MATHS AND STATS

Absolute Values

An absolute value is the magnitude of a value without caring about the direction. The result of an absolute value cannot be negative. It can also be thought of as a distance. Absolute values are written as a pair of vertical lines which behave like brackets, taking the positive value of the result of what is inside them. For example:

$$|4| = 4$$
$$|-5| = 5$$
$$|10 - 7| = 3$$
$$|7 - 10| = 3$$

Both of these last two examples give the distance between 7 and 10.

Note that it is still possible to get a negative number as an answer to a question involving absolute values.

$$|-2| - |-5| = 2 - 5$$
$$= -3$$

In general:

- If the expression inside the absolute value lines is **positive**, the expression is not changed.
- If the expression inside the absolute value lines is **negative**, then we take the negative of the expression (as the negative of a negative is positive)

Sometimes this means a question with absolute values may have two solutions:

Consider $|5 - x| = 6$

If $5 - x$ is positive this becomes

$$5 - x = 6$$
$$-x = 6 - 5$$
$$-x = 1$$
$$x = -1$$

If $5 - x$ is negative this becomes

$$-(5 - x) = 6$$
$$-5 + x = 6$$
$$x = 6 + 5$$
$$x = 11$$

So $x = -1$ or $x = 11$. You can check these solutions by substituting them into the original equation. You may also notice that 6 is the distance between 5 and x when $x = -1$ or when $x = 11$.

A question may have no solutions, as well:

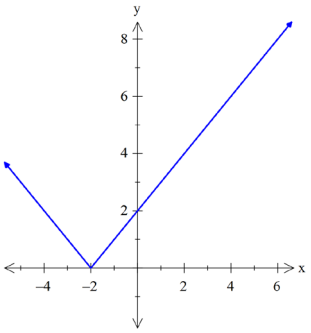
$$|4 + x| = -2$$

Since the result of an absolute value cannot be negative, it cannot equal -2 no matter what the x is.

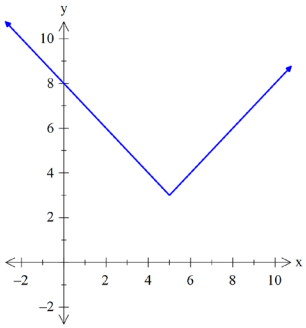
Graphs involving absolute values

Graphs involving absolute values usually have immediate changes of direction, appearing to "bounce" off a line at points where an absolute value evaluates to 0.

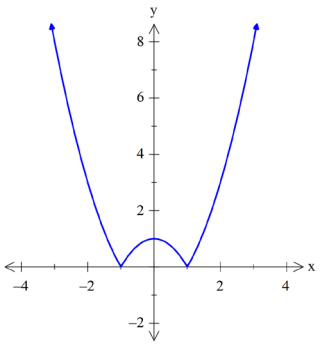
Examples:



$y = |x + 2|$



$y = 3 + |5 - x|$



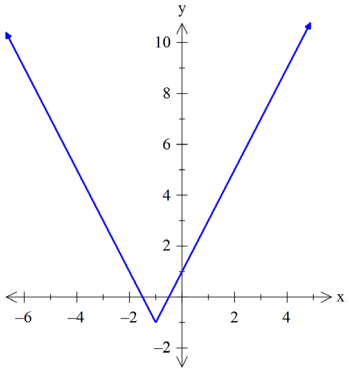
$y = |x^2 - 1|$

Exercises

1. Find $|1 - 9|$
2. Evaluate $|-4| - |2 - 7|$
3. Evaluate $5 \times |-7 + 1| - 2$
4. For what two values of x does $|x + 3| = 5$
5. Simplify $|a - b| - 3 \times |b - a|$
6. Draw the graph of $y = |2x + 2| - 1$

Answers

1. 8
2. -1
3. 28
4. $x = 2$ or $x = -8$
5. $-2 \times |b - a|$
   $= \begin{cases} 2a - 2b & \text{if } a < b \\ 2b - 2a & \text{if } a > b \end{cases}$

6. 

$y = |2x + 2| - 1$