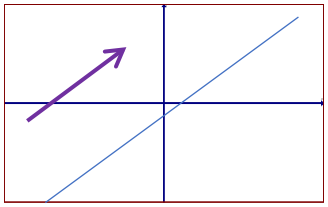
MATHS AND STATS

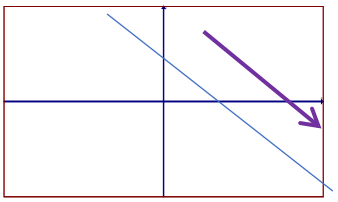
Straight lines and linear equations

Gradients

A **gradient is the slope of the line** and mathematically represents a **rate of change**.
The steeper the slope, the larger the number. If a line is flat, it has no slope or slope = 0.
A gradient is negative if it runs in the same direction as the sloping line in capital N



Positive or increasing slope (**not** negative)

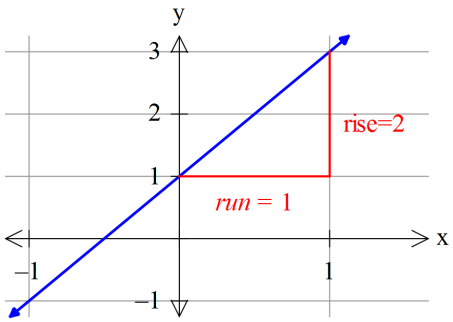


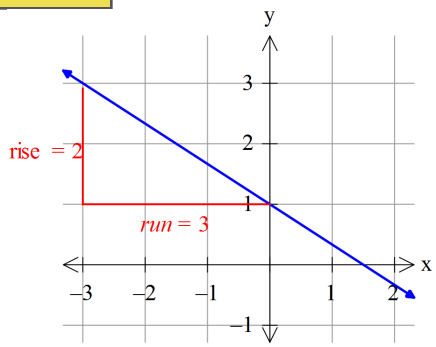
Negative or decreasing slope

Finding the gradient

1. **From a graph**

$$gradient = [sign +/-] \frac{rise}{run}$$





Create **any right angle triangle** which makes it easy to find the point coordinates on the line.
Use the point coordinates to work out the triangle side lengths for the rise and run.

$$gradient = [+]\frac{rise}{run} = \frac{2}{1} = 2$$

$$gradient = [-]\frac{rise}{run} = -\frac{2}{3}$$

2. **Given 2 points**

$$gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the gradient of the line joining the points (2, 3) and (4, -2)

(x_1, y_1) (2, 3) and (x_2, y_2) (4, -2)

$$gradient = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{4-2} = -\frac{5}{2}$$

3. **From the line equation** – more details on the line equation follow

Make sure the equation has y as the subject. The gradient is always the number next to the *x*

$y = \boxed{3}x - 5$ Gradient = 3

$y = \boxed{-\frac{2}{3}}x + 4$ Gradient = $-\frac{2}{3}$

Drawing a graph

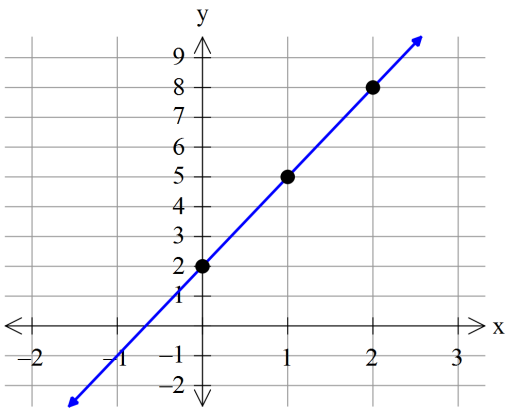
1. **Box method** – choose easy *x*-values

$y = 3x + 2$

x	0	1	2
y			

When $x = 0$ $y = 3 \times 0 + 2 = 2$
When $x = 1$ $y = 3 \times 1 + 2 = 5$
When $x = 2$ $y = 3 \times 2 + 2 = 8$

x	0	1	2
y	**2**	**5**	**8**



Plot the points and join them to form the line.

2. **Using the *y*-intercept and *x*-intercept**

$y - x = 2$

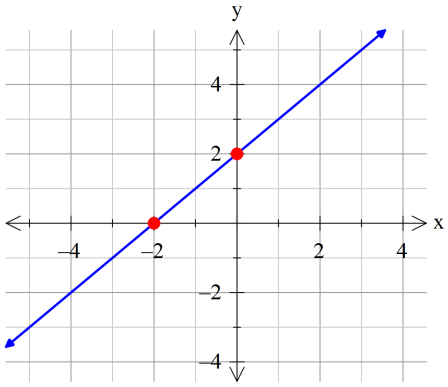
To find the y-intercept put $x = 0$

$y - 0 = 2$ so $y = 2$ **(0, 2)**

To find the x-intercept put $y = 0$

$0 - x = 2$ or $-x = 2$

So $x = -2$ **(-2, 0)**



Plot the points and join them to form the line.

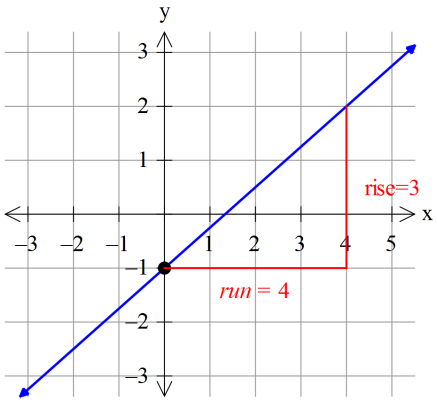
3. **Using gradient and y-intercept**

Graph $y = \frac{3}{4}x - 1$

Mark on the y-intercept at -1

Then draw the triangle to find a second point to join up.

$gradient = \frac{3}{4} = \frac{rise}{run}$



Gradient – intercept form of equation, note that y always needs to be the subject

$$y = mx + b$$

Gradient — y-intercept

Finding the equation of a line

1. Given the gradient and the y-intercept and using $\boldsymbol{y = mx + b}$

Substitute the values into the equation

Find the equation of the line with a **gradient of 3** and y-intercept of -1?

$\boldsymbol{y = mx + b}$

We know that $m = 3$ and $b = -1$

So $y = \mathbf{3}x - \mathbf{1}$

2. Given the gradient and a point on the line and using $\boldsymbol{y = mx + b}$

Find the equation of the line with gradient $\frac{2}{3}$ and passing through (6, 1)

We know that $\boldsymbol{m = \frac{2}{3}}$ so put that into our equation $\boldsymbol{y = mx + b}$

It becomes $y = \frac{2}{3}x + b$

Now substitute in the *x* and *y* values from our point (x, y) (6, 1) to obtain the value of *b*

$$1 = \frac{2}{3} \times 6 + b$$
$$1 = 4 + b$$
$$b = -3$$

Put this into our equation and it becomes $y = \frac{2}{3}x - 3$

3. Using the coordinates of two points

Find the equation of the line through the points (x_1, y_1) (1, 5) and (x_2, y_2) (2, 3)

First find the gradient using the formula

$$\boldsymbol{gradient} = \frac{\boldsymbol{y_2 - y_1}}{\boldsymbol{x_2 - x_1}} = \frac{3 - 5}{2 - 1} = -\frac{2}{1} = -2$$

The question is now like 2. above

Put the **gradient** into the line equation $\boldsymbol{y = mx + b}$

$$y = \mathbf{-2}x + b$$

Substitute the *x* and *y* values from a point (x, y) (2, 3) to find the value of *b*

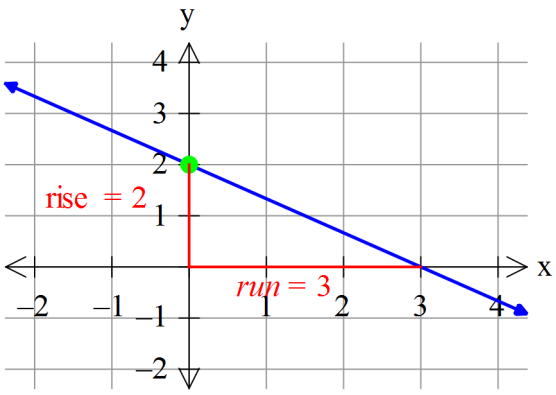
(You could use either point for this step.)

$$\mathbf{3} = -2 \times \mathbf{2} + b$$
$$3 = -4 + b$$
$$\boldsymbol{b = 7}$$

Put this into our equation.

The equation is $y = -2x + 7$

4. **Given a graph** and using $y = mx + b$
Find the equation of the line drawn.



Locate the y-intercept. $b = 2$

Draw on a triangle to find the gradient

$$m = -\frac{rise}{run} - \frac{2}{3}$$

Put the gradient and y-intercept in the equation

$$y = mx + b$$
$$y = -\frac{2}{3}x + 2$$

Is a point on a line?

- Is the point (x, y) (2, -3) on the line $y = -3x + 5$?

Substitute the *x* and *y* values into the equation and see if it satisfies the line equation.

$$-3 = -3 \times 2 + 5$$
$$-3 = -6 + 5$$
$$-3 = -1$$ No, it does not satisfy the equation

The point (2, -3) **does not lie on the line** $y = -3x + 5$

- Is the point (x, y) (1, 2) on the line $y = -3x + 5$?

Substitute the *x* and *y* value into the equation

$$2 = -3 \times 1 + 5$$
$$2 = -3 + 5$$
$$2 = 2$$ True, the values on the LHS = RHS

So the point (1, 2) **is on the line** $y = -3x + 5$

Write the equation in gradient-intercept form

Use algebra techniques to get the *y* term on the left on its own.

- Write $4x + 3y - 16 = 0$ (equation in general from) in gradient-intercept form.

$4x + 3y - 16 = 0$ leave the *3y* term alone and $-4x$ *both sides*

$3y - 16 = -4x$ $+16$ *both sides*

$3y = -4x + 16$ $\div 3$ *both sides*

The equation in gradient – intercept from is $y = -\frac{4}{3}x + \frac{16}{3}$