MATHS AND STATS

Inequalities

Inequalities are like equations, but they do not use $=$ to separate the sides. Instead they use:

$<$	less than	$>$	greater than
$\leq$	less than or equal to	$\geq$	greater than or equal to

$x \geq 4$ means that x can either be equal to 4 exactly, or any value above 4.

Equations involving inequalities

As an example, say we are trying to solve the inequality $4x - 3 < 5$

We are after a description of all values of x which satisfy this statement, i.e. that make it true when we substitute the value into the inequality. Our answer will be an inequality describing this set of x values.

If this was an equation, $4x - 3 = 5$, we would solve it as follows:

$$\begin{aligned} 4x - 3 &= 5 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

We do the same for the $<$ version:

$$\begin{aligned} 4x - 3 &< 5 \\ 4x &< 8 \\ x &< 2 \end{aligned}$$

So our answer is $x < 2$, meaning x can be any number below 2 (test some).

For simple equations, we can solve an inequality just as if we were solving a regular equation, with the following rule added:

> If you swap the sides of the equation around, or if you multiply or divide both sides by a negative number, then change the direction of the arrow

This is because:

- If $x < y$, then $y > x$
- If $x < y$, then $-x > -y$

You can test these with some real numbers:

- $5 < 8$ $\quad 8 > 5$
- $4 > 2$ $\quad -4 < -2$

Examples

1)

$$3 + 5x \geq x - 7$$
$$3 + 4x \geq -7$$
$$3x \geq -10$$
$$x \geq -\frac{5}{2}$$

2)

$$2 - 7x < x + 6$$
$$2 - 6 < x + 7x$$
$$-4 < 8x$$
$$-\frac{1}{2} < x$$
$$x > -\frac{1}{2}$$

3)

$$2x \leq 4x + 18$$
$$-2x \leq 18$$
$$x \geq \frac{18}{-2}$$
$$x \geq -9$$

4)

$$-\frac{x}{5} > 3$$
$$x < 3 \times (-5)$$
$$x < -15$$

Inequalities with cases

When we are solving an inequality with a variable on the bottom of a fraction, there is a problem. To get rid of the fraction, we need to know if we are multiplying both sides by a positive or negative value. Solution: split the problem into two cases and try both.

Example:

$$\frac{4x}{x-5} \leq 3$$

We split this into two cases. We know that $x \neq 5$ as that would give us division by 0.

Case 1: If $x - 5 < 0$, i.e. $x < 5$

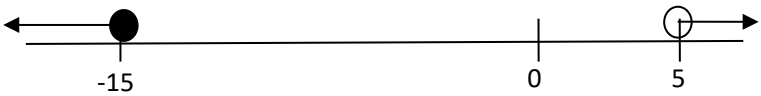
$$4x \geq 3x - 15$$
$$x \geq -15$$

Case 2: If $x - 5 > 0$, i.e. $x > 5$

$$4x \leq 3x - 15$$
$$x \leq -15$$

From case 1, the answer $x \geq -15$ only applies if $x < 5$, so combining those two inequalties we know that $-15 \leq x < 5$. Any x within that overlapping range should be a solution.

From case 2, we have $x \leq 15$, but this only applies if $x > 5$. These can't both be true for any value of x, so there is no solution for case 2.



This means $-15 \leq x < 5$ is our final answer.

Exercises

Find the range of x values which satisfy the following inequalities

1) $-2x + 3 < 7$

2) $3x - 5 > 6x + 1$

3) $2(x + 5) \geq 5x$

4) $\frac{4-3x}{2} \leq 7$

5) $\frac{x}{x+1} > 2$

6) $\frac{4}{3-x} \leq 7$

Answers

1) $x > 2$

2) $x < -\frac{4}{3}$

3) $x \leq \frac{10}{3}$

4) $x \geq -\frac{10}{3}$

5) $x < -2$

6) $x \leq \frac{17}{7}$ or $x > 3$