MATHS AND STATS

Word problems

The first step in solving any word problem is to choose a letter to represent an unknown quantity within the scenario (e.g., the amount of something). If there is a second unknown quantity, then you can either:

- represent the second unknown quantity in terms of the first to create one equation, or
- represent the second unknown quantity with a second letter to create two equations.

The following example shows how to solve a word problem using each of the above methods.

Examples

I visited a bike shop which sells both bicycles and tricycles. The shop assistant reported that they had 8 models available in store, and I counted 19 wheels among them. How many bicycles and how many tricycles were on display?

Method 1: using one variable and a single equation

Let x be the number of bicycles in the shop. Since there were 8 models in total, then the number of tricycles can be expressed as $(8-x)$.

Now each bicycle has two wheels, which means that x bicycles have $2x$ wheels among them, and each tricycle has three wheels, which means that $(8-x)$ tricycles have $3(8-x)$ wheels among them. All of this should total to give 19 wheels, which allows us to form the equation $2x+3(8-x)=19$.

We can then solve this equation as follows:

$$\begin{aligned} 2x+24-3x &= 19 && \text{Expand the brackets} \\ 24-x &= 19 && \text{Collect like terms} \\ -x &= -5 && \text{Subtract 24 from both sides} \\ x &= 5 && \text{Divide both sides by } -1 \end{aligned}$$

Therefore, there are 5 bicycles on display, and also 3 tricycles.

NOTE: If you had chosen to let x be the number of tricycles in the shop, then you would have instead obtained the equation $3x+2(8-x)=19$, whose solution is $x=3$. While this might look different, it's still the same answer! (i.e., 3 tricycles and 5 bicycles).

Method 2: using two variables and simultaneous equations

Let x be the number of bicycles in the shop, and let y be the number of tricycles in the shop. Since there were 8 models in total, then we can form the equation $x+y=8$.

Now each bicycle has two wheels, which means that x bicycles have $2x$ wheels among them, and each tricycle has three wheels, which means that y tricycles have $3y$ wheels among them. All of this should total to give 19 wheels, which allows us to form the equation $2x + 3y = 19$.

We can then solve these simultaneous equations as follows:

$3x + 3y = 24$ ... (1) $2x + 3y = 19$ ... (2)	Multiply the first equation by 3 to match the coefficients of y in equations (1) and (2)
$x = 5$	Subtract the two equations to eliminate y: (1)-(2)
$10 + 3y = 19$	Substitute the solution $x = 5$ into equation (2)
$3y = 9$	Subtract 10 from both sides
$y = 3$	Divide both sides by 3

Therefore, there are 5 bicycles on display, and also 3 tricycles.

NOTE: The above steps solve the simultaneous equations using the method of elimination to first eliminate y, but there were many possibilities. We could have alternately chosen to first eliminate x, or we could have chosen to use the method of substitution instead.

Exercises

1. George found some loose change in the car made up of 50 cent pieces and 10 cent pieces. There were 15 coins and they totalled to \$4.70. How many of each coin were there?
2. I found 25 coins in my pockets when I cleaned out my wardrobe. They were all 5 cent coins and 20 cent coins. The total value was \$3.20, how many of each type of coin were there?
3. The tip jar contained \$23 made up from \$2 coins and 50 cent coins. There were 22 coins in total how many of each coin were there?
4. We paid our \$125 bill at the restaurant using only \$5 notes and \$10 notes. There were 18 notes in total how many of each size did we use?
5. The zoo enclosure contained zebras and emus. Jan counted 26 heads and Eric counted 88 legs. How many of each species were there?
6. At present Erin is twice as old as her cousin Fred. In 8 years' time their ages will add to 61 years. How old are they both now?
7. Laura is three times as old as her sister Mary. In 7 years' time their ages will total 38 years. How old are they both now?
8. Harold's mother is twice his age. 14 years ago she was four times his age. How old are they both now?
9. Sam is 25 years older than Luke. In 20 years, Sam will be twice as old as Luke. How old are they now?

ANSWERS – using one variable and a single equation

NOTE: If you have used x to represent something different you will get a different equation but you should still get the same answers.

Abridged solutions are given – not all working steps are included

1. Let $x =$ the number of 50cent coins. $50x + 10 \times (15 - x) = 470$
   So $x = 8$. There are 8 x 50cent coins and 7 x 10cent coins.

2. Let $x =$ the number of 5cent coins. $5x + 20 \times (25 - x) = 320$
   So $x = 12$. There are 12 x 5cent coins and 13 x 20cent coins.

3. Let $x =$ the number of \$2 coins. $200x + 50 \times (22 - x) = 2300$
   So $x = 8$ There are 8 x \$2 coins and 14 x 50cent coins.

4. Let $x =$ the number of \$5 notes. $5x + 10 \times (18 - x) = 125$
   So $x = 11$. There are 11 x \$5 notes and 7 x \$10 notes.

5. Let $x =$ the number of zebras. $4x + 2 \times (26 - x) = 88$
   So $x = 18$. There are 18 zebras and 8 emus.

6. Let $x =$ Fred's age now. $(2x + 8) + (x + 8) = 61$
   So $x = 15$. Fred is now 15 years old and Erin is 30 years old.

7. Let $x =$ Mary's age now $(3x + 7) + (x + 7) = 38$
   So $x = 6$. Mary is now 6 years old and Laura is 18 years old.

8. Let $x =$ Harold's age now. $2x - 14 = 4 \times (x - 14)$
   So $x = 21$. Harold is now 21 years old and his Mum is 42 years old.

9. Let $x =$ Luke's age now. $x + 25 + 20 = 2 \times (x + 20)$
   So $x = 5$. Luke is now 5 years old and Sam is 30 years old.

ANSWERS using two variables and simultaneous equations

Abridged solutions are given – not all working steps are included

1. Let $x =$ the number of 10c coins. And $y =$ the number of 50c coins.

   The two equations are : $x + y = 15$

   $10x + 50y = 470$

   Answer: $x = 7$ and $y = 8$ So, there are 8 x 50cent coins and 7 x 10cent coins.

2. Let $x =$ the number of 5c coins. And $y =$ the number of 20c coins.

   The two equations are : $x + y = 25$

   $5x + 20y = 320$

   Answer: $x = 12$ and $y = 13$ So, there are 12 x 5cent coins and 13 x 20cent coins.

3. Let $x =$ the number of 50c coins. And $y =$ the number of \$2 coins.

   The two equations are : $x + y = 22$

   $50x + 200y = 2300$

   Answer: $x = 14$ and $y = 8$ So, there are 8 x \$2 coins and 14 x 50cent coins.

4. Let $x =$ the number of \$5 notes. And $y =$ the number of \$10 notes.

   The two equations are : $x + y = 18$

   $5x + 10y = 125$

   Answer: $x = 11$ and $y = 7$ So, there are 11 x \$5 notes and 7 x \$10 notes.

5. Let $x =$ the number of emus. And $y =$ the number of zebras.

   The two equations are : $x + y = 26$

   $2x + 4y = 88$

   Answer: $x = 8$ and $y = 18$ So, there are 18 zebras and 8 emus.

6. Let $x =$ Erin’s age. And $y =$ Fred’s age.

   The two equations are : $x = 2y$

   $x + 8 + y + 8 = 61$

   Answer: $x = 30$ and $y = 15$ So, Fred is now 15 years old and Erin is 30.

7. Let $x =$ Laura’s age. And $y =$ Mary’s age.

   The two equations are : $x = 3y$

   $x + 7 + y + 7 = 38$

   Answer: $x = 18$ and $y = 6$ Mary is now 6 years old and Laura is 18.

8. Let $x =$ Harold’s age. And $y =$ His mother’s age.

   The two equations are : $y = 2x$

   $y - 14 = 4(x - 14)$

   Answer: $x = 21$ and $y = 42$ Harold is now 21 years old and his Mum is 42.

9. Let $x =$ Sam’s age And $y =$ Luke’s age.

   The two equations are : $x = y + 25$

   $x + 20 = 2(y + 20)$

   Answer: $x = 30$ and $y = 5$ So, Luke is now 5 years old and Sam is 30.