A SEMI-ANALYTICAL SOLUTION FOR SWELLING AROUND A BOREHOLE

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SUMMARY

Previous field and laboratory studies have shown that the creation of a borehole in a saturated cohesive soil mass induces significant pore water suction in the vicinity of the hole. The dissipation of these pore water suction (swelling) leads to local increases in the moisture content and hence a softening of the soil around the hole. This softening may have important consequences for the stability of the hole and also for the ultimate load capacity of any foundation elements (bored piles or drilled shafts) constructed in these holes. This paper presents a semi-analytical solution for the radial dissipation of pore water pressure around a freshly created, vertical hole. It is assumed the soil deforms elastically during the swelling process. The solutions are presented in the form of isochrones of excess pore water pressure and may be used to obtain estimates of the time required for the soil around the hole to swell and therefore to soften. Both permeable and impermeable borehole interfaces have been considered, together with either no support for the hole or partial support provided by hydrostatic pressure within the hole.

INTRODUCTION

When a vertical shaft or borehole is drilled into a deposit of saturated clay, significant changes to the pore water pressures may occur in the region close to the hole. Initially, pore water pressure reductions will occur in response to the reduction of the total stress acting across the cylindrical wall of the opening. Once drilling is complete, the excess pore water suction will gradually dissipate, allowing the soil around the hole to increase its voids ratio by taking up any available free water. During this swelling process the strength and stiffness of the soil will decrease and, if the hole is not supported, collapse (or closure) of the hole may result. Even if the hole is filled with wet concrete, to form a ‘cast-in-place’ pile, some swelling may also occur and thus the final strength of the soil immediately adjacent to the pile will be somewhat less than the strength of the undisturbed deposit.

This swelling and softening phenomenon has been observed in field and laboratory tests by a number of investigators (see, for example, References 1–4). Meyerhof and Murdock1 excavated a bored pile in London clay and discovered that the water content of the soil next to the shaft was 2–7 per cent higher than the natural water content of the clay and that the softened zone extended to a radius of about \( r = 1.33r_0 \), where \( r_0 \) was the radius of the bored pile (pile radius = 6 in., thickness of softened zone = 2 in. approx). In a series of careful laboratory experiments, Beech and Kulhawy4 measured the changes in pore water pressure around model drilled shafts. These tests indicated that significant pore water suction were induced in the clay around the hole immedia-

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tely after it was augered and that these suctions eventually dissipated after the placement of concrete in the hole to form the shaft. Beech and Kulhawy also measured the final moisture content of the soil around the model shafts, but no clear trend was observed. However, the initial water contents of the various deposits used in their tests were all well above the plastic limit and, as they and others (see, for example, References 5 and 6) have pointed out, the greatest increase in soil moisture will occur at a liquidity index of approximately zero.

In this paper a semi-analytical solution is presented for the process of consolidation (most of the soil will in fact swell) around a vertical borehole drilled into saturated clay. Closed form expressions are found for the field quantities, but their evaluation requires some numerical integration. To obtain this solution a number of simplifying assumptions have to be made. The major assumptions are listed below.

1. The creation of the borehole occurs rapidly and so the soil behaves in an undrained manner during borehole drilling.
2. During the subsequent consolidation the soil skeleton deforms elastically.
3. The flow of pore water during consolidation is governed by Darcy's law.
4. The most significant gradients of pore water pressure occur in the radial direction and so vertical flow of pore water is ignored.
5. Similarly, only radial movements of the soil skeleton occur during consolidation. Thus the deformations occur under conditions of plane strain and axial symmetry.
6. The distribution of excess pore water pressure immediately after drilling is known, i.e. it can be obtained independently of the consolidation analysis.
7. The entire soil body surrounding the hole remains saturated during the consolidation/swelling process.

Assumptions 4, 5 and 7 considerably reduce the complexity of the problem, in that the behaviour of the soil can be described in terms of only one spatial co-ordinate, i.e. radius \( r \). Further discussion of assumption 2 is given later in the paper.

The problem treated in this paper is of course an extreme idealization of the real situation. Nevertheless, it does allow an uncluttered look at the basic physical processes in operation in this type of problem and provides an assessment of the likely severity of the various effects, e.g. shear strength of the soil, support conditions in the hole, and permeability of the soil. An additional benefit of the solution is its potential for use as a benchmark solution for checking more complex numerical procedures. The latter may be used to solve problems of this type where some or many of the restricting assumptions adopted here are relaxed.

The analytical techniques used here could also be applied to the problem of consolidation around a driven pile or an expanded pressuremeter in clay. However, these problems have been solved previously by Randolph and Wroth \(^7\) using an alternative procedure, and thus the driven pile and pressuremeter applications are not pursued here.

GOVERNING EQUATIONS

It is well known that the consolidation of a saturated, elastic soil under conditions of plane strain and axial symmetry is governed by the diffusion equation (see, for example, Reference 7), namely

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - \frac{1}{c} \frac{\partial p}{\partial t} = 0
\]  

for \( r \geq 0 \) and \( t > 0 \), where \( p \) is the excess pore water pressure at time \( t \) and radius \( r \). Compression is considered to be positive. The symbol \( c \) is used to represent the coefficient of consolidation which,
for an elastic soil, is given by

\[ c = 2G \left( \frac{1 - \nu}{1 - 2\nu} \right) \left( \frac{k}{\gamma_w} \right) \]  

(2)

where \( v \) and \( G \) are the Poisson's ratio and the elastic shear modulus of the soil skeleton, \( k \) is the permeability coefficient, and \( \gamma_w \) is the unit weight of the pore fluid.

Taking Laplace transforms of equation (1) gives

\[ \frac{\partial^2 \bar{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{p}}{\partial r} - \frac{q^2 \bar{p}}{c} = -\frac{p_0}{c} \]  

(3)

where

\[ q^2 = \frac{s}{c} \]

\[ \bar{p} = \int_0^\infty e^{-st} p \, dt \]

\[ p_0 = \text{the excess pore water pressure distribution at time } t=0, \text{ immediately after the hole is created.} \]

The initial condition for the problem is defined by \( p_0 \) which is a function of \( r \) only. The precise form of the function \( p_0 \) will depend on the type of problem, e.g. driven pile or bored pile, and this is discussed later.

The solution of equation (2) is expressible in terms of modified Bessel functions \( I_0 \) and \( K_0 \), namely

\[ \bar{p} = A_1 K_0(qr) + A_2 I_0(qr) + P \]  

(4)

where \( P \) is a particular integral of equation (3), whose precise form depends on \( p_0 \). The coefficients \( A_1 \) and \( A_2 \) are determined from the boundary conditions of the problem.

**INITIAL CONDITION**

For the problem of a borehole or a bored pile of radius \( r_0 \), it is possible to identify a number of different initial conditions for the consolidation process, which may have practical significance. In all cases examined here it has been assumed that, prior to the drilling of the vertical hole, the in situ stress conditions in the soil at any given horizon are as follows:

- \( \sigma'_v \) = the vertical effective stress
- \( K_0 \sigma'_v \) = the horizontal effective stress (the same in all horizontal directions)
- \( p_i \) = the initial pore water pressure

\( K_0 \) is the coefficient of earth pressure at rest. In some overconsolidated soils, negative values of the ambient pore water stress \( p_i \) may exist, especially near the surface, but it is assumed that these pore water stresses are in equilibrium and thus increase linearly with depth along a hydrostatic gradient. Furthermore, it is assumed that during the drilling of the borehole the surrounding soil behaves in an undrained manner, with a constant undrained shear strength \( c_u \) and an elastic shear modulus \( G \).

As the total radial stress acting across the cylindrical boundary of the borehole is removed, the soil will respond elastically at first, until its shear strength is reached. As the total stress is further reduced, plastic yielding will occur and this will be contained within an annular zone around the borehole. The soil within this zone then behaves as an elastic, perfectly plastic material. Once
yielding occurs, excess pore water suctions are generated. In contrast to this, in the elastic region beyond the plastic interface, unloading causes no change to the mean total stress and so no excess pore water pressures are generated.

Consider the general case where some support is provided to the borehole in the form of a hydrostatic pressure, and specifically, let us assume that at the end of construction and during the subsequent period of consolidation the total stress acting across the cylindrical surface of the hole has a magnitude of \( \lambda |p_i| \), where \( \lambda \) is a positive quantity. Thus \( \lambda = 0 \) corresponds to the case of an unsupported hole and when \( \lambda = 1 \) the support pressure is equal in magnitude to the original \textit{in situ} pore water stress. The special case where \( \lambda = 1 \) and \( p_i > 0 \) corresponds to a supported hole with zero excess pore pressure \( (p=0) \) and zero effective radial stress at the boundary \( r = r_0 \), for \( t > 0 \).

Hence the overall change in total radial stress at \( r = r_0 \), resulting from borehole creation and support, is

\[
\Delta \sigma_r = -K_0 \sigma_r' - p_i + \lambda |p_i| 
\]

During unloading of the cavity wall, yielding of the soil may occur; as long as the largest shear stress occurs in the horizontal plane, the assumption of axisymmetric, plane strain deformation remains valid. It is shown in Appendix I that, for yielding to be initiated in the horizontal plane, the following conditions must apply:

\[
-1/2 \leq f \leq 1/2
\]

where \( f = (1 - K_0)/(2c_w/\sigma_v) \) is the initial shear stress ratio defined by D'Appolonia \textit{et al.} \(^8\) Excess pore water suctions will be generated once yielding commences. At the point where the support pressure is equal to \( \lambda |p_i| \) the initial distribution of excess pore water stress is given by (see Appendix I):

\[
p_0(r) = 2c_u \ln \left( \frac{r}{r_0} \right) - \sigma_R + \lambda |p_i| \quad r_0 \leq r \leq R
\]

\[
p_0(r) = 0 \quad r > R
\]

where

\[
\sigma_R = K_0 \sigma_r' + p_i - c_u
\]

= the total radial stress at first yield

The radius of the interface between elastic and plastic soil at this stage is given by

\[
R = r_0 \exp \left( \frac{\sigma_R - \lambda |p_i|}{2c_u} \right)
\]

It is obvious from equation (7d) that plastic yielding will occur only when

\[
\sigma_R > \lambda p_i
\]

or (using 7c),

\[
K_0 + \left( \frac{p_i}{\sigma_v'} \right) - \left( \frac{\lambda |p_i|}{\sigma_v'} \right) - \left( \frac{c_u}{\sigma_v'} \right) > 0
\]

**BOUNDARY CONDITIONS**

From equation (7a) it can be seen that the initial condition for the consolidation problem is dependent on the plastic radius \( R \). In the annular zone of yielded material around the borehole,
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$r_0 \leq r \leq R$, the initial excess pore pressure distribution is non-trivial. Thus within this region the solution for $\bar{p}$ has the general form of equation (4).

However, in the region $r > R$, the initial condition is trivial, i.e. $p_0(r)=0$, and since the solution must remain bounded as $r \to \infty$, the Laplace transform of the transient pore water pressure must take the form

$$\bar{p} = A_3 K_0(qr), \quad r > R$$

(8)

The constants $A_1, A_2$ and $A_3$ of equations (4) and (8) are determined from the continuity and boundary conditions. Both $p$ and $\frac{\partial p}{\partial r}$ must be continuous across the interface $r = R$ and the hydraulic condition at the borehole $r = r_0$ must be specified. If the excess pore water pressure at $r = r_0$ is maintained at a value $p = p_c$ for $t > 0$, then $A_1, A_2$ and $A_3$ are determined from the solution of

$$
\begin{bmatrix}
K_0(qr_0) & I_0(qr_0) & 0 \\
K_0(qR) & I_0(qR) & -K_0(qR) \\
-K_1(qR) & I_1(qR) & K_1(qR)
\end{bmatrix}
\begin{pmatrix}
A_1 \\
A_2 \\
A_3
\end{pmatrix}
= \begin{pmatrix}
-P(r_0) + \left(\frac{1}{s}\right)p_c \\
0 \\
-\left(\frac{1}{q}\right)\phi(R)
\end{pmatrix}
$$

(9)

The functions $\phi$ and $P$ appearing in equation (9) are defined as

$$
\phi(r) = \frac{\partial p}{\partial r} \quad \text{and} \quad P(r) = p_0(r)/s
$$

For the general case of a supported borehole with a permeable interface, the excess pore pressure, $p_c$, at $r = r_0$ is given by

$$p_c = \lambda |p| - p_i$$

Alternatively, if the interface at $r = r_0$ is impermeable, then $A_1, A_2, A_3$ can be obtained from

$$
\begin{bmatrix}
-K_1(qr_0) & I_1(qr_0) & 0 \\
K_0(qR) & I_0(qR) & -K_0(qR) \\
-K_1(qR) & I_1(qR) & K_1(qR)
\end{bmatrix}
\begin{pmatrix}
A_1 \\
A_2 \\
A_3
\end{pmatrix}
= \begin{pmatrix}
\frac{-1}{q}\phi(r_0) \\
0 \\
-\frac{1}{q}\phi(R)
\end{pmatrix}
$$

(10)

INVERSION OF TRANSFORMS

The solution of either equations (9) or (10) gives the constants $A_1, A_2, A_3$ and these may be substituted into either equation (4) or (8) to obtain the Laplace transform of the excess pore water pressure. Inversion of the transforms is then carried out by numerical integration using the efficient scheme devised by Talbot.9

APPLICATION

It was indicated in the sections above that at least two conditions must be met for this theoretical analysis to be valid, namely relations (6) and (7e). Data presented by Ladd et al.10 give a guide to
Figure 1. Relationship between initial shear stress ratio and OCR (after Ladd et al.)

The type of clay soils in which these conditions are likely to be found. Figure 1 shows the relationship between the initial shear stress ratio $f$ and the overconsolidation ratio (OCR) for six different clays, covering a range from low to high plasticity. For these soils it can be noted that condition (6) will almost always be met whenever OCR is greater than about 1.5, and will often be met at smaller values. Data of the type shown in Figure 2 can be used to estimate the ratio $c_u/\sigma_{vc}$ which, together with a value of $K_o$, allows a determination of $K_o$. From this type of information it is found that condition (7e) is also met whenever $K_o$ is greater than about 1.5 or perhaps at even smaller values. Hence the solutions presented here for the initial excess pore water pressures around a borehole are likely to be valid for all but possibly normally and very lightly overconsolidated clays. Bored piles are more commonly used in overconsolidated clays and thus conditions (6) and (7e) should hold in many practical situations.

**EXAMPLES**

Isochrones of excess pore water pressures have been evaluated for four different cases and these correspond to both permeable and impermeable boreholes, which are either unsupported ($\lambda = 0$) or supported by groundwater pressure ($\lambda = 1$). The cases considered and their appropriate pore water pressure boundary conditions are summarized in Table I. It should be noted that, although cases I and II correspond to a 'dry' borehole, it has been assumed that all the soil around the hole remains saturated and that no desiccation occurs. Cases II and IV, where the borehole is assumed
impermeable, have been included here to provide some assessment of the influence of a relatively impermeable smear zone at the cylindrical surface of the hole.

Isochrones of excess pore water pressure have been evaluated for two soil types. In the first, designated as type (a), it has been assumed that \(|p|/\sigma_v' = 1\), \(c_u/\sigma_v' = 0.4\), \(K_0 = 0.8\), while in the second, designated as type (b), it is assumed that \(|p|/\sigma_v' = 1\), \(c_u/\sigma_v' = 1.2\), and \(K_0 = 1.6\). Reference to Figures 1 and 2 indicates that the first set of values is typical of a clay soil with OCR ~1.5 to 2, while the second is typical of a clay with OCR ~8.

The results of the consolidation analyses for the four sets of boundary and initial conditions and the two different soil types are plotted in Figures 3–10, where isochrones of excess pore pressure have been plotted for each of the cases considered. Figures 3–6 show the results for a
Figure 3. Isochrones of excess pore pressure

Soil Type (a)

\[ \frac{\varphi}{\sigma'_{v0}} = 1, \quad \frac{\varphi}{\sigma'_{v0}} = 0.4, \quad \varphi = 0.8 \]

No support, zero total pore water pressure at borehole (Case I)

Figure 4. Isochrones of excess pore pressure

Soil Type (b)

\[ \frac{\varphi}{\sigma'_{v0}} = 1, \quad \frac{\varphi}{\sigma'_{v0}} = 12, \quad \varphi = 16 \]

No support, zero total pore water pressure at borehole (Case I)
completely unsupported hole ($\lambda = 0$) and these apply to the case where the original pore water stress may be either compressive ($p_i > 0$) or in suction ($p_i < 0$). Figures 7–10 show results for a hole supported by a fluid which, at any depth, exerts a pressure equal in magnitude to the original pore water stress ($\lambda = 1$). For the special case of a permeable borehole, this means that the results in Figures 7 and 8 apply only to the case where $p_i > 0$, i.e. the ambient pore water stress is compressive.
For the case of an unsupported hole with zero pore water pressure at the boundary (Figures 3 and 5), the isochrones indicate that the flow of pore water always occurs in the direction away from the hole, i.e. the hole is acting as a source mechanism. Thus, these solutions will remain valid only if sufficient water is available at the interior boundary of the soil. The solutions also indicate that significant swelling of the soil continues around the hole for long periods, although in terms
of non-dimensional time the dissipation process is 'faster' in soil type (b) than in type (a), cf. Figures 4 and 5. At large times the excess pore water pressures approach a uniform, non-zero distribution, indicating that the actual pore water pressures are approaching zero values.

If the interior boundary of the soil is completely impermeable, then both the transient and steady-state flow patterns are quite different from those described above. The soil immediately around the hole swells, but the flow of water is inwards from outer regions (see Figures 5 and 6). In this case the excess pore water pressures completely dissipate at large time, and this corresponds to a return to the ambient pore water pressure conditions. Again, the swelling occurs 'faster' for the soil type (b) than for (a).

Isochrones for various cases for which some support for the hole is provided are plotted in Figures 7-10. The results for boreholes considered to be permeable are shown in Figures 7 and 8. The interior boundary condition in each of these cases was zero excess pore water pressure for \( t > 0 \), and this implies continuity (at least in terms of pressure) of the pore water and the fluid medium providing support within the hole. It can be noted that the magnitude of the initial excess pore water pressures at all radii are smaller than for the corresponding cases of an unsupported hole. (The fact that, in the case of the supported holes, the initial excess pore water pressure at \( r = r_0 \) is the same for both soil types is merely fortuitous, and a result of the selection of parameter values.) Providing some support for the hole also reduces the size of the zone of yielded soil but, whether support is provided or not, this plastic zone is more extensive in soil type (a) than in type (b). Some swelling occurs even for the supported holes, and in general the process is 'quicker' in soil type (b) than in (a). For the impermeable hole, the flow of water into the swelling zone can only occur in the inward direction, but for the permeable hole, flow occurs in both radial directions.

**STRESS PATHS**

One of the important assumptions made in this analysis was that the soil behaves elastically during swelling. It was necessary to make such an assumption so that an analytical solution for the Laplace transforms could be obtained. As the moisture content of the soil increases, the mean effective stress \( \sigma'_{\text{me}} \), defined as

\[
\sigma'_{\text{me}} = \frac{1}{3}(\sigma'_r + \sigma'_\theta + \sigma'_z)
\]

will decrease. However, attention should also be focused on the changes in the deviator stress \( \sigma_d \), during the swelling process. This stress component is defined here as

\[
\sigma_d = \left( \frac{1}{\sqrt{2}} \right) \sqrt{\{(\sigma'_r - \sigma'_\theta)^2 + (\sigma'_\theta - \sigma'_z)^2 + (\sigma'_z - \sigma'_r)^2\}}
\]

where it is assumed that \( \sigma'_r, \sigma'_\theta \) and \( \sigma'_z \) are all principal stress components. It is a fairly simple operation to show that, if the soil behaves elastically during swelling, then at the edge of the borehole (at least) the deviator stress must increase as the mean effective stress decreases. However, the soil at the edge of the hole already will have reached its maximum shear strength during the undrained unloading of the hole, and thus during the swelling process a further increase in deviator stress with a decrease in mean stress is not physically possible. Thus the assumption that a material with finite shear strength deforms elastically during swelling around a borehole is not strictly valid.

However, this does not necessarily invalidate the use of the analytical solutions presented here for the prediction of the rate of swelling around a borehole. If much of the soil around the hole...
continues to behave plastically during the swelling, then its volumetric stiffness, and hence the coefficient of consolidation $c$, will be less than for an elastic material. It is suggested that these solutions may still provide useful predictions if used together with an appropriate choice of the coefficient of consolidation. A similar situation exists in the problem of consolidation around a driven pile. Randolph \textit{et al.} have shown that reliable use may be made of solutions for elastic consolidation in a problem where plastic deformations (work hardening) actually occur as soil consolidates back around a driven pile.

**CONCLUSIONS**

A semi-analytical solution has been presented for the swelling of soil around a borehole, based on radial flow of pore water and radial movement of soil particles (plane strain). The soil skeleton has been assumed to deform elastically during the swelling process.

These solutions may be used to obtain estimates of the time required for the soil around the hole to swell and therefore to soften. These times will depend on the hydraulic and stress boundary conditions assumed in the problem. Both a permeable and an impermeable borehole interface have been considered, together with either no support for the hole or partial support provided by hydrostatic pressure within the hole. In all cases it has been assumed that the soil remains saturated during the swelling. The analytical results have been presented in the form of isochrones of excess pore water pressure around the hole for two clay soils with different consolidation histories.

Prediction of effective stress changes around the hole have not been given because a more sophisticated soil model is required for realistic prediction of these quantities. The use of such a model would almost certainly also require numerical solution techniques.

**APPENDIX I**

In this appendix a solution is derived for the excess pore water pressures induced in a cohesive soil, immediately following the creation of a long cylindrical cavity. It is assumed that the axis of the hole coincides with the vertical co-ordinate axis and that only constant volume, radial deformations occur, i.e. conditions of plane strain and axial symmetry are assumed, and the soil material behaves in an undrained manner.

Prior to the creation of the vertical cylindrical hole, the state of effective stress in the ground at any depth can be described in cylindrical components by

$$
\sigma'_r = K_0 \sigma'_v \\
\sigma'_\theta = K_0 \sigma'_v \\
\sigma'_z = \sigma'_v
$$

(13)

where $\sigma'_v$ is the effective overburden pressure and $K_0$ is the coefficient of earth pressure at rest. The initial pore water stress in the ground at the same depth is $p_i$, so that the components of total stress are

$$
\sigma_r = K_0 \sigma'_v + p_i \\
\sigma_\theta = K_0 \sigma'_v + p_i \\
\sigma_z = \sigma'_v + p_i
$$

(14)

In some clay deposits, particularly near the surface, the ambient pore water stress may actually be a suction, i.e. $p_i < 0$ is possible.
The instantaneous creation of the hole may be modelled reasonably as the removal of total radial stress acting across the boundary \( r = r_0 \). Initially the soil responds as an undrained elastic material, so that the changes in total stress and excess pore water pressure are as follows:

\[
\begin{align*}
\Delta \sigma_r &= - \Delta \sigma_\theta \\
\Delta \sigma_z &= 0 \\
\Delta p &= 0 
\end{align*}
\]  

(15)

Yielding of the soil will occur whenever the difference between the major and minor principal stress is equal to twice the undrained shear strength. Adopting the convention that compressive stresses are positive, the yield condition may be written as

\[ \sigma_\theta - \sigma_r = 2c_u \]  

(16)

It is important to realize that (16) will only be valid as long as the following conditions hold:

\[ \sigma_r \leq \sigma_z \leq \sigma_\theta \]

Thus, for yield to first occur in the horizontal plane, this requires

\[ K_0 \sigma'_v + p_i + \Delta \sigma_r \leq \sigma'_v + p_i \leq K_0 \sigma'_v + p_i + \Delta \sigma_\theta \]

or

\[ \Delta \sigma_r \leq (1 - K_0)\sigma'_v \leq \Delta \sigma_\theta \]  

(17)

If condition (17) is met, then the stress changes at first yield are given by

\[ \Delta \sigma_\theta = - \Delta \sigma_r = c_u \]  

(18)

and thus the total radial stress at first yield is

\[ \sigma_R = K_0 \sigma'_v + p_i - c_u \]  

(19)

When equation (18) is substituted into (17), the requirement that yield first occurs in the horizontal \((r - \theta)\) plane becomes

\[ -\frac{1}{2} \leq (K_0 - 1) \left( \frac{\sigma'_v}{2c_u} \right) \leq \frac{1}{2} \]  

(20)

As \( \sigma_r \) is reduced below the value \( \sigma_R \), the yielded zone spreads beyond \( r = r_0 \), and the elastic–plastic interface is at \( r = R \). Consider the case where the radial stress at the cavity has been reduced to a magnitude \( \lambda |p_i| \). (The case where \( \lambda = 0 \) corresponds to a completely unsupported borehole, while \( \lambda = 1 \) corresponds to support being provided by a fluid pressure equal in magnitude to the original groundwater pressure.)

As the total stress at the cavity boundary is reduced from the first yield stress \( \sigma_R \) to \( \lambda |p_i| \), two conditions must hold within the spreading elastoplastic region; the stresses must obey both the equilibrium and yield conditions, i.e.

\[ \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \]  

(21)

and

\[ \sigma_\theta - \sigma_r = 2c_u \]  

(22)

Substituting equation (22) into (21) and applying the boundary condition \( \sigma_r = \sigma_R \) at \( r = R \), it is
found that

\[ \sigma_r = \sigma_R + 2c_u \ln \left( \frac{r}{R} \right) \]  

(23)

throughout the plastic region \((r_0 \leq r \leq R)\).

When the total radial stress at \(r = r_0\) is reduced to \(\sigma_r = \lambda |p|\), equation (23) gives the size of the plastic zone as

\[ \left( \frac{R}{r_0} \right)^{\frac{\sigma_R - \lambda |p|}{2c_u}} = \exp \left( \frac{\sigma_R - \lambda |p|}{2c_u} \right) \]  

(24)

and the distribution of total radial stress throughout the region \(r_0 \leq r \leq R\) as

\[ \sigma_r = \lambda |p| + 2c_u \ln \left( \frac{r}{r_0} \right) \]  

(25)

The total circumferential stress throughout the plastic region is given by (25) and (22) as

\[ \sigma_\theta = \lambda |p| + 2c_u \left( 1 + \ln \left( \frac{r}{r_0} \right) \right) \]  

(26)

The changes in these stress components are thus determined as

\[ \Delta \sigma_r = 2c_u \ln \left( \frac{r}{r_0} \right) - K_0 \sigma_e + \lambda |p| - p_i \]  

(27)

\[ \Delta \sigma_\theta = 2c_u \left( 1 + \ln \left( \frac{r}{r_0} \right) \right) - K_0 \sigma_e + \lambda |p| - p_i \]  

(28)

The change in the total vertical stress is given by Hooke’s law, with \(v = v_u = 0.5\), as

\[ \Delta \sigma_z = 0.5(\Delta \sigma_r + \Delta \sigma_\theta) \]

\[ = c_u \left( 1 + 2 \ln \left( \frac{r}{r_0} \right) \right) - K_0 \sigma_e + \lambda |p| - p_i \]  

(29)

Thus the change in the mean total stress \(\Delta \sigma_m\) is

\[ \Delta \sigma_m = \frac{1}{3}(\Delta \sigma_r + \Delta \sigma_\theta + \Delta \sigma_z) \]

\[ = c_u \left( 1 + 2 \ln \left( \frac{r}{r_0} \right) \right) - (K_0 \sigma_e - c_u) + \lambda |p| - p_i \]  

(30)

If it is assumed that no consolidation takes place during cavity unloading, then the mean effective stress remains unchanged \((\Delta \sigma'_m = 0)\), and the change in mean total stress is equal to the excess pore water pressure generated during yielding. Hence, at the instant that the total radial stress at \(r = r_0\) first reaches a magnitude of \(\lambda |p|\), the excess pore water pressure distribution around the hole is given by

\[ p = 2c_u \ln \left( \frac{r}{r_0} \right) - \sigma_R + \lambda |p|, \quad r_0 \leq r \leq R \]  

(31)

and

\[ p = 0, \quad r > R \]  

(32)

where \(R\) is given by (24).
REFERENCES