

# Binomial Theorem

Use Pascal's Triangle to get the coefficient for each term. Do you see how the triangle is formed? Can you add in the next line?

$$\begin{array}{cccccccc}
 & & & & 1 & & & & \\
 & & & & & 1 & & 1 & \\
 & & & & & & 1 & & 2 & & 1 & \\
 & & & & & & & 1 & & 3 & & 3 & & 1 & \\
 & & & & & & & & 1 & & 4 & & 6 & & 4 & & 1 & \\
 & & & & & & & & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
 & & & & & & & & & & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
 & & & & & & & & & & & & 1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 & 
 \end{array}$$

The rows of Pascal's Triangle make up the coefficients of a binomial expansion. These numbers can also be found as combinations, or how many ways can you choose  $k$  elements from  $n$  elements. This is often written  $\binom{n}{k}$ , which gives the  $k^{\text{th}}$  number on the  $n^{\text{th}}$  row (both counts start at 0). Calculators often write this as "nCr"

The powers count down for one term and up for the other, always adding up to  $n$ .

Written as a formula:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

### Example 1

Expand  $(x + y)^5$  5<sup>th</sup> power so use the 5 row: 1, 5, 10, 10, 5, 1

$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

### Example 2

$$\begin{aligned}
 \text{Expand } (2x + 3y)^4 & \\
 &= (2x)^4 + 4(2x)^3(3y) + 6(2x)^2(3y)^2 + 4(2x)(3y)^3 + (3y)^4 \\
 &= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4
 \end{aligned}$$

### Example 3

$$\begin{aligned}
 \text{Expand } (x - y)^6 & \\
 &= x^6 + 6x^5(-y) + 15x^4(-y)^2 + 20x^3(-y)^3 + 15x^2(-y)^4 + 6x(-y)^5 + (-y)^6 \\
 &= x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6
 \end{aligned}$$



## Binomial Theorem Exercises

1.  $(x + 2)^2$
2.  $(5x + 3y)^3$
3.  $(1 - 2x)^4$
4.  $(2 + 3a)^5$
5.  $(3x - 2y)^6$

Challenge questions:

6.  $(x^2 + x)^4$
7.  $\left(2x^2 - \frac{3}{x}\right)^5$

## Answers

1.  $x^2 + 4x + 4$
2.  $125x^3 + 225x^2y + 135xy^2 + 27y^3$
3.  $1 - 8x + 24x^2 - 32x^3 + 16x^4$
4.  $32 + 240a + 720a^2 + 1080a^3 + 810a^4 + 423a^5$
5.  $729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$
6.  $x^8 + 4x^7 + 6x^6 + 4x^5 + x^4$
7.  $32x^{10} - 240x^7 + 720x^4 - 1080x + 810x^{-2} - 243x^{-5}$