MATHS AND STATS

# Logarithms

What do we mean when we ask, "what is $\log_2 16$ equal to"?

We are asking what power 2 must be raised to in order to get 16.
Another way of writing this is: If $2^x = 16$, what is $x$?
Since $2^4 = 16$, we have $\log_2 16 = 4$.

| | | |
|---|---|---|
| $\log_a(b) = c$ | is equivalent to | $a^c = b$ |

In general, $\log_a(b) = c$ means that $a^c = b$. The $a$ in this example is called the 'base' of the log.

## Example

Evaluate $\log_3(9)$. Since $3^2 = 9$, we have that $\log_3(9) = 2$

## Exercises A

1) Rewrite $x^y = a$ in log form (using the statement in the shaded rectangle).

2) Rewrite $\log_p(r) = t$ in index form

3) Determine the following

a. $\log_4(16) =$

b. $\log_{10}(1000) =$

c. $\log_6(36) =$

d. $\log_2(16) =$

e. $\log_3(27) =$

f. $\log_5(125) =$

g. $\log_2(32) =$

h. $\log_{10}(10000) =$

i. $\log_3\left(\frac{1}{9}\right) =$

j. $\log_{10}\left(\frac{1}{1000}\right) =$

k. $\log_3(27) + \log_5(1) - \log_2(8)$.

Here are some rules for working with logarithms. Notice that the base doesn't change in any of these rules. The main rule we will need in ACFI is (C).

$$\log_a(pq) = \log_a(p) + \log_a(q) \qquad (A)$$

$$\log_a\left(\frac{p}{q}\right) = \log_a(p) - \log_a(q) \qquad (B)$$

$$\log_a(p^q) = \mathrm{q} \times \log_a(p) \qquad (C)$$

$\log_\mathrm{a}(a) = 1$ and $\log_\mathrm{a}(1) = 0$

## Example

Use a log law to rewrite $\log_{10}(41^{x/2})$ without a power

$$\log_{10}(41^{x/2}) = \frac{x}{2}\log_{10}(41) \qquad \text{(using rule C)}$$

## Exercises B

1) Evaluate $\log_{10}(10)$
2) Evaluate $\log_2(1)$
3) Evaluate $\log_3(3)$
4) Evaluate $\log_7(1)$
5) Rewrite $\log_b(b^a)$ without a power
6) Rewrite $\log_4(4^{10})$ without a power
7) Rewrite $\log_{10}(0.03^n)$ without a power
8) Rewrite $\log_{10}(1.12^{n/3})$ without a power
9) Give the value of $\log_{10}(25)$ accurate to 3 decimal places (using a calculator, the "log" button is specifically a log base 10 button).
10) Without a calculator, determine the first digit of $\log_3(20)$

### Answers A

1) $\log_{x}(a) = y$

2) $p^{t} = r$

3)

a) $\log_4(16) = 2$
b) $\log_{10}(1000) = 3$
c) $\log_6(36) = 2$
d) $\log_2(16) = 4$
e) $\log_3(27) = 3$
f) $\log_5(125) = 3$
g) $\log_2(32) = 5$
h) $\log_{10}(10\,000) = 4$
i) $\log_3\left(\frac{1}{9}\right) = -2$
j) $\log_{10}\left(\frac{1}{1000}\right) = -3$
k) $0$

### Answers B

1) $1$
2) $0$
3) $1$
4) $0$
5) $a$
6) $10$
7) $n\log_{10}(0.03)$
8) $\frac{n}{3}\log_{10}(1.12)$
9) $1.398$
10) $2$