MATHS AND STATS

Scientific Notation, Measurement & Accuracy

When working with very large or very small numbers it is often easier to write them in scientific notation. This means writing the number as the product of a number between 1 and 10 and an integer power of 10.

e.g. 23 000 000 000 is written as 2.3×10^{10} in scientific notation

0.0000257 is equal to 2.57×10^{-5} in scientific notation

Calculations with scientific notation

Addition and subtraction: you must first make sure that the powers of 10 are the same, and then you can simply add or subtract.

Example 1

Calculate $1.25 \times 10^2 + 2.84 \times 10^4$.

We make the powers of 10 the same:
$2.84 \times 10^4 = 284 \times 10^2$

Now the powers are the same we do the addition:

$$\begin{array}{r} 1.25+ \\ \underline{284.00} \\ 285.25 \end{array}$$

We now write it in scientific notation:
$285.25 \times 10^2 = 2.85 \times 10^4$

Multiplication: Multiply the decimal parts and the powers of ten separately

Example 2

Calculate $1.05 \times 10^{-3} \times 3.0 \times 10^{6}$.

We rearrange and multiply the decimal parts and powers separately.
$1.05 \times 3.0 \times 10^{-3} \times 10^{6} = 3.15 \times 10^{3}$

Division: divide the decimal parts and the powers of ten separately

Example 3

Calculate $8.4 \times 10^{6} \div (4.2 \times 10^{7})$

We rearrange and divide the decimal parts and powers of ten separately

$$8.4 \times 10^{6} \div (4.2 \times 10^{7}) = \left(\frac{8.4}{4.2}\right) \times \left(\frac{10^{6}}{10^{7}}\right)$$
$$= 2 \times 10^{-1}$$

Significant figures

The number of significant figures in a measurement is the same as the number of digits needed to write it in scientific notation. The number of significant figures reflects the accuracy of the measurement.

Example 4

a) 125000m = 1.25×10^{5}m so it has 3 significant figures
b) 0.0023 = 2.3×10^{-3} has 2 significant figures
c) 105.26 = 1.0526×10^{2} has 5 significant figures

Rounding

Numbers can be rounded to any number of significant figures or to a particular degree of accuracy. Method for rounding:

- Find the last digit you wish to include.
- If the digit immediately to the right of it is 5 or above, then increase your digit by 1. If the digit immediately to the right of it is 4 or less, then do not change it.

Example 5

(a) 2.5482 rounded to three significant figures is 2.55
(b) 251370 rounded to three significant figures is 251000 (or 2.51×10^5)
(c) 23.58 rounded to the nearest unit is 24 (or 2.4×10^1)

Adding and subtracting

When *measured values* are to be added or subtracted, we add or subtract them and then round the answer to the least number of decimal places present in the question.

Example 6

2.81 m + 12.557 m

We add the numbers first and then round to the least number of decimal places present in the question:

2.81 m + 12.557 m = 15.367 m = 15.37 m (2 dec. pl.)

Multiplying and dividing

With multiplication and division the number of significant figures in your answer should be no greater than the number of significant figures in the measurement with the fewest significant figures.

Example 7

A rectangular plate measuring 15.3cm by 8.4cm has an area of 128.52cm^2. This should be rounded to 130cm^2 (2 sig. figs) as the width 8.4m has only two significant figures.

Exercises

1. Write in scientific notation
   (a) 6250000 (b) 0.0038 (c) 0.05066

2. Calculate $1.27 \times 10^{-3} \times 2.0 \times 10^{5}$

3. How many significant figures in
   1205.3 0.000251 23000

4. Round: (a) 1236 to the nearest hundred
   (b) 35.58 to the nearest 10
   (c) 23.681 to the nearest tenth

5. Write in scientific notation and then calculate
   1125000000 × 0.0003

6. Calculate $2.54 \times 10^{4} + 2.184 \times 10^{3}$

7. Round to 3 significant figures

   (a) 12.54 (b) 1035700 (c) 0.01811

8. Express in decimal notation
   (a) 2.51×10^{5} (b) 6.24×10^{-8} (c) 1.08×10^{4}

9. Calculate these measurements considering precision and the number of significant figures.
   (a) 12.85m × 12.5m
   (b) 15.85m – 8.505m

Answers

1 (a) 6.25×10^{6} (b) 3.8×10^{-3} (c) 5.006×10^{-2}
2. 2.54×10^{2}
3. (a) 5 (b) 3 (c) 2
4. (a) 1200 (b) 40 (c) 23.7
5. 3.377×10^{5}
6. 2.7584×10^{4}
7. (a) 12.5 (b) 1040000 (c) 0.0181
8. (a) 251000 (b) 0.0000000624
9. (a) $161\ m$ (b) 7.35 m