MATHS AND STATS

Graphing functions

Any function $y = f(x)$ can be graphed by making a table of x-values and corresponding y-values.

Key points in graphing a function are the *y*-intercept(s) and *x*- intercept(s).

- To find the *x*- intercept(s) set *y* = 0 and solve for *x*.
- To find the *y*-intercept(s) set *x* = 0 and solve for *y*.

Here are some common functions and their graphs

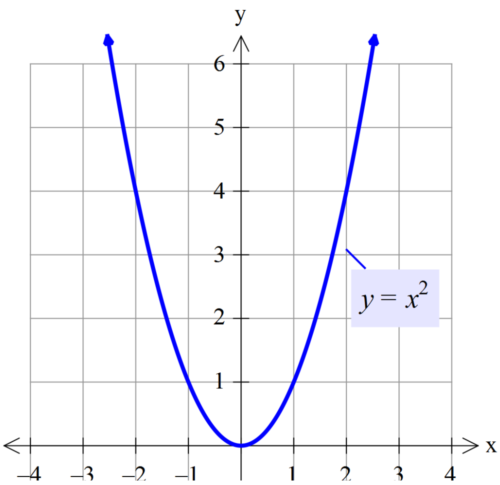
- Quadratics
- Cubics
- Hyperbolas
- Exponentials
- Logarithms
- Circles (not actually a function)

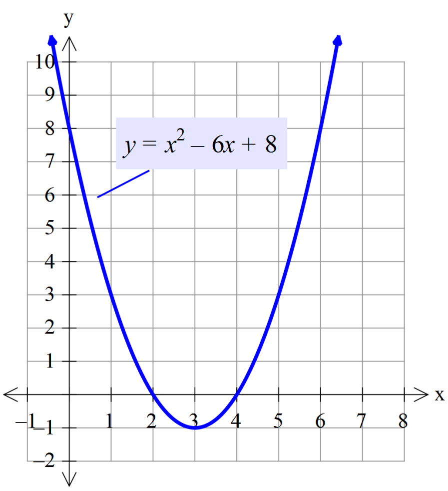
Quadratics

The highest power of *x* is 2 for all Quadratic functions. They are of the form

$$\boldsymbol{y = ax^2 + bx + c}$$

Their graph shape is called a **parabola**.





- c is the *y*-intercept.
- Solve the quadratic at $y = 0$ to get the **roots** (or **zeros**) of the equation which are the *x*- intercepts.
- a affects the steepness of the parabola.
- a being negative means it's upside down.
- The axis of symmetry is at $x = -\frac{b}{2a}$
- The vertex is $\left(-\frac{b}{2a}, c - \frac{b^2}{2a}\right)$
- Alternatively, complete the square so

  that the equation is in the form

  $$y = (x - h)^2 + k$$
  then the vertex is (h, k)

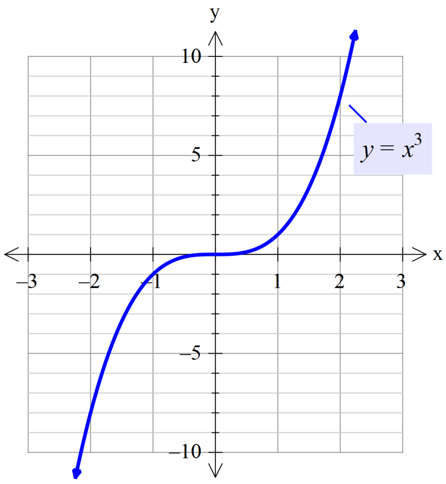
Exercises

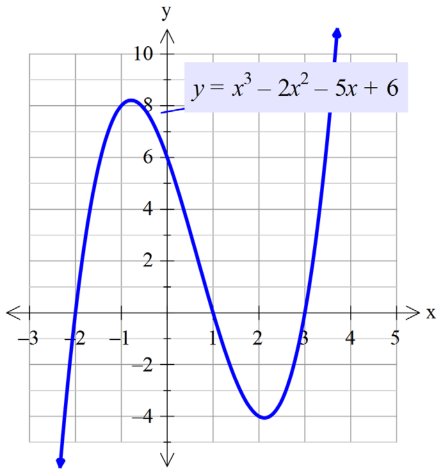
1) Graph (a) $y = (x - 3)(x + 1)$ (b) $y = x^2 + 2x - 8$

Cubics

The highest power of x is 3. Cubic functions are of the form $\boldsymbol{y = ax^3 + bx^2 + cx + d}$

- The y-intercept is d.





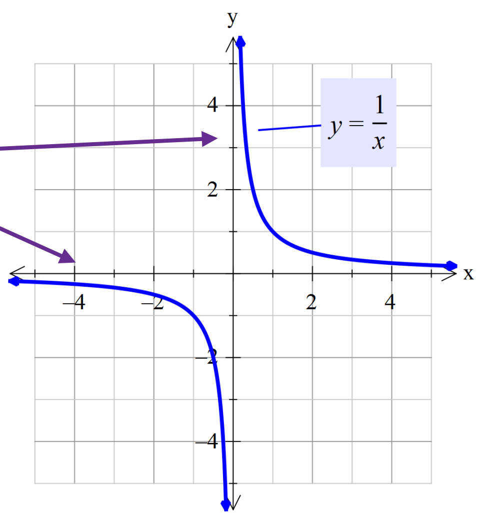
- A cubic in factorised form $y = (x - a)(x - b)(x - c)$ means the x- intercepts are a, b, and c.

Exercises

2) Graph (a) $y = (x - 1)(x + 1)(x - 3)$ (b) $y = x^3 - 2$

Hyperbolas

Simple hyperbola equations $y = \frac{a}{bx}$ or $y = \frac{a}{bx + c}$ where a, b, c can be positive or negative



- Know the **basic shape**
- $\boldsymbol{\frac{a}{b}}$ can be positive or negative
  - **+** the hyperbola is in quadrants 1 & 3
  - **-** the hyperbola is in quadrants 2 & 4
- The hyperbola has two asymptotes,
  - The **vertical asymptote** is when the denominator = 0
  - The **horizontal asymptote** is often the x-axis (here the x and y axes are the asymptotes)
- x and y - intercepts may not be useful or even exist

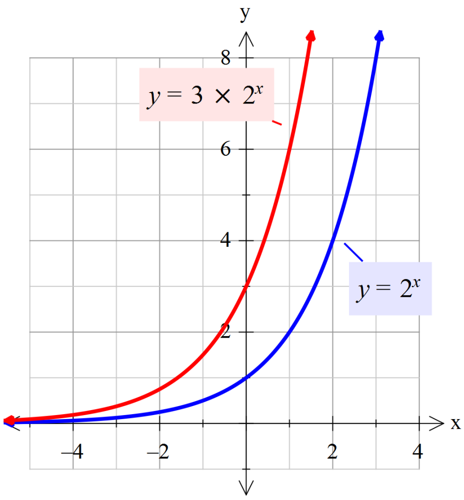
(Asymptotes are lines that the graph runs closer and closer to but generally never crosses. They are usually drawn as dashed lines when the asymptote is not an axis.)

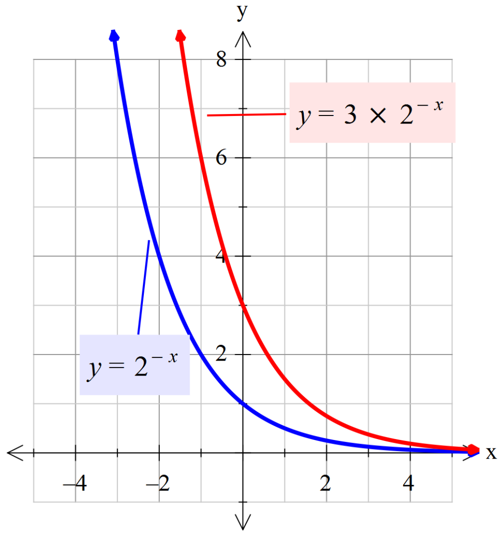
Exercises

3) Graph (a) $y = \frac{-3}{x}$ (b) $y = \frac{4}{x - 1}$

Exponentials

Have equations of the form $y = b \times a^x$





- Know the **basic shape**
- The **horizontal asymptote** is often the x-axis
- The y-intercept is b
- When $y = b \times a^{-x}$ the curve is reflected in the y-axis (ie flipped)
- a and b affect the steepness of the curve

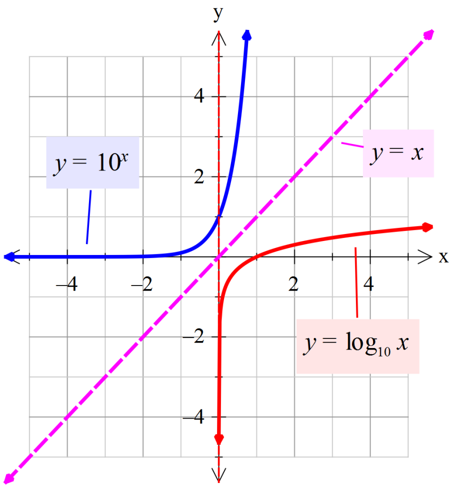
Exercises

4) Graph (a) $y = 3^x$ (b) $y = 4 \times 3^x$ (c) $y = -2 \times 3^x$ (d) $y = 2 \times 3^{-x}$

Logarithms

Have equations of the form $y = \log_a f(x)$

The graph of a Logarithmic Function is the reflection of the exponential graph in the line $y = x$ (as shown here)



- Know the **basic shape**
- The **vertical asymptote** is often the y-axis
- Find the x-and y intercepts, where they exist

Exercises

5) Graph (a) $y = \log_2 x$ (b) $y = 2\log_3 x$ (c) $y = 1 + \log_3 x$

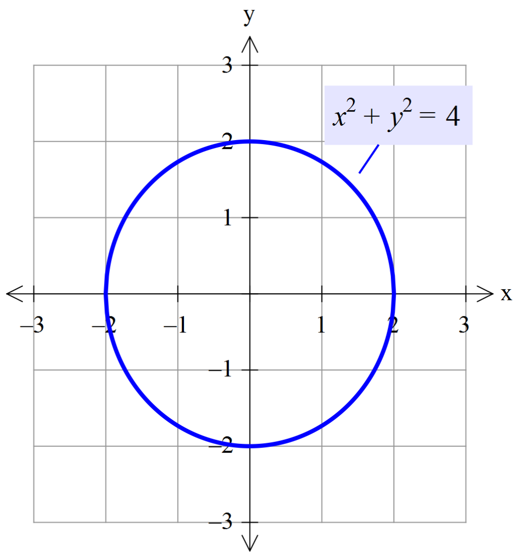
Circles

Have equations where both x and y have degree 2

$$\boldsymbol{x^2 + y^2 = r^2}$$

This is a **circle with centre** $(0, 0)$ **and radius** r

Example: $x^2 + y^2 = 4$

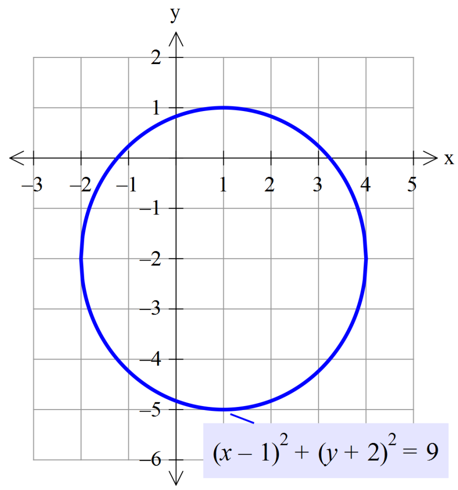


If the circle is not centred on (0, 0) the equation is of the form

$$(x - h)^2 + (y - k)^2 = r^2$$

This is a **circle with centre** $(\boldsymbol{h}, \boldsymbol{k})$ **and radius** r

Example $(x - 1)^2 + (y + 2)^2 = 9$
has centre$(1, -2)$ and radius 3.



If your equation is not in this form complete the square for x and y to put it into this form.

Eg $\quad x^2 - 4x + y^2 + 6y + 12 = 0$ $\quad$ complete the squares for x and y

$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -12 + 13$ $\quad$ to complete the square we added 4 + 9 =13 to the LHS. We must therefore add 13 to the RHS

$$(x - 2)^2 + (y + 3)^2 = 1$$

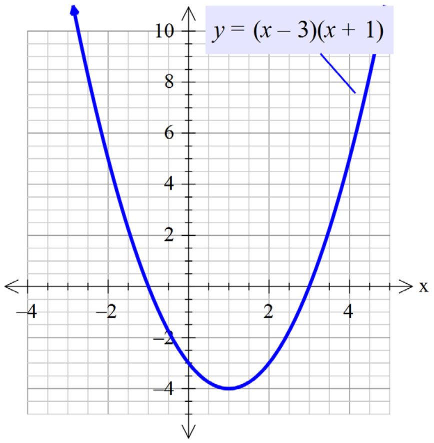
So this circle is centred at $(2, 3)$ and has a radius of 1

Exercises

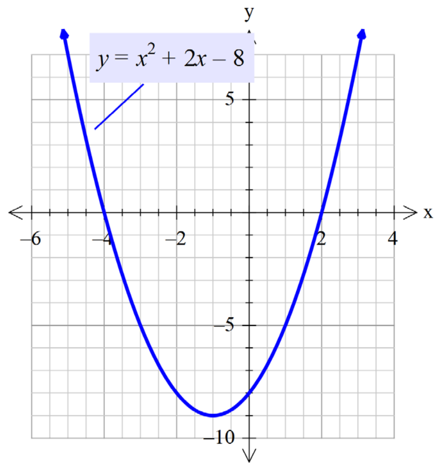
6) Graph (a) $(x - 3)^2 + (y - 1)^2 = 16$ $\quad$ (b) $x^2 - 6x + y^2 + 2y + 5 = 4$

Answers

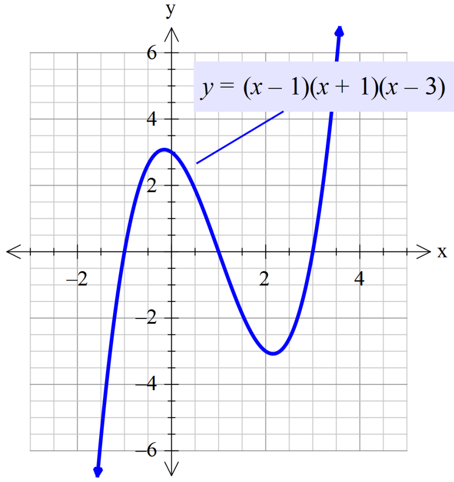
1. (a)



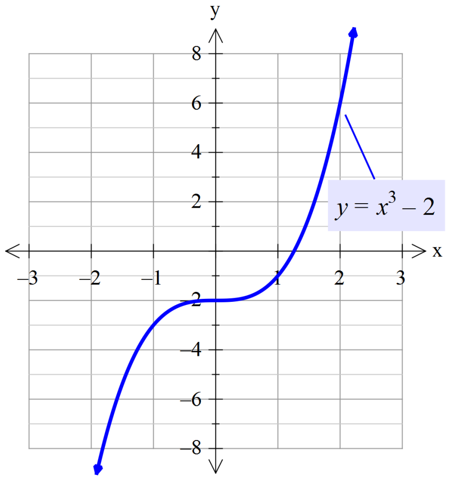
(b)



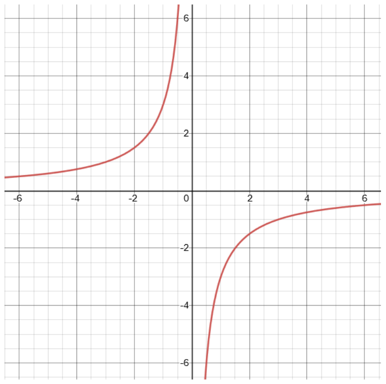
2. (a)



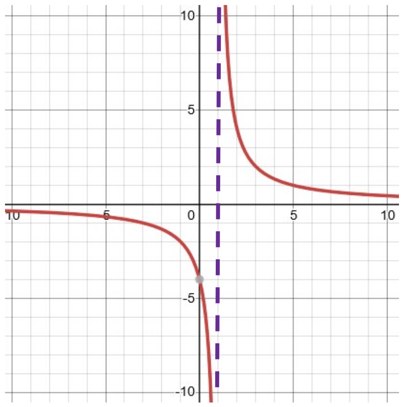
(b)



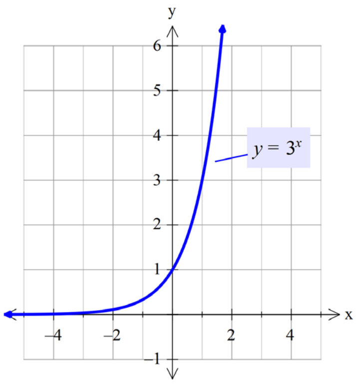
3. (a)



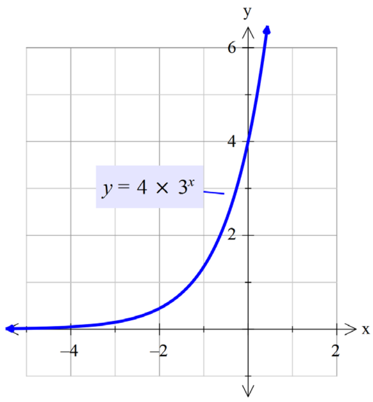
(b)



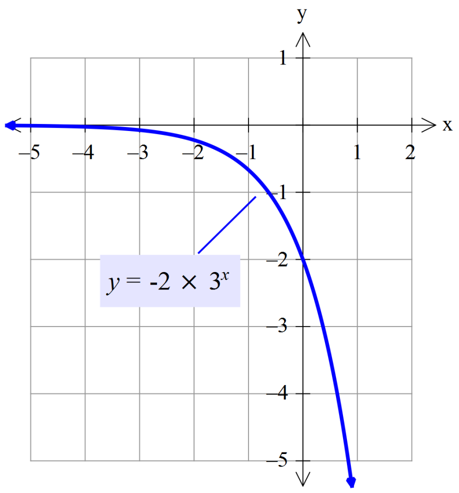
4. (a)



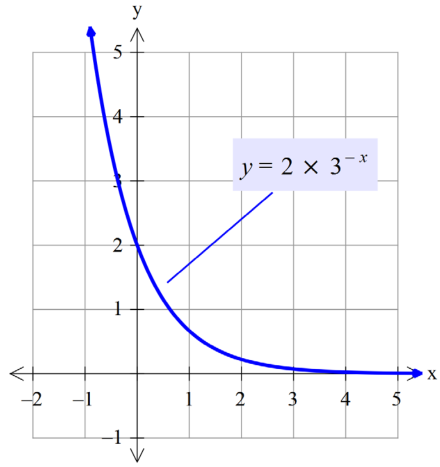
(b)



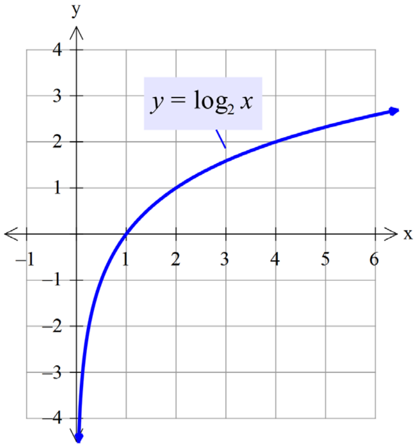
4. (c)



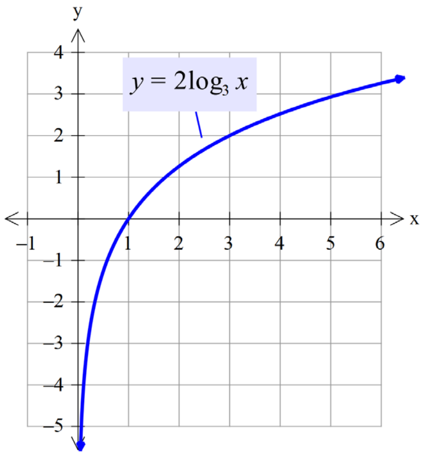
(d)



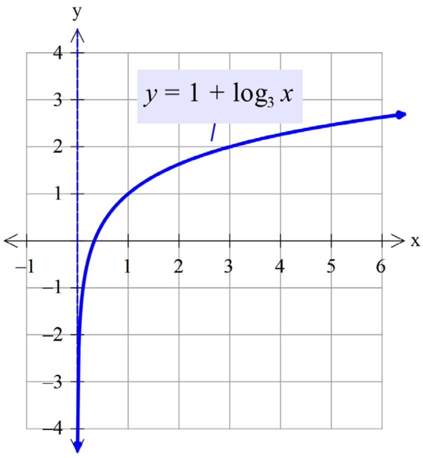
5. (a)



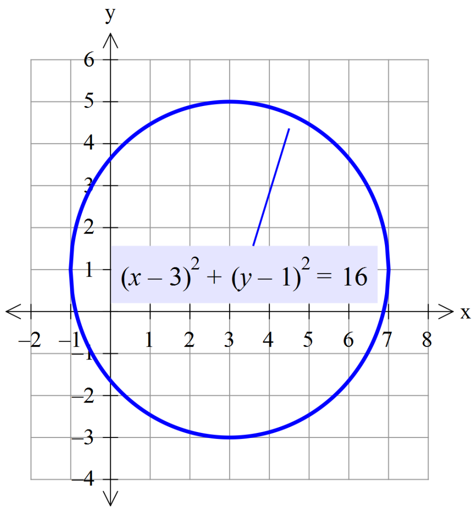
(b)



(c)



6. (a)



(b) Completing the square gives the equation as

$$(x-3)^2 + (y-1)^2 = 9$$