MATHS AND STATS

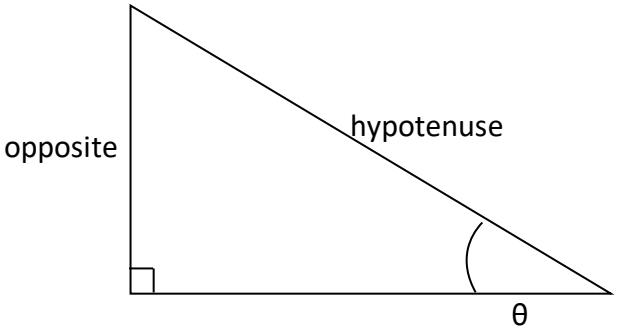
Trigonometric ratios

In right-angled triangles

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

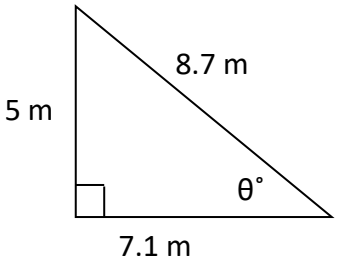
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



The common mnemonic for the above three equations is **SOH CAH TOA**.

Examples

1) Given the following triangle determine cos θ, sin θ and tan θ.

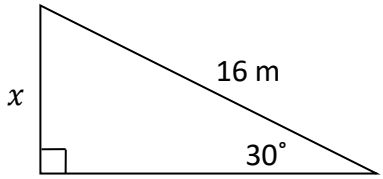


$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{7.1}{8.7} = 0.8161$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{8.7} = 0.5747$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{7.1} = 0.7042$$

2) Find the value of x



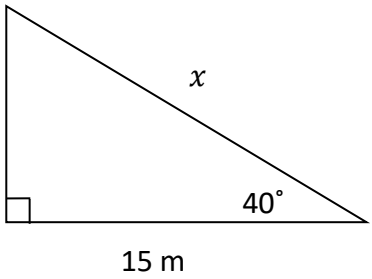
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30 = \frac{x}{16}$$

$$x = 16\sin 30$$

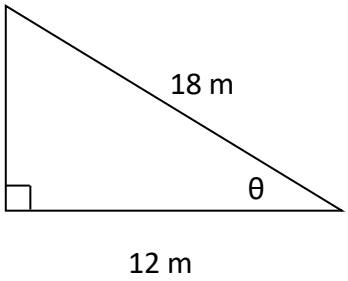
$$= 8m$$

3) Find the value of x



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 40 = \frac{15}{x}$$
$$x\cos 40 = 15$$
$$x = \frac{15}{\cos 40}$$
$$x = 19.6m$$

4) Find the value of θ to the nearest minute

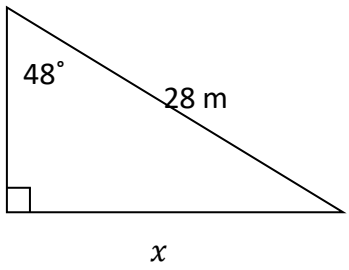


$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{12}{18}$$
$$\theta = \cos^{-1}\left(\frac{12}{18}\right)$$
$$= 48.19°$$
$$= 48°11'$$

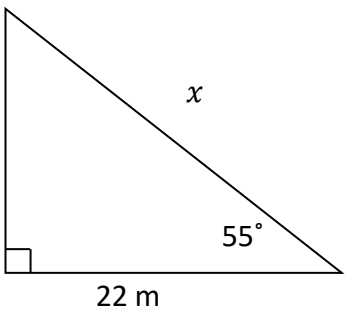
Exercises

1. Find the length of the side

(a)

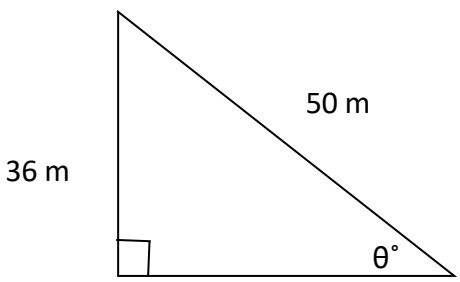


(b)

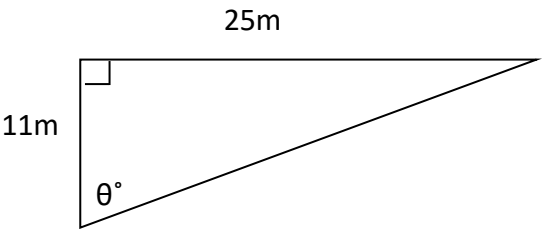


2. Find the angle θ.

(a)



(b)



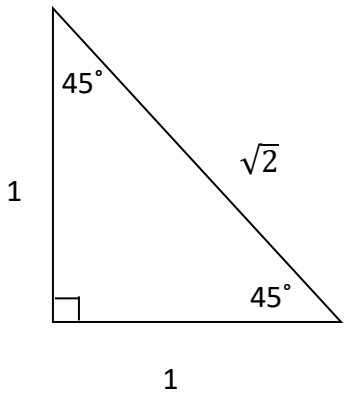
Answers

1. (a) 20.8 m (b) 38.4 m
2. (a) 46˚ 3’ (b) 66˚ 15’

Special triangles

The trig ratios of the angles in these special triangles can be expressed as exact values.

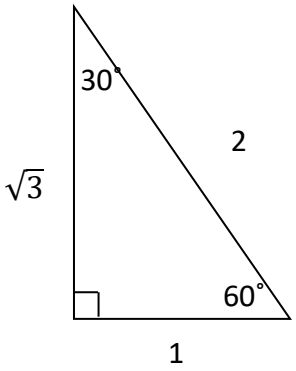
Determine the exact values of these angles. These exact values are used repeatedly in trigonometry.



$\sin 45^\circ = \frac{\text{opposite}}{\text{hypotenuse}} =$

$\cos 45^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} =$

$\tan 45^\circ = \frac{\text{opposite}}{\text{adjacent}} =$



$\sin 60^\circ = \frac{\text{opposite}}{\text{hypotenuse}} =$

$\cos 60^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} =$

$\tan 60^\circ = \frac{\text{opposite}}{\text{adjacent}} =$

$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} =$

$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} =$

$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} =$