MATHS AND STATS

Logarithms

What do we mean when we ask what $\log_2 16$ is equal to?

We are asking what power 2 must be raised to to get 16.
Another way of writing this is: If $2^x = 16$, what is x?
So, since $2^4 = 16$, we have $\log_2 16 = 4$

$$\log_a(b) = c \quad \text{is equivalent to} \quad a^c = b$$

In general, $\log_a b = c$ means that $a^c = b$. The a in this example is called the 'base' of the log. Here are some rules for working with logarithms. Notice that the base doesn't change in any of these rules.

$$\log_x(pq) = \log_x(p) + \log_x(q) \qquad \text{(A)}$$

$$\log_x\left(\frac{p}{q}\right) = \log_x(p) - \log_x(q) \qquad \text{(B)}$$

$$\log_x(p^q) = q\log(p) \qquad \text{(C)}$$

Examples

1. Solve $\log_3(9)$	$= 2$	(since $3^2 = 9$)
2. $\log_2(24) - \log_2(6)$	$= \log_2(24 / 6)$	(using rule B)
	$= \log_2(4)$	
	$= 2$	(since $2^2 = 4$)
3. Solve $\log_x(36)$	$= \log_x(6^2)$	
	$= 2\log_x(6)$	(using rule C)
4. Simplify $\log_x(216)$	$= \log_x(2^3 . 3^3)$	(as $216 = 2\times2\times2\times3\times3\times3$)
	$= \log_x(2^3) + \log_x(3^3)$	(using rule A)
	$= 3\log_x(2) + 3\log_x(3)$	(using rule C)
5. $\log_5(x) + \log_3(x)$	cannot be simplified as it uses different bases	

Logarithms on calculators

Most calculators have 2 log buttons, labelled '*log*', and '*ln*'. These calculate logarithms in base 10, and base *e*, which is approx. 2.718. *ln* is called the natural logarithm, and *e* is an irrational number which pops up in mathematics in several areas, much like π. If you see a log without a base specified, eg log(100), it usually means base 10.

To calculate a different log value on the calculator, you need to convert to another base.

Change of Base formula:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

To get $\log_5(15)$, we use the change of base formula which says

$$\log_5(15) = \frac{\log_{10}(15)}{\log_{10}(5)}$$

(Note the original base, 5, becomes the number in the log on the bottom of the fraction)

On the calculator we type: [log] 15 [÷] [log] 5 [=] 1.682

Note that this numerical value is only an approximation, and generally it is considered better to write answers in exact form rather than a calculator answer. It could be useful though to find these approximate values when, for example, plotting on a graph.

For any positive value of *b*

$$\log_b(b) = 1 \quad \text{and} \quad \log_b(1) = 0$$

Exercises

1) $\log_{10}(10)$ = ___
2) $\log_b(b^a)$= ___
3) $\log_4(16)$ is equal to: ____
4) Express ln(6) – ln(10) as a single logarithm.
5) Give the value of ln(25) accurate to 3 decimal places.
6) Express 2log(3) + log(7) as a single logarithm.

7) $\log_3(27) + \log_5(1) - \log_2(8) =$ ___

8) Simplify $5\log_2(4) - \log_2(16)$

9) Write $\log_{10}(42)$ in terms of the function ln.

10) Find a numerical approximation for $\log_2 15$ correct to 3 decimal places.

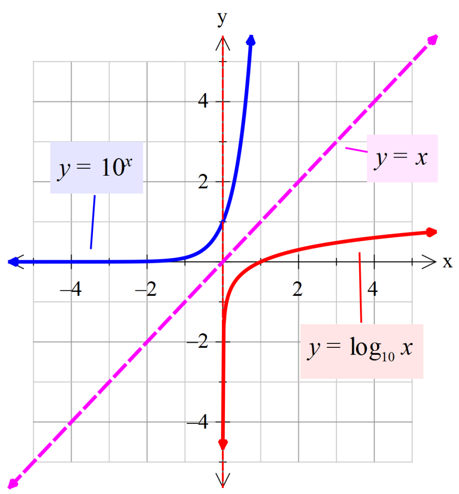
11) Express $a \log b - c \log d + b \log a$ as a single logarithm.

12) Evaluate $10^{\log_{10} 53}$ (challenge question)

Answers

1) 1
2) a
3) 2
4) $\ln\left(\frac{3}{5}\right)$
5) 3.219
6) $\log(63)$
7) 0
8) 6
9) $\frac{\ln(42)}{\ln(10)}$
10) 3.91
11) $\log \frac{b^a a^b}{d^c}$
12) 53

Graphing log functions



$y = \log_a(x)$ is a reflection of the exponential graph in the line $y = x$

The *x*-intercept is 1.

The graph is asymptotic to the *y*-axis.

A table of *x* and *y* values can be used to graph the function

Recall $\log_b b = 1$ for all positive b. This should allow you to find a point on the graph. Also recall that $\log_b(b^n) = n$ and so this will allow you to find as many points as you wish. These points along with an x intercept and a general idea of the shape of a logarithm graph will help you graph the log functions.

Exercises

Graph (a) $y = \log_2 x$ (b) $y = 2\log_3 x$ (c) $y = 1 + \log_3 x$

Solving log equations

Examples

1. Solve $\log_3 81 = x$ (convert it to an exponential question)

$$3^x = 81$$
$$3^x = 3^4$$
$$x = 4$$

2. Solve $\log_2(4x) = 8$

$$\log_2 4 + \log_2 x = 8 \quad \text{(using log rule A)}$$
$$2 + \log_2 x = 8$$
$$\log_2 x = 6$$
$$x = 2^6 = 64$$

3. Solve $\log_3\left(\frac{x}{27}\right) = -1$ (using log rule B)

$$\log_3 x - \log_3 27 = -1$$
$$\log_3 x - 3 = -1$$
$$\log_3 x = 2$$
$$x = 3^2 = 9$$

Exercises

Solve the following equations for x

1) $\log_5(x) = 2$
2) $\log_3(20x) = 3$
3) $\log_{10}(4x) = 2$
4) $\log_4\left(\frac{x}{2}\right) = 6$
5) $\log_2\left(\frac{2x}{5}\right) = 1$
6) $\log_3\left(\frac{3x}{4} + 1\right) = 8$
7) $\log_x(4) = 2$
8) $\log_x(125) = 3$

Answers

1) $x = 25$
2) $x = \frac{27}{20}$
3) $x = 25$
4) $x = 8192$
5) $x = 5$
6) $x = 8746\frac{2}{3}$
7) $x = 2$
8) $x = 5$