MATHS AND STATS

Logarithms

A logarithm is a type of operation (you know other operations, for example: multiplying by 2, dividing by 5, adding 8, subtracting 3, squaring, taking the cube root etc). It is an operation that can be applied to positive numbers and results in a number. Logarithms have a nice property that make them very useful when trying to solve a certain type of equation (exponential equations). The property is that "logs turn powers into multiplication" in that:

$$\log(a^x) = x\log(a)$$

This means, for example, $\log(10^2)$ or $\log(100)$, isn't ten times the size of $\log(10)$ but in fact, it is only two times the size of $\log(10)$ since

$$\log(100) = \log(10^2) = 2\log(10)$$

Example

Use a log law to rewrite $\log(41^{x/2})$ without a power.

$$\log(41^{x/2}) = \frac{\mathrm{x}}{2}\log(41)$$

Exercises A (non-calculator)

1) Rewrite $\log(b^a)$ without a power
2) Rewrite $\log(4^{10})$ without a power
3) Rewrite $\log(0.03^n)$ without a power
4) Rewrite $\log(1.12^{n/3})$ without a power
5) If $\log(10) = 1$, then what would the value of $\log(10^5)$ be?

If $\log(10) = 1$, then what would the value of $\log(10^{-3})$ be?

Our main purpose of using logs in accounting and finance is to solve equations. There are however, other properties (or laws) of logs that are useful for simplifying and rewriting expressions:

$$\log(a \times b) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$\log(a^b) = b\log(a)$$

$$\log(1) = 0$$

The truth is there are lots of logs, there is a log for every number. For example, there is a "log base 10" which is denoted $\log_{10}$ and a "log base 2" which is denoted $\log_2$. Knowing the base of the log gives us another property (or law):

$$\log_b(b) = 1$$

You may (or may not) be interested in understanding what these logs actually do. How are they defined? Well $\log_b(x)$ is the number such that b raised to that number would be equal to x.

Examples

a) Recall that $7^2 = 49$. This means $\log_7(49) = 2$.
b) Recall that $10^4 = 10000$. This means $\log_{!0}(10000) = 4$.
c) Since $2^3 = 8$ and $2^4 = 16$, $\log_2(11)$ is a number between 3 and 4.
d) Recall that $0.1 = \frac{1}{10} = 10^{-1}$. This means that $\log_{!0}(0.1) = -1$.

$\log_{\mathrm{b}}(a) = c$ is equivalent to $b^c = a$

The "log" button on calculators is specifically a log base 10 button. The following exercises are to make sure you are comfortable with using the log button on the calculator.

Exercises b (calculator)

1) Determine the value of $\log_{10}(25)$ accurate to 3 decimal places.
2) Confirm that $\log_{10}(500)$ is a number between $\log_{10}(100)$ and $\log_{10}(1000)$.
3) Calculate $\log_{10}(144)/\log_{10}(12)$.
4) Compare the value of $\log_{10}(5^3)$ with $3\log_{10}(5)$ and finally with $\log_{10}(5)^3$
5) Calculate $\log_{10}(0.000123)$ to 3 decimal places.
6) Can you determine the value of $\log_{10}(-1)$?

Answers A

1) $a\log(b)$
2) $10\log(4)$
3) $\mathrm{n}\log(0.03)$
4) $\frac{n}{3}\log(1.12)$
5) 5
6) -3

Answers B

1) 1.398
2) It is equal to 2.7 (to 1 d.p) so is between 2 and 3.
3) 2
4) You should find that the first two are the same but the third is smaller (this is a demonstration that brackets really are important)
5) -3.910
6) This is undefined. Logs can only act on a positive number since 10 to any power is always positive.