MATHS AND STATS

Solving exponential equations

Equations that involve a variable (or unknown) as a power (exponent or index) are known as exponential equations. For instance,

$$3^x = 20$$

In general, we can't just use addition, subtraction, multiplication, division and taking roots to solve such equations (for the power), we require another operation and that is taking the logarithm (of both sides).

Example

Solve $3^x = 20$ for x.

We should expect our answer to be between 2 and 3 since $3^2 = 9$ and $3^3 = 27$. But to get an accurate answer we apply a log to both sides:

$$\log(3^x) = \log(20)$$

Then we apply that our super handy log law to get:

$$x \log(3) = \log(20)$$

Which means we can now divide both sides by $\log(3)$ to get:

$$x = \frac{\log(20)}{\log(3)}$$

Which we can put into a calculator to get an approximate answer of 2.7268

> We can't forget our standard techniques to solve equations, taking the log is just another tool in our toolbox, a tool that has the useful property $\log(a^x) = x \log(a)$

A calculator gives approximate answers (e.g. 2.7268) but you should always use the exact answer (e.g. $\log(20) / \log(3)$) in further calculations to avoid compounding rounding errors.

Example

Solve $5000 = 1500(1.08)^{4n}$

$$\frac{5000}{1500} = 1.08^{4n} \qquad \text{(dividing both sides by 1500)}$$

$$\log\left(\frac{10}{3}\right) = \log(1.08^{4n}) \qquad \text{(take log of both sides)}$$

$$\log\left(\frac{10}{3}\right) = 4n\log(1.08) \qquad \text{(using log rule C)}$$

$$\frac{\log\left(\frac{10}{3}\right)}{\log(1.08)} = 4n \qquad \text{(dividing both sides by } \log(1.08))$$

$$\frac{\log\left(\frac{10}{3}\right)}{4\log(1.08)} = n \qquad \text{(dividing both sides by 4)}$$

$$n \approx 3.9110 \qquad \text{(4 decimal places)}$$

Exercises

Solve the following equations

a) $50 = 5^n$

b) $1000 = 100(1.02)^n$

c) $2500 = 500(1.005)^n$

d) $1200 = 60(1.007)^{12n}$

e) $4500 = 2000(1.004)^{52n}$

f) $1500 = 1000(1.01)^{4n}$

g) $4000 = 4000(1.05)^{6n}$

h) $10000 = \frac{1000(1.04^n - 1)}{0.04}$

i) $20000 = \frac{600(1.08^n - 1)}{0.08}$

j) $123456 = \frac{789(1.1^n - 1)}{0.1}$

k) $32000 = \frac{2000(1.05^n - 1)}{0.05}$

Answers

a) $n \approx 2.43$

b) $n \approx 116.28$

c) $n \approx 322.69$

d) $n \approx 35.79$

e) $n \approx 3.91$

f) $n \approx 10.19$

g) $n = 0$

h) $n \approx 8.58$

i) $n \approx 16.88$

j) $n \approx 29.51$

k) $n \approx 12.05$