MATHS AND STATS

Scientific notation, significant figures and rounding

Scientific or Standard Notation is best used to express very large or very small numbers in a compact, easy to read form, but can be used on any numbers.

Simply, the basic format of the notation is

$a \times 10^{n}$

+ n -> a **positive** index indicates a **large** number

– n -> a **negative** index indicates a **small** number

where "a" is always a number between 1 and 10

10^{n} indicates the **magnitude or size** of the number.

$10^5 = 10 \times 10 \times 10 \times 10 \times 10$ or 100000 (1 and 5 zeros)
$10^3 = 10 \times 10 \times 10$ or 1000 (1 and 3 zeros)
$10^8 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ or 100000000 (1 and 8 zeros)

SN makes it is easy to compare sizes of numbers. **Compare magnitude (n) FIRST**
2.3×10^6 *is bigger than* 8.97×10^5 since the index 6 > 5
6.7×10^6 *is bigger than* 5.2×10^6 with the same index of 6, compare 6.7 and 5.2

a indicates the **accuracy or precision** of the number. It is determined by the number of **Significant Figures**

3.11 is more accurate than 3.1 and 5.6027 is more accurate than 47

- In general the more digits the number has the more accurate or precise the measurement.
- Significant figures are different to decimal places
- Non-zero digits are significant
- The digit zero is ONLY significant if contained between non-zero digits or it is after the decimal point, at the end of a number

eg

3.11	(3 sf)	0.00005621	(4 sf)
3.1	(2 sf)	0.8	(1 sf)
5.6027	(5 sf)	6701000	(4 sf)
47	(2 sf)	0.00350	(3 sf)

More examples at
http://www.purplemath.com/modules/rounding2.htm

Rounding

Numbers are rounded for many reasons including

- Avoiding false precision eg 3.647382 mm
- Estimation required rather than precision
- Convenience

When rounding, the last retained digit rounds up only if the digit immediately following is 5 or greater.

Lets look at the number 18.60235

Precision	Significant figures	Decimal places
5	18.602	18.60235
4	18.60	18.602**4** (rounded up)
3	18.6	18.602
2	1**9** (rounded up)	18.60
1	**2**0 (rounded up)	18.6
0	n/a	1**9** (rounded up)

Tip – be clear on whether you are rounding according to the number of decimal places or the number of significant figures

Converting from scientific notation

Examples

1) 3.4×10^9

This form tells us it is a big number and makes it easy to compare to other big numbers
$3.4 \times 10^9 = 3.4 \times 1\,000\,000\,000$ (10^9 indicates the magnitude or size of the number)

$\times 1\,000\,000\,000$ means the decimal point moves 9 places to make the number 3.4 bigger
= 3 400 000 000 ← the decimal point is now here and not usually written

9 places – '4' takes one place then fill with 8 zeros to the decimal point

2) 7.85×10^3

Well $7.85 \times 10^3 = 7.85 \times 1000$ and x1000 means the decimal point moves 3 places to make the number 7.85 bigger
= 7 850

3 places – '85' takes 2 places then fill with one zero

$6.7 \times 10^6 = 6\,700\,000$	(move 6 pls, 1 place then fill 5 x 0's)
$6.7421 \times 10^6 = 6\,742\,100$	(move 6 pls, 4 places then fill 2 x 0's)
$1.364 \times 10^8 = 136\,400\,000$	(move 8 pls, 3 places then fill 5 x 0's)
$7.34 \times 10^4 = 73\,400$	(move 4 pls, 2 places then fill 2 x 0's)

3) 4.72×10^{-8}

This form tells us it is a small number as the index is negative
$4.72 \times 10^{-8} = 4.72 \div 100\,000\,000$

The negative index means divide by 10^8
÷ 100 000 000 means the decimal point moves 8 places to make the number 4.72 smaller
= 0.000 000 0472

8 places – '4' takes one place then fill with 7 zeros to the decimal point

4)

Lets look at some more numbers in SN and convert them back into decimal numbers
$7.85\times10^{-3} = 7.85 \div 1000$
÷ 1000 means the decimal point moves 3 places to make the number 7.85 smaller
= 0.007 85

3 places – '7' takes 1 place then fill with 2 zeros

$6.7\times10^{-6} = 0.000\ 006\ 7$ (move 6 pls, 1 place then fill 5 x 0's)

$6.7421\times10^{-6} = 0.000\ 006\ 742\ 1$ (move 6 pls, 1 place then fill 5 x 0's)

$1.364\times10^{-8} = 0.000\ 000\ 013\ 64$ (move 8 pls, 1 place then fill 7 x 0's)

$7.34\times10^{-4} = 0.000\ 734$ (move 4 pls, 1 place then fill 3 x 0's)

On your calculator

EXP 10^x

Look for the <EXP> or <10^n> buttons which can be used to enter numbers in scientific notation directly into your calculator.

eg 2.31 EXP 6 displays as 2 310 000 or 2.31×10^6 on your calculator

Converting to scientific notation

Count the number of places or digits between the decimal point and where the decimal point needs to be in order to create a number between 1 and 10

Examples

1) 85 312 000

The decimal point is at the end of this number (as is the case for all whole numbers). We need a number between 1 and 10, given the number above we require 8.5312, this requires the decimal point to move 7 places or digits.

So we write 85 312 000 = 8.5312×10^7

(note we had a big number and so we have a positive index)

2) 0.0312

3.12 is the number between 1 and 10 we require for scientific notation. This requires moving the decimal point 2 places or digits.

$$0.0312 = 3.12 \times 10^{-2}$$

(note we had a small number and so we have a negative index)

3) 780 = 7.8×10^{2}

4) 470 000 000 000 = 4.7×10^{11}

5) 0.000 000 02 = 2×10^{-8}

6) 0.000 906 6 = 9.066×10^{-4}

Exercises

- Be careful to ensure whether to use a negative or positive index

1 Where possible, round the following to
   I. 3 significant figures
   II. 2 decimal places

a 56210233	f 9.2917	k 4006.283
b 0.00052834	g 384.728	l 86254000
c 176.25	h 1.0009	m 0.566666
d 13.8816	I 0.0203	n 34000
e 0.4625	j 9738.8925	o 0.005006

2 Express the following as decimal numbers

a 8.71×10^{6}	f 6.39×10^{-6}	k 5.017×10^{-8}
b 5.2478×10^{4}	g 4.7115×10^{3}	l 3.7×10^{-5}
c 8.04×10^{5}	h 3.22×10^{-2}	m 1.6×10^{2}
d 8.32158×10^{-4}	I 9.305×10^{5}	n 4.7×10^{0}
e 2.0×10^{-3}	j 7×10^{8}	o 6.480382×10^{4}

3 Express the following in Scientific notation (you may round to 3 sf for convenience)

a 56210233	f 9.2917	k 4006.283
b 0.00052834	g 384.728	l 86254000
c 176.25	h 1392.0009	m 0.566666
d 13.8816	I 0.0203	n 34000
e 0.4625	j 0.000097	o 0.005006

Answers

1 I 3 Sig figs / II 2 dec places

a	56200000	n/a	f	9.29	9.29	k	4010	4006.28
b	0.000528	0.00	g	385	384.73	l	86300000	n/a
c	176	176.25	h	1.00	1.00	m	0.567	0.57
d	13.9	13.88	I	0.0203	0.02	n	34000	n/a
e	0.463	0.46	j	9740	9738.89	o	0.00501	0.01

2

a	8 710 000	f	0.000 006 39	k	0.000 000 050 17	
b	52 478	g	4 711.5	l	0.000 037	
c	804 000	h	0.0322	m	160	
d	0.000 832 158	I	930 500	n	4.7	
e	0.002	j	700 000 000	o	64 803.82	

3

a	5.6210233×10^{7}	f	9.2917×10^{0}	k	4.006283×10^{3}
b	5.2834×10^{-4}	g	3.84728×10^{2}	l	8.6254×10^{7}
c	1.7625×10^{2}	h	1.3920009×10^{3}	m	5.66666×10^{-1}
d	1.38816×10^{1}	I	2.03×10^{-2}	n	3.4×10^{4}
e	4.625×10^{-1}	j	9.7×10^{-5}	o	5.006×10^{-3}