniaxial tensile yield stress k_{t} are functions of some internal variable, e.g. the effective plastic strain $\mathrm{d}\varepsilon_{\mathrm{eff}}^{\mathrm{p}} = \sqrt{\frac{2}{3}\mathrm{d}\mathrm{e}_{ij}^{\mathrm{p}}\mathrm{d}\mathrm{e}_{ij}^{\mathrm{p}}}$ (e_{ij} is the strain deviator

tensor). In Eq. (1), the first invariant $J_1^{\mathrm{o}} = \sigma_{ii}$ of the spherical stress tensor σ_{ij}^{o} and the second invariant $J_2^{'} = \frac{1}{2} s_{ij} s_{ij}$ of the deviator stress tensor s_{ij} can be introduced to obtain after rearranging the following form:

$$F = \frac{3}{1+\frac{\alpha^2}{9}} \cdot J_2^{'} + \frac{\frac{1}{9}\alpha^2}{1+\frac{\alpha^2}{9}} \cdot \left(J_1^{\mathrm{o}}\right)^2 - k_{\mathrm{t}}^2 = 0. \quad (2)$$

It might be more appropriate for mathematical transformations to replace the tensile yield stress k_{t} by the shear stress k_{s}. To this end, a σ–τ stress space is considered. This stress space is defined by a single normal stress σ and a single shear stress τ. The invariants reduces in such a stress space to $J_1^{\mathrm{o}} \to \sigma$ and $J_2^{'} \to \frac{1}{3}\sigma^2 + \tau^2$. Introducing these definitions into Eq. (2) results in the yield criterion in the $\sigma - \tau$ stress space as:

$$F_{\sigma-\tau} = \frac{3}{1+\frac{\alpha^2}{9}} \cdot \left(\frac{1}{3}\sigma^2 + \tau^2\right) + \frac{\frac{1}{9}\alpha^2}{1+\frac{\alpha^2}{9}} \cdot \sigma^2 - k_{\mathrm{t}}^2 = 0. \quad (3)$$

The relationship between the tensile and shear yield stress follows from the conditions $\sigma \to 0 \;\Rightarrow\; \tau \to k_{\mathrm{s}}$ as:

$$\frac{3}{1+\frac{\alpha^2}{9}} k_{\mathrm{s}}^2 = k_{\mathrm{t}}^2. \quad (4)$$

Introducing the last relationship into Eq. (2) gives finally:

$$F = \frac{1}{27}\alpha^2 \left(J_1^{\mathrm{o}}\right)^2 + J_2 - k_{\mathrm{s}}^2 = 0, \quad (5)$$

which is much more appropriate for mathematical derivations. Equation (5) can generally be expressed in the form

$$F = g_1(\alpha) \cdot f_1(J_1^{\mathrm{o}}) + f_2(J_2) + f_3(k_{\mathrm{s}}) = 0, \quad (6)$$

where f_i (i = 1,2,3) and g_1 are arbitrary scalar functions. The general form (6) includes many different yield criteria. Some examples are given in Tab. 1.

g_1	f_1	f_2	f_3
Deshpande and Fleck[1,2]			
$\frac{1}{27}\alpha^2$	$\left(J_1^{\mathrm{o}}\right)^2$	J_2	k_{s}^2
Mahrenholtz and Ismar[9,10]			
α	$\left(J_1^{\mathrm{o}}\right)^2$	J_2	k_{s}^2
Drucker and Prager[11]			
α	J_1^{o}	$\sqrt{J_2}$	k_{s}
von Mises[11]			
0	0	$\sqrt{J_2}$	k_{s}

Table 1 Yield criteria of the form (6).

The experimental task is the determination of the plasticity parameter α and the yield stress k_s (or k_t). While the uniaxial yield stress k_t can be determined for example based on uniaxial tensile or compression tests, the measurement of α is much more complicated. In the following, a plane strain and a uniaxial strain state is considered to determine both parameters.

2.2 States of uniaxial strain and plane strain

Under the classical assumption of small strains, the total strain increment $\mathrm{d}\varepsilon_{ij}$ is assumed to be the sum of the elastic strain increment $\mathrm{d}\varepsilon_{ij}^{\mathrm{e}}$ and the plastic strain increment $\mathrm{d}\varepsilon_{ij}^{\mathrm{p}}$. Furthermore, the elastic strain increment can be obtained from Hooke's law

$$\varepsilon_{ij} = \frac{1+\nu}{E}\left(\sigma_{ij} - \frac{\nu}{1+\nu}\delta_{ij}\sigma_{kk}\right) = C_{ijkl}^{-1}\sigma_{kl}\,, \quad (7)$$

and the plastic strain increment from the associated flow rule

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \mathrm{d}\lambda \tfrac{\partial F}{\partial \sigma_{ij}}. \quad (8)$$

Application of the chain rule with respect to the stress tensor gives the derivative of the general scalar yield criterion (6) as:

$$\frac{\partial F}{\partial \sigma_{ij}} = \frac{\partial F}{\partial J_1^{\mathrm{o}}}\frac{\partial J_1^{\mathrm{o}}}{\partial \sigma_{ij}} + \frac{\partial F}{\partial J_2^{'}}\frac{\partial J_2^{'}}{\partial \sigma_{ij}} = g_1(\alpha)\frac{\partial f_1}{\partial J_1^{\mathrm{o}}}\cdot\delta_{ij} + \frac{\partial f_2}{\partial J_2}\cdot s_{ij}\,. \quad (9)$$

The terms $\partial F/\partial J_1^{\mathrm{o}}$ and $\partial F/\partial J_2^{'}$ are normally simple derivatives of polynomial expressions. In the following, only the yield criterion according to Eq. (5) is considered. The derivative of F with respect to σ_{ij} is obtained as:

$$\frac{\partial F}{\partial \sigma_{ij}} = \frac{2}{27}\alpha^2\cdot J_1^{\mathrm{o}}\cdot\delta_{ij} + 1\cdot s_{ij}\,. \quad (10)$$

If Hooke's law (7) is applied for the elastic component and the associated flow rule (8) for the plastic component, the complete stress-strain relationship for a material obeying a yield criterion of form (5) is expressed as:

$$\mathrm{d}\varepsilon_{ij} = \mathrm{d}\lambda\left(\frac{2}{27}\alpha^2 J_1^{\mathrm{o}}\delta_{ij} + s_{ij}\right) + \frac{1+\nu}{E}\left(\mathrm{d}\sigma_{ij} - \frac{\nu}{1+\nu}\delta_{ij}\mathrm{d}\sigma_{kk}\right). \quad (11)$$

For a state of plane strain (no wall friction, principal directions are I, II, III), we have (cf. Fig. 1 a))

$$\sigma_I < 0 \;\wedge\; \varepsilon_I \neq 0, \quad (12)$$

$$\sigma_{II} \neq 0 \;\wedge\; \varepsilon_{II} = 0, \quad (13)$$

$$\sigma_{III} = 0 \;\wedge\; \varepsilon_{III} \neq 0, \quad (14)$$

while for the characterisation of a state of uniaxial strain (cf. Fig. 1 b)) Eq. (14) has to be replaced by :

$$\sigma_{III} \neq 0 \ \wedge \ \varepsilon_{III} = 0. \quad (15)$$

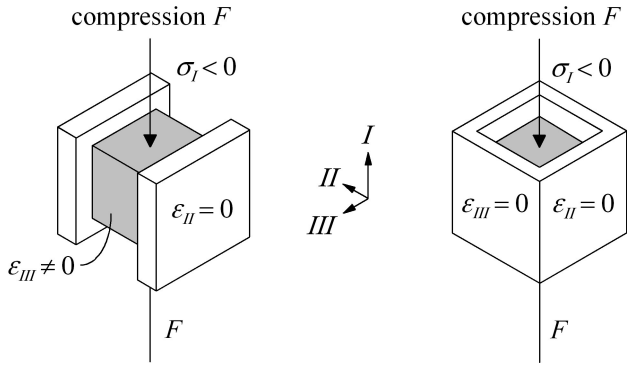

Fig. 1 Schematic state of plane strain (left) and a state of uniaxial strain (right).

Using Eqs. (12)-(15) with the associated flow rule (8) and (11), the increments of the plastic strain are given by ($i = I, II$):

$$\mathrm{d}\varepsilon_i^{\mathrm{p}} = \mathrm{d}\lambda \left[\frac{2}{27} \alpha^2 \cdot J_1^o + s_i \right]. \quad (16)$$

Introducing the principal stresses in Eqs. (16), one can obtain:

$$\mathrm{d}\varepsilon_I^{\mathrm{p}} = \mathrm{d}\lambda \left[\tfrac{2}{27} \alpha^2 (\sigma_I + \sigma_{II} + \kappa\sigma_{III}) + \tfrac{1}{3}\left(2\sigma_I - \sigma_{II} - \kappa\sigma_{III}\right) \right], \quad (17)$$

$$\mathrm{d}\varepsilon_{II}^{\mathrm{p}} = \mathrm{d}\lambda \left[\tfrac{2}{27} \alpha^2 (\sigma_I + \sigma_{II} + \kappa\sigma_{III}) + \tfrac{1}{3}\left(-\sigma_I + 2\sigma_{II} - \kappa\sigma_{III}\right) \right], \quad (18)$$

where parameter κ distinguishes between the plane strain and uniaxial strain case: $\kappa = 0$: plane strain; $\kappa = 1$: uniaxial strain. Dividing Eq. (18) by (19) and rearranging for α yields to the following equation:

$$\alpha^2 = \frac{9}{2} \cdot \frac{\sigma_I \left(1 + 2\frac{\mathrm{d}\varepsilon_{II}^{\mathrm{p}}}{\mathrm{d}\varepsilon_I^{\mathrm{p}}}\right) - \sigma_{II}\left(2 + \frac{\mathrm{d}\varepsilon_{II}^{\mathrm{p}}}{\mathrm{d}\varepsilon_I^{\mathrm{p}}}\right) + \kappa\sigma_{III}\left(1 - \frac{\mathrm{d}\varepsilon_{II}^{\mathrm{p}}}{\mathrm{d}\varepsilon_I^{\mathrm{p}}}\right)}{(\sigma_I + \sigma_{II} + \kappa\sigma_{III})\left(1 - \frac{\mathrm{d}\varepsilon_{II}^{\mathrm{p}}}{\mathrm{d}\varepsilon_I^{\mathrm{p}}}\right)}. \quad (19)$$

For small plastic strains (respectively at the beginning of the yielding), the quotient is $\mathrm{d}\varepsilon_{II}^{\mathrm{p}} / \mathrm{d}\varepsilon_I^{\mathrm{p}} \approx 1$ (respectively = 0) and the quantity α can be approximated by

$$\alpha^2 \approx \frac{9}{2} \cdot \frac{\sigma_I - 2\sigma_{II} + \kappa\sigma_{III}}{\sigma_I + \sigma_{II} + \kappa\sigma_{III}}. \quad (20)$$

Evaluation of Eq. (19) requires the determination of the plastic strain increments $\mathrm{d}\varepsilon_I^{\mathrm{p}}$ and $\mathrm{d}\varepsilon_{II}^{\mathrm{p}}$. These increments can be determined from the sum of the elastic and plastic strain increments as

$$\mathrm{d}\varepsilon_I^{\mathrm{p}} = \mathrm{d}\varepsilon_I - \mathrm{d}_I^{\mathrm{e}}, \quad (21)$$

$$\mathrm{d}\varepsilon_{II}^{\mathrm{p}} = -\mathrm{d}\varepsilon_{II}^{\mathrm{e}}, \quad (22)$$

$$\mathrm{d}\varepsilon_{III}^{\mathrm{p}} \begin{cases} \kappa = 1: \ \underbrace{\mathrm{d}\varepsilon_{III}}_{0} - \mathrm{d}\varepsilon_{III}^{\mathrm{e}} = -\mathrm{d}\varepsilon_{III}^{\mathrm{e}}, \\ \kappa = 0: \ \mathrm{d}\varepsilon_{III} - \mathrm{d}\varepsilon_{III}^{\mathrm{e}}, \end{cases} \quad (23)$$

while the elastic strain increments can be obtained from Hooke's law (7). It should be noted here that in the case of a uniaxial stress state $(\sigma_{II} = \sigma_{III} = 0)$, Eqs. (17) and (18) can be reduced to

$$-\frac{\mathrm{d}\varepsilon_{II}^{\mathrm{p}}}{\mathrm{d}\varepsilon_{I}^{\mathrm{p}}} = \frac{(1/2)-(\alpha/3)^2}{1+(\alpha/3)^2} \quad .^{[1]} (24)$$

However, Eq. (24) is difficult to apply in experimental practice for the determination of α since the lateral strains are extremely hard to measure.

In the hardening theory of plasticity, the hardening parameter in the yield criterion must be related to the experimental uniaxial stress-strain curve. To this end, one needs to define a stress variable, called effective stress, which is a function of the stresses and some strain variables, called effective strain, which is a function of the plastic strains, so that they can be plotted and used to correlate the test results obtained by different loading conditions.[11] Since the effective stress should reduce to the stress σ_I in a uniaxial test, it follows that the function $f = g_1(\alpha) \cdot f_1(J_1^{\mathrm{o}}) + f_2(J_2^{'})$ must be some constant c multiplied by the effective stress σ_{eff} to some power n

$$f(\sigma_{ij}, \varepsilon_{\mathrm{eff}}^{\mathrm{p}}) = c\sigma_{\mathrm{eff}}^{\mathrm{n}}. \quad (25)$$

For the uniaxial test with $\sigma_{\mathrm{eff}} = \sigma_I$, $\sigma_{II} = \sigma_{III} = 0$ and $J_1^{\mathrm{o}} = \sigma_I; J_2^{'} = \frac{1}{3}\sigma_I^2$, coefficient comparison, i.e.

$$g_1(\alpha) f_1(\sigma_I) + f_2(\sigma_I) = c\sigma_I^n, \quad (26)$$

gives the parameters c and n. The effective plastic strain increment $\mathrm{d}\varepsilon_{\mathrm{eff}}^{\mathrm{p}}$ can be defined in terms of the plastic work per unit volume in the form

$$\mathrm{d}w^{\mathrm{p}} = \sigma_{\mathrm{eff}} \mathrm{d}\varepsilon_{\mathrm{eff}}^{\mathrm{p}} = \sigma_{ij} \mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \sigma_I \mathrm{d}\varepsilon_I^{\mathrm{p}} + \sigma_{II} \mathrm{d}\varepsilon_{II}^{\mathrm{p}} + \kappa\sigma_{III} \mathrm{d}\varepsilon_{III}^{\mathrm{p}}. \quad (27)$$

It follows from Eq. (27) using Eq. (7) and the results from Eq. (26) that in the case of the axial compaction the effective plastic strain increment is given by the following equation

$$\begin{aligned} \mathrm{d}\varepsilon_{\mathrm{eff}}^{\mathrm{p}} = \tfrac{1}{\sigma_{\mathrm{eff}}} \Big[& \sigma_I \mathrm{d}\varepsilon_I^{\mathrm{p}} - \sigma_{II} \tfrac{1}{E}\left(-\nu \mathrm{d}\sigma_I + \mathrm{d}\sigma_{II} - 2\kappa\nu \mathrm{d}\sigma_{III}\right) \\ & -\kappa\sigma_{III} \tfrac{1}{E}\left(-\nu \mathrm{d}\sigma_I - \nu \mathrm{d}\sigma_{II} + \mathrm{d}\sigma_{III}\right) \Big]. \end{aligned} \quad (28)$$

It should be mentioned here that the elastic range is independent of the yield criterion and the following incremental relation can be derived for the uniaxial strain case:

$$\frac{\mathrm{d}\sigma_I}{\mathrm{d}\varepsilon_I} = K + \frac{4}{3}G, \quad (29)$$

where $K + \frac{4}{3}G$ is known as the constrained modulus. The last equation may play an important role for the experimental determination of the elastic material parameters of cellular materials. If Young's modulus E is obtained from a uniaxial tensile or compression test, then Poisson's ratio ν can be calculated from the slope $\frac{\mathrm{d}\sigma_I}{\mathrm{d}\varepsilon_I}$ in the elastic range of an axial compression test. Using the well-known relationships between elastic constants (e.g. [11]), Eq. (29) can be rewritten as:

$$\frac{\mathrm{d}\sigma_I}{\mathrm{d}\varepsilon_I} = \frac{E}{3(1-2\nu)} + \frac{4}{3}\cdot\frac{E}{2(1+\nu)}. \quad (30)$$

The plane strain test reveals also an interesting relationship for the elastic range. Introducing $\varepsilon_{II}^{\mathrm{e}} = 0$ and $\sigma_{III} = 0$ into Hooke's law, a new equation for the determination of Poisson's ratio is obtained:

$$\nu = \frac{\sigma_{II}}{\sigma_I}. \quad (31)$$

It should be highlighted here that the application of Eq. (31) requires only force measurement to determine the stresses and is independent of any strain measurement.

3. Finite Element Approach

A foamed material which exhibits a certain number of cells in every direction is usually regarded as isotropic and the microstructure of this material can be homogenised. Accordingly, the finite element models can be assembled by solid elements, simulating the macroscopic behaviour of the microstructure.[12,13] In the scope of the finite element simulation, the mechanical properties of the commercial aluminium foam Alporas® (cf. Tab. 2) are applied. Poisson's ratio is calculated from the initial α(0) as shown in [14].

Relative Density	$\rho_{\mathrm{rel}} = 8.4$ %
Young's Modulus	$E = 1100$ N/mm^2
Poisson's ratio	$\nu = 0.13$
Yield stress (ideal plasticity) σ_{y}	$= 1.6$ N/mm^2
Initial α[1]	$\alpha(0) = 2.08$

Table 2 Mechanical properties of the aluminium foam Alporas®; manufacturer information GLEICH GmbH, [1].

The evolution of the plasticity parameter α in dependence of the equivalent plastic strain $\varepsilon_{\text{eff}}^{\text{p}}$ is described by the polynomial

$$\alpha(\varepsilon_{\text{eff}}^{\text{p}}) = 0.1242 + 0.32046 \cdot 10^{-5} / (\varepsilon_{\text{eff}}^{\text{p}} + 0.0001) + 0.103884 \cdot 10^{-10} / (\varepsilon_{\text{eff}}^{\text{p}} + 0.0001)^2 . \quad (32)$$

Two different models are generated, simulating a plane strain state and uniaxial compaction (cf. Fig. 2).

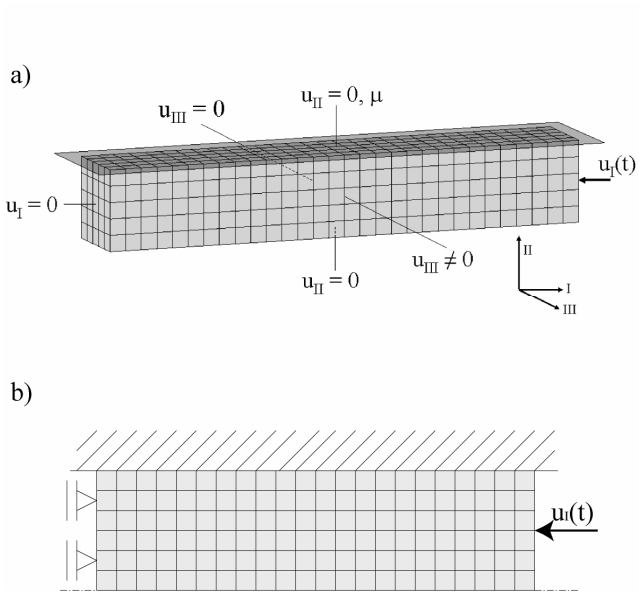

Fig. 2 Finite element models: a) plane strain state; b) uniaxial compaction.

In order to investigate the influence of friction, the constraint in the positive *II*-direction (plane strain), respectively axial direction (uniaxial compaction) is modelled by means of a stiff contact body. The created tribological system is described by the friction coefficient μ (stick-slip friction model). Two different coefficients for aluminium/steel contact are considered ($\mu = 0.01$ and $\mu = 0.053$ for the lubricant LP308).[15] For post processing, macroscopic stress and strain values are calculated by summing up the nodal values of a surface (nodal force or displacement) and dividing the result by the area of the surface, respectively corresponding length of the specimen. This operation is performed by a user-defined Fortran subroutine.

4. Results

According to Eq. (19), two different stress states, namely plane strain and uniaxial compaction, can be utilised to determine the plasticity parameter α based on the macroscopic strains and stresses (respective curves are labelled FEM) in dependence on the equivalent plastic strain. The results are visualised in Fig. 3. The evaluated parameters α are plotted together with the input polynomial defining the behaviour of the homogenised elements. As expected, the curves coincide disregarding minor deviations due to the numerical approximation.

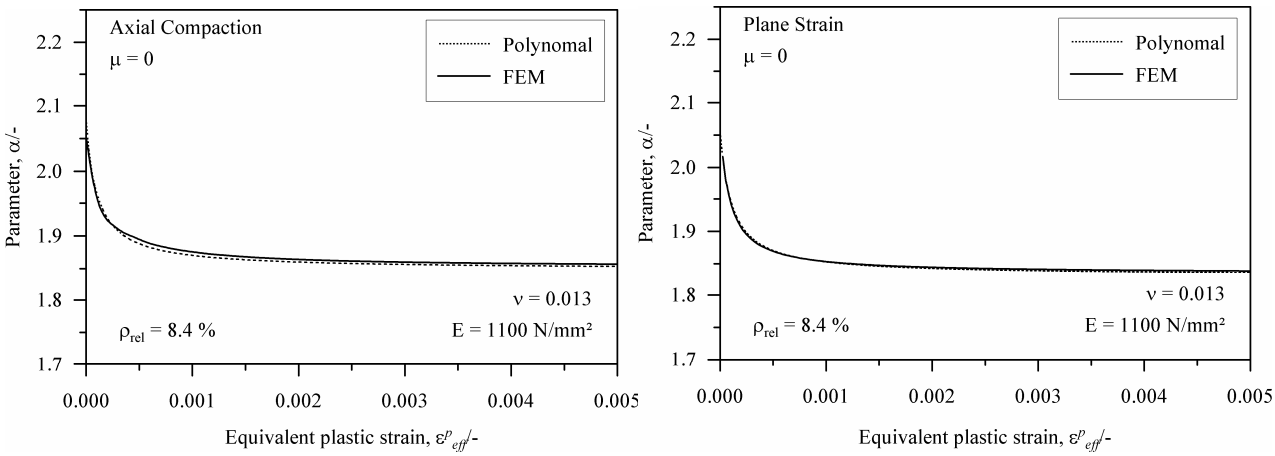

Fig. 3 Material parameter α in dependence on the equivalent plastic strain.

The practical realisation of the suggested experimental procedure introduces unavoidable effects which affect the evaluated values. Therefore, the influence of friction between the specimen and the test fixture on the measured value is exemplarily investigated. Two different values of the friction coefficient μ are considered (cf. Fig. 4). With increasing friction coefficient, parameter α is shifted to higher values for increasing equivalent plastic strain. This effect is slightly less emphasised in the case of the plane strain state. Exemplarily, at $\varepsilon^{p}_{eff} = 0.001$ the deviation to the exact solution (Polynomal) amounts 1.6 % for $\mu = 0.01$ and 2.73 % for $\mu = 0.053$. This deviation increases to 2.54 % ($\mu = 0.01$) respectively 3.67 % ($\mu = 0.053$) at $\varepsilon^{p}_{eff} = 0.005$. One possible explanation of this phenomenon is the smaller contact surface formed by specimen and fixture,

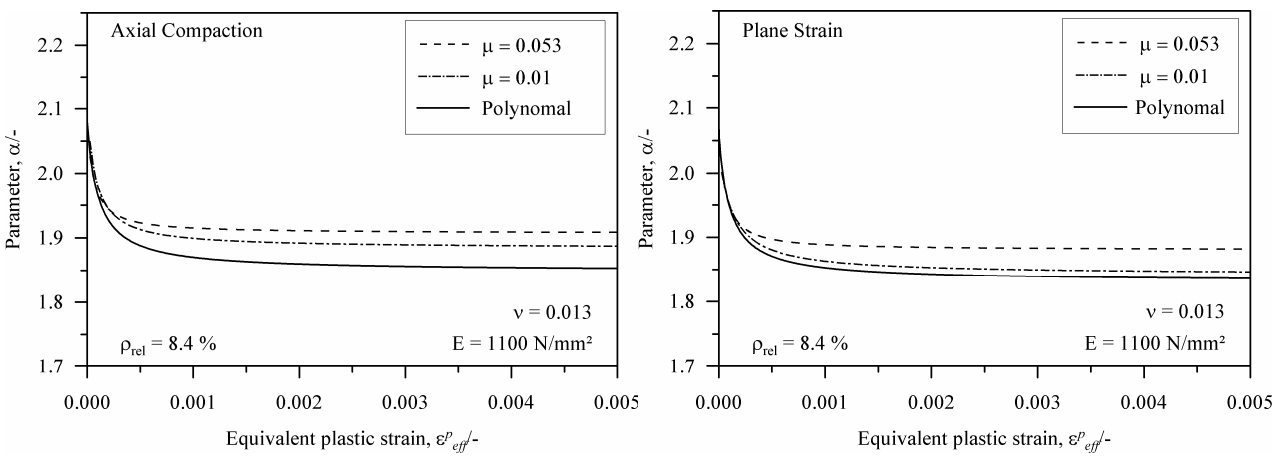

Fig. 4 Influence of friction on the material parameter α.

normalised by the volume of the specimen, of the plane strain experiment compared to the case of uniaxial compaction. Consequently, for an accurate determination of α, the friction in the contact zone must be minimised, e.g. by the application of appropriate lubricants. Furthermore, the experimental setups allow for the determination of a second elastic constant, e.g. Poisson's ratio. Figure 5 visualises the slope of the stress σ_I in dependence on σ_{II}. Inside the elastic region, the graph exhibits a constant

gradient and Eq. (31) is valid. The result of this equation in the considered case is similar to the input value of Poisson's ratio of the FE model ($\nu = 0.013$). It should be highlighted that the determination of the elastic constant requires only the measurement of stresses, no strain measurement is required.

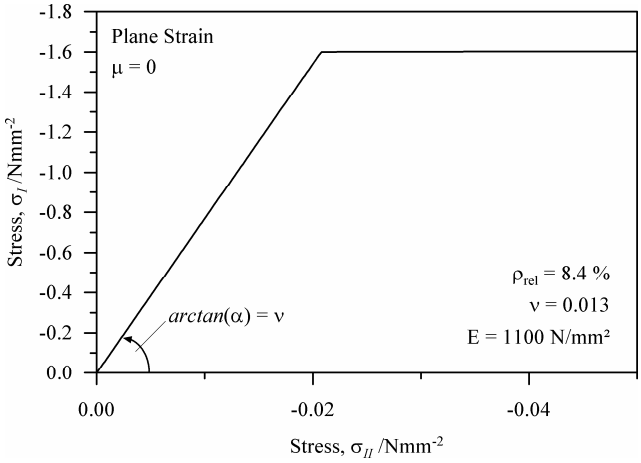

Fig. 5 Evaluation of the elastic constant ν.

5. Remarks on Experimental Realisation

The experimental realisation of a plane strain test requires for example a biaxial testing machine as discussed in [16]. Such an experimental setup results in a extremely homogeneous stress state and the required measuring values, i.e. σ_I, σ_{II} and ε_I, can be easily determined. However, standard equipment of testing laboratories does normally not comprise such a machine. Therefore, the uniaxial strain test is an interesting alternative. Realisation of this test should be based on cylindrical specimens $(I \rightarrow z, II \rightarrow r, III \rightarrow \varphi)$ for a direct measurement of σ_I and ε_I. A direct measurement of $\sigma_r = \sigma_\varphi$ is not possible. Instead of this, the peripheral strain ε_φ on the outer surface of the tube might be obtained with the aid of strain gauge measurements. Based on this value, analytic equations (e.g. [17]) for a thick-walled cylinder allow for the determination of the internal pressure p_i, which is identical to the radial stress of the specimen σ_r for $r = r_i$ (r_i is the inner radius of the tube).

6. Conclusion

In the scope of this article, the applicability of two experimental procedures is numerically analysed. Furthermore, the influence of friction is investigated and the necessity of the minimisation of friction is shown. In the case of a plane strain experiment, a lower influence of friction, caused by a smaller normalised contact zone, can be observed. However, the biaxial testing machines, required for realisation of this strain state, are not available in most laboratories and the axial compaction might be an interesting

alternative. In any case, the application of lubricants is recommended. Finally, the proposed experiments provide the possibility for the determination of a second elastic constant, e.g. Poisson's ratio.

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