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Non-convexity of the Barcelona Basic Model — Comment on S. J. Wheeler, D. Gallipoli and M. Karstunen (2002;26:1561–1571)

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Wheeler *et al*. (2002) recently published a paper in this journal (**26**:1561–1571) on the use of the Barcelona Basic Model (BBM) by Alonso *et al*. (1990). This paper provides some interesting insights into the BBM. The points they raised are particularly useful for those who implement the model into finite element codes. While I share most of the views of Professor Wheeler and his colleagues, one comment made in the paper seems to be at variance with my understanding of the model.

One of the limitations of the BBM, according to the paper, is the non-convexity of the LC yield surface. However, it seems to me that the non-convexity of the LC yield surface is inevitable if the isotropic yielding stress $\bar{p}_0(s)$ is going to increase with suction s. This is rather clear if we look at the LC yield surface in the plane of $p - u_{\mathrm{w}}$ *versus* u_{w} as shown in Figure 1. Note the isotropic yielding stress $p_0'(s) = p_0(s) - u_{\mathrm{w}}$ for saturated states is independent of the pore water pressure u_{w}.

Looking at the yield surface in the $\bar{p}$–s plane can be misleading, as the suction axis is never negative. Since, in most geotechnical problems involving unsaturated soils, the soil will change between unsaturated and saturated states and the pore water pressure u_{w} hence changes between negative and positive. Therefore, we need to look at the yield surface across the negative and positive regions of the u_{w} axis. In fact, the derivatives with respect to the pore water pressure are required both at constitutive and global levels in implementation of unsaturated soil models into finite element codes where displacements and pore pressures are primary unknowns. In the case when the pore air pressure u_{a} can be neglected (as u_{a} is static under common geotechnical applications), the suction is of course simply the negative pore water pressure.

It is clear from Figure 1 that imposing the convexity for strictly unsaturated states will only cause a vertex in the yield surface at the transition point between saturated and unsaturated states (see the LC yield surface denoted by the dashed line). This is of course not desirable.

In addition, the Drucker postulate which states the dissipation of energy is always positive in a closed cycle of stress, implies the convexity of the yield surface only for materials whose instantaneous elastic modulus does not depend on the accumulated plastic strain (the so-called elastoplastically decoupled materials), see Reference [1] or Reference [2]. In the framework of critical state models, an elasto-plastically decoupled model requires linear relationships between

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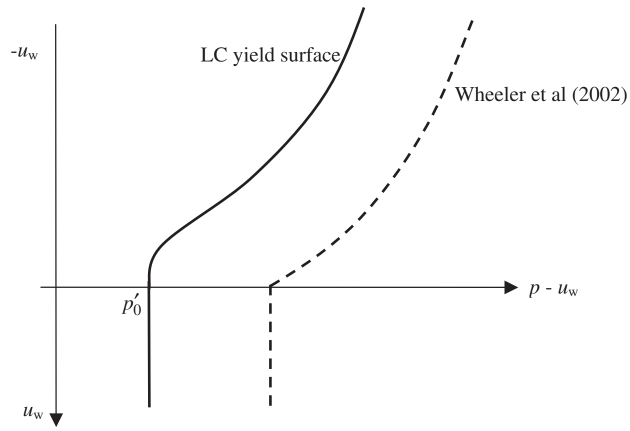

Figure 1. Transition of the load–collapse yield surface across the full saturation.

the logarithmic specific volume and the logarithmic mean stress ($\ln v - \ln p$). The original BBM is an elasto-plastically coupled model where the instantaneous elastic modulus does depend on the accumulated plastic strain (the elastic bulk modulus is also a function of the specific volume which in turn a function of the accumulated plastic strain). In such a case, one can not prove the convexity of the yield surface from the Drucker postulate.

However, I fully agree with Professor Wheeler and his colleagues on the consequence of the non-convexity of the yield surface, namely it will cause problems in stress integration, in particular when the implicit returning mapping scheme is used. Let us look at a hypothetical stress path shown in Figure 2. If an elastic trial stress increment during one single increment of strain and suction crosses the yield surface more than once, the only possible way to solve the incremental stress–strain equations is to use an explicit scheme that incorporates the substepping technique and is able to locate multi-intersection points with the yield surface (see References [3,4]). Implicit schemes that evaluate the gradients of the yield surface and plastic potential and hardening laws at unknown stress states and then solve the resulting nonlinear equations by iteration, will have difficulty here.

While the non-convexity inevitably exists, the vertex of the yield surface of the original BBM at the transition point between saturated and unsaturated states can be avoided by smoothing the function defining the negative slope of the normal compression line at constant suction, for example, as [4]

$$\lambda_s = \begin{cases} \lambda_0((1-r)\exp(\beta s) + r) & s \geqslant s_2 \\ \lambda_0\left((1-r)\left(0.875 + \sqrt{0.015675 + (\beta s - s_1)^2}\right) + r\right) & s_1 \leqslant s < s_2 \\ \lambda_0 & s < s_1 \end{cases}$$

where the constants s_1 and s_2 can be solved by requiring the continuity of the gradient $d\lambda_s/ds$. With this smoothed λ_s, the LC yield surface defined in the BBM using the net stress:

$$\bar{p}_0(s) = p_r \left(\frac{\bar{p}_0(s=0)}{p_r}\right)^{(\lambda_0 - \kappa)/(\lambda_s - \kappa)}$$

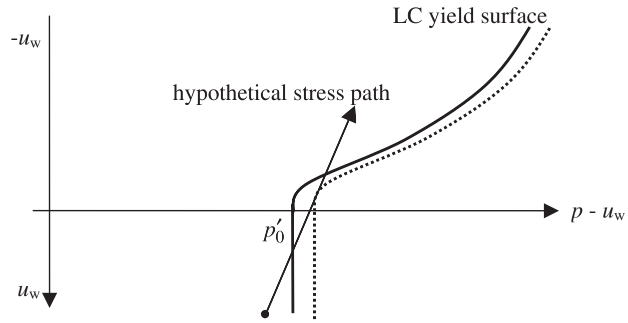

Figure 2. A hypothetical stress path that crosses the load–collapse yield surface twice.

or using the average stress:

$$p'_0(s) = p_0(s) - S_r u_w - (1 - S_r)u_a = p_r\left(\frac{p'_0(s=0)}{p_r}\right)^{(\lambda_0-\kappa)/(\lambda_s-\kappa)} + S_r s$$

will be smooth across the transition point between saturated and unsaturated states.

REFERENCES

1. Lubarda VA, Mastilovic S, Knap J. Some comments on plasticity postulates and non-associative flow rules. *International Journal of Mechanical Sciences* 1996; **38**:247.
2. Palmer AC, Maier G, Drucker DC. Normality relations and convexity of yield surfaces for unstable materials or structures, *Journal of Applied Mechanics* 1967; **34**:464.
3. Sloan SW, Abbo AJ, Sheng D. Refined explicit integration of elastoplastic models with automatic error control. *Engineering Computations* 2001; **18**:121–154.
4. Sheng D, Sloan SW, Gens A, Smith DW. Finite element formulation and algorithms for unsaturated soils. Part I: Theory. *International Journal for Numerical and Analytical Methods in Geomechanics*, 2003; **27**:745–765.