Some questions about unsaturated soil modelling

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Abstract: The last two decades or so have seen significant advances in unsaturated soil mechanics. Nevertheless, a number of fundamental questions have not been fully answered. These questions include: (1) the difference in modelling reconstituted and compacted soils and the implication of double porosity in constitutive modelling, (2) the relationship between volume change, yield surface and shear strength, (3) the implications of using a Bishop effective stress, and (4) the effect of deformation (void ratio) on soil water retention behaviour. This paper attempts to discuss these questions and in particular their implications in constitutive modelling of unsaturated soils.

Keywords: unsaturated soils, constitutive modelling, reconstituted soils, compacted soils

1 INTRODUCTION

Soils that are partially saturated with water are often referred to as unsaturated soils. Some soils exhibit distinctive volume, strength and hydraulic properties when become unsaturated. For these soils, a change in the degree of saturation can cause significant changes in the volume, shear strength and hydraulic properties. Nevertheless, the distinctive volume, strength and hydraulic behaviour for unsaturated states should be treated as material nonlinearity and modelled consistently and coherently. After all, all soils can become partially saturated with water and an unsaturated soil is only a state of the soil, not a new soil.

In the last two decades or so, significant advances have been made on constitutive modelling of unsaturated soils. A large number of constitutive models can be found in the literature. Recent reviews of these models can be found in Gens (2010) and Sheng (2011). Notwithstanding these advances, some fundamental questions remain unanswered or not fully answered:

1. *Reconstituted soil versus compacted soil*: What are the main differences in the hydro-mechanical behaviour of these soils? What are the implications of a unimodal and a bimodal pore size distribution (PSD) in constitutive modelling? Can a reconstituted soil become collapsible?
2. *Relationship between volume change, yield stress and shear strength*: Can the constitutive equations for volume change, yield stress and shear strength be defined separately? What are consequences if the loading-collapse yield surface does not recover the apparent tensile strength surface? Do we need the suction-increase surface to capture possible plastic volume change when a soil is dried to a historically high suction? What are the implications of stress state variables in defining volume change and shear strength equations?
3. *Implications of using a Bishop effective stress*: Can we use a Bishop effective stress in constitutive modelling of unsaturated soils and what are the implications? Can the stress-path dependency in the elastic zone be avoided?
4. *Effects of deformation on soil water retention behaviour*: How do we capture the deformation effects on soil water characteristic (retention) curves (SWCC or SWRC)? Can we use the conventional SWCC equations to fit data from both constant stress and constant volume tests? Is the plastic volume change always synchronised with 'plastic' change of degree of saturation?

These questions represent the most fundamental issues in unsaturated soil mechanics. There are currently no full answers to these questions. This paper presents the authors' understanding of these questions and their thoughts on possible answers. It is their wish that the paper can serve as a springboard leading to more in-depth discussion and perhaps improved understanding of these most fundamental issues in unsaturated soil mechanics.

2 RECONSTITUTED SOILS VERSUS COMPACTED SOILS

A soil can become partially saturated in different ways. Two types of unsaturated soil samples are often used in laboratory: (1) dry soil powders mixed at specified water contents are statically or dynamically compacted, (2) saturated samples reconstituted from slurry (at moisture contents in excess of the liquid limit) are dried to unsaturated states. The first type of samples (compacted soils) is far more common than the second type of samples (reconstituted soils), because it is more difficult to desaturate a slurry sample than a compacted sample. It has been noted that most constitutive models for unsaturated soils are based on experimental data for compacted soils (Sheng *et al.* 2008). Compacted soil samples can be prepared dry of optimum or wet of optimum. Reconstituted samples can be air-dried, heat-dried, freeze-dried or osmotically-dried. Different sample preparation methods usually result in different soil microstructures. For example, soils compacted dry of optimum usually have a double-porosity microstructure, meaning that the pore size distribution curve exhibits two or more peaks. In these soils, there usually exist two types of pores: large inter-aggregates pores which are collapsible upon wetting and small intra-aggregates pore which are more stable. On the other hand, soils air-dried from slurry usually exhibit a unimodal pore size distribution, at least at low stresses. Nevertheless, as recently pointed out by Tarantino (2010), the boundary between compacted and reconstituted soils is not always clear and the microstructure of a soil can change with stress and hydraulic paths.

Compacted unsaturated soils with a bimodal pore size distribution are usually collapsible at certain stress ranges. Wetting of such a soil can collapse the inter-aggregates pores and result in a unimodal pore size distribution when the soil becomes saturated. In terms of a constitutive model employing elastoplasticity such as the Barcelona Basic Model (BBM) developed by Alonso et al. (Alonso *et al.* 1990), a collapsible soil is typically characterised by a loading-collapse (LC) yield surface where the yield stress increases with increasing suction. This LC yield surface evolves with stress and/or suction changes. As shown in Figure 1, the compacted soil at point A is unsaturated (provided that suction is greater than the air entry suction) and has a bimodal pore size distribution. At point B the soil is saturated and can have a unimodal pore size distribution. Experimental data seems to support such an evolution of pore structure (Li & Zhang, 2009). The wetting path $A \rightarrow B$ causes the LC yield surface to evolve from LC_A to LC_B, as the inter-aggregate pores collapse and the soil volume decreases. However, drying the unimodal soil at point B to point A and then compressing it to point C, i.e. stress path $B \rightarrow A \rightarrow C$, should regenerate the bimodal pore size distribution, because the soil at point C is collapsible again according to BBM (Alonso *et al.* 1990). Some experimental data also support such a development of soil collapsibility

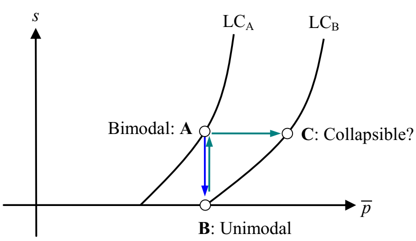


Figure 1. Evolution of pore size distribution and loading-collapse yield surface (s: matric suction, $\overline{p}$: net mean stress).

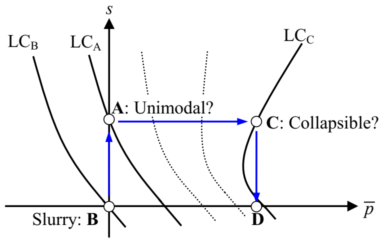


Figure 2. Evolution of the yield surface for a reconstituted soil according to the SFG model (Sheng *et al.* 2008).

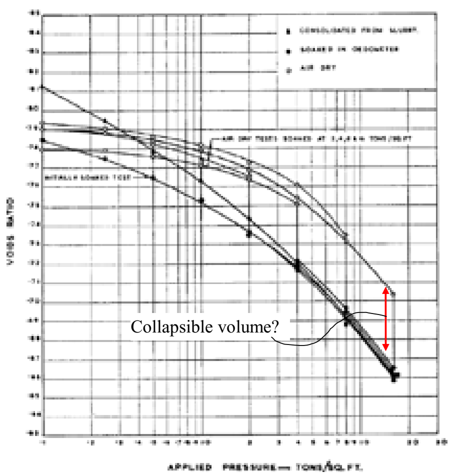


Figure 3. Oedometer curves for air-dry silt soaked at various constant applied pressures (Jennings & Burland, 1962)

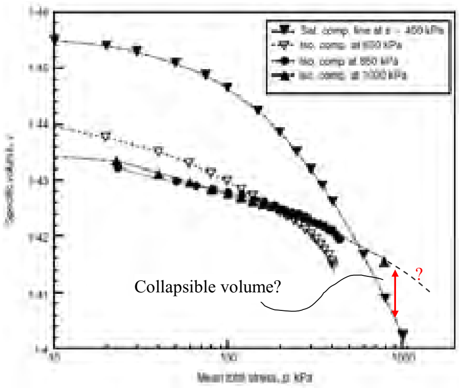


Figure 4. Isotropic compression curves for a reconstituted silty clay at various suctions (Cunningham *et al.* 2003)

(Suriol *et al.* 1998). In other words, the pore size distribution can evolve with stress and hydraulic paths (history). Most constitutive models in the literature are based on data for compacted soils and hence predict an evolution of the LC yield surface as shown in Figure 1.

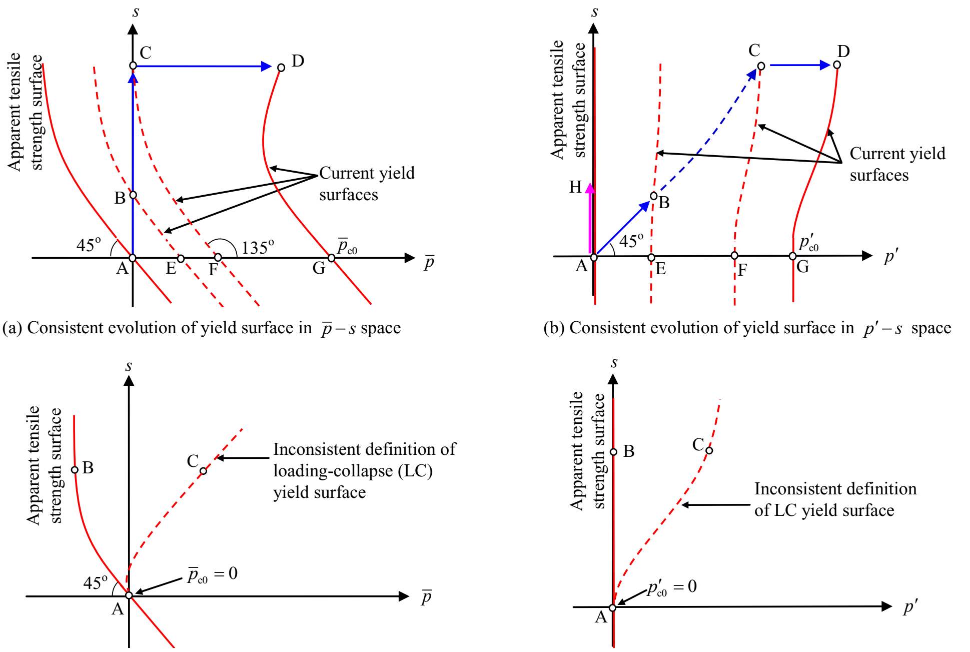
The question is: does this kind of yield surface evolution apply to reconstituted soils as well? A soil air-dried from slurry is characterised by a yield surface where the yield stress decreases with increasing suction (Point A in Figure 2). Since such a soil usually has a unimodal pore size distribution, wetting it under constant stress usually does not cause volume collapse. However, can such a soil become collapsible if it is compressed to high stresses? According to the SFG model (Sheng *et al.* 2008), the yield surface for an air-dried soil can evolve into a LC yield surface if the soil is compressed to sufficiently high stresses (Figure 2), which means that compressing a unimodal reconstituted soil at unsaturated states can generate a collapsible soil (stress path $A \rightarrow C$). Is there experimental evidence for such an evolution? Unfortunately there is very few data on reconstituted soils in the literature. Nevertheless, the classic reference by Jennings and Burland (1962) seems to support such an evolution. Figure 3 is a replot from Jennings and Burland (1962) for an air-dry silt. It is clear that the air-dry soil can become collapsible if it is compressed to sufficiently high stresses. This collapsibility is purely due to the degradation of soil stiffness during wetting. In other words, the volume decrease alone stress path BAC is smaller than the volume decrease along path BD. Therefore, wetting the soil from C to D will cause volume decrease (collapse). This collapse is not necessarily related to the microstructural change of the soil, rather to the overall stiffness change of the soil with suction. Another set of data reported by Cunningham *et al.* (2003) for a reconstituted silty clay also seem to suggest that compressing a soil at sufficiently high suction can make the soil collapsible (Figure 4). Therefore, the available data in the literature seem to support the evolution of the LC yield surface as suggested by the SFG model (Figure 2). However, experimental data on reconstituted soils are generally too few to be conclusive on this specific characteristic.

In summary, the microstructure and particularly the pore size distribution of a soil is usually reflected in the yield surface and volume change behaviour of the constitutive model for the soil. A bimodal pore size distribution can evolve to a unimodal pore size distribution under appropriate stress and hydraulic paths, and vice versa.

3 RELATIONSHIP BETWEEN VOLUME CHANGE, YIELD STRESS AND SHEAR STRENGTH

The volume change equation that defines the volume change caused by a stress or suction change also underpins the yield stress – suction and shear strength – suction relationships (Sheng *et al.* 2008). It is the most fundamental element that is needed to extend a saturated soil model to unsaturated states. The so-called LC yield surface can be derived from the volume change equation, as done in the BBM (Alonso *et al.* 1990) and in the SFG model (Sheng *et al.* 2008).

The shear strength – suction relation is related to the volume change equation, as recently shown by Sheng (2011). In constitutive models, the shear strength of an unsaturated soil is determined by (1) the apparent tensile strength function, and (2) the friction angle of the soil. The apparent tensile strength is usually a function of suction and this function can be derived from the volume change equation. The apparent tensile strength surface in the stress-suction space also represents the zero shear surface, as the soil has no shear strength when suction and stress change along this surface. For saturated soils, this surface follows the 135^o line in the net mean stress – suction space, but the 90^o (vertical) line in the effective mean stress – suction space. In fact, effective stress definitions for unsaturated soils in the literature are essentially the definitions of this zero shear strength surface. The discussion below will show that the zero shear strength surface is also the initial yield surface of a soil that has a zero preconsolidation stress and that the LC yield surface must recover this zero shear strength surface if the preconsolidation stress is set to zero.



(a) Consistent evolution of yield surface in $\bar{p}-s$ space

(b) Consistent evolution of yield surface in $p'-s$ space

(c) Inconsistent evolution of yield surface in $\bar{p}-s$ space

(d) Inconsistent evolution of yield surface in $p'-s$ space

Figure 5. Evolution of yield surface for a slurry soil (s: suction; $\bar{p}$: net mean stress; p' : effective mean stress; p_{c0} : preconsolidation stress at zero suction; ABC: stress path under constant net mean stress, CD: stress path under constant suction,).

The relationship between the shear strength and the LC yield surface has often been overlooked in some existing models for unsaturated soils. In the BBM (Alonso *et al.* 1990) and early models based on the BBM, the volume change equation has a singularity when the suction becomes zero. As a consequence, the LC yield surface becomes undefined when the preconsolidation stress at zero suction is zero, and hence the apparent tensile strength surface can not be derived from the volume change equation. Therefore, a separate apparent tensile strength surface was introduced in the BBM. However, it should be noted that this is solely due to the singularity in the volume change equation. In later models based on the Bishop effective stress and in the SFG model, the volume change equation is continuously defined through saturated and unsaturated regions. As such, the apparent tensile strength surface is automatically recovered from the LC yield surface by setting the preconsolidation stress ($\bar{p}_{c0}$ or p'_{c0}) to zero at zero suction (Figures 5a, 5b). Hence a separate function for the apparent tensile strength surface is not required. In fact, the original suction-increase (SI) yield surface in BBM, which is a horizontal line in the $\bar{p}-s$ space is used to capture the possible plastic volume change when a soil is dried to a historically high suction. The SI yield surface is not needed if the volume change equation is properly defined, as shown in Figures 5a and 5b. On the other hand, if the LC yield surface does not recover the apparent tensile strength surface (as shown in Figures 5c and 5d) when the preconsolidation stress ($\bar{p}_{c0}$ or p'_{c0}) is set to zero, the yield stress and shear strength functions would become non-unique. The LC yield surfaces shown in Figures 5c and 5d are also in conflict with the definition of yield surface, which are contours of the hardening parameter in the stress space. Obviously, the hardening parameter (e.g., the plastic volumetric strain) can not be constant along these LC yield surfaces. The hardening parameter at points B and C should have the exactly same value as point A in Figures 5c and 5d, but plastic volumetric strain must occur in order to expand the yield surface from B to C. Therefore, the LC yield surface must recover the apparent tensile strength surface when the preconsolidation stress at

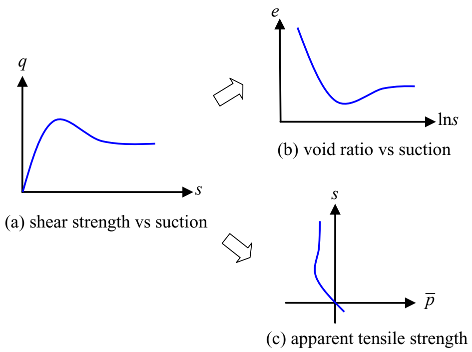


Figure 6. Implications of peak shear strength at intermediate suctions.

zero suction is set to zero, irrespective of the stress space used.

It is sometimes observed that the shear strength of an unsaturated soil exhibits a peak value at an intermediate suction level (Figure 6a). Such a peak shear strength has implications in the volume change equation and hence in the yield surface as well. To capture peak shear strength, the equation that defines the volume change caused by suction changes should predict a minimum value at the intermediate suction (Figure 6b). As a consequence, the apparent tensile strength should also change (Figure 6c).

The shear strength of an unsaturated soil is often considered to be sufficiently defined by a single effective stress (Wheeler, 2006; Alonso *et al.*, 2010; Sheng, 2011). However, this is only true when the slope of the critical state line or the friction angle of the soil does not change with suction. When the friction angle of the soil changes with suction, the shear strength cannot be defined by a single effective stress. This is clear from the general shear strength equation proposed by Fredlund *et al.* (1978):

$$\begin{aligned} \tau &= \left[c' + \bar{\sigma}_{\mathrm{n}} \tan\phi'(s)\right] + \left[s \tan\phi^{\mathrm{b}}\right] \\ &= c' + \left(\bar{\sigma}_{\mathrm{n}} + s\frac{\tan\phi^{\mathrm{b}}}{\tan\phi'(s)}\right)\tan\phi'(s) \qquad (1) \\ &= c' + \sigma'_{\mathrm{n}} \tan\phi'(s) \end{aligned}$$

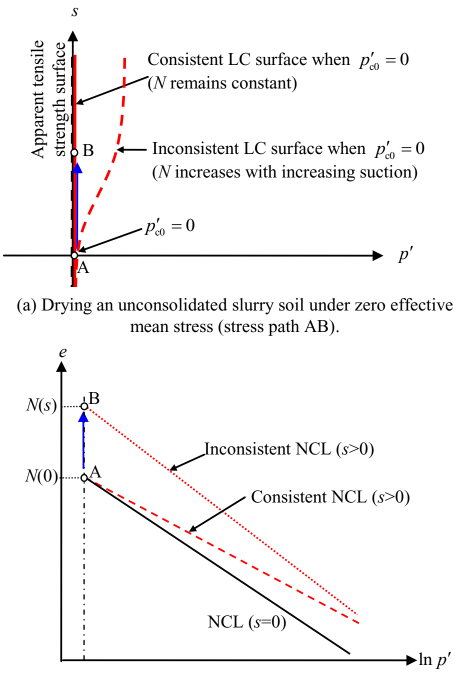
where τ is the shear strength, c' is the effective cohesion for saturated states, σ'_{n} and $\bar{\sigma}_{\mathrm{n}}$ are respectively the effective and net normal stress on the failure plane, and ϕ' is the friction angle of the soil. In the equation above, the Bishop effective stress parameter (χ) is set to $\tan\phi^{\mathrm{b}} / \tan\phi'$. It is clear that the variable (s) can not be eliminated from equation (1) no matter how the effective stress is defined. While there are experimental data supporting that the slope of the critical state line is independent of suction (e.g., Ng & Chiu 2001; Thu *et al.* 2007), there are perhaps equal amount of data supporting the opposite (e.g., Toll 1990; Futai & de Almeida 2005; Merchán *et al.* 2008). Therefore, there is no conclusive evidence that the shear strength of an unsaturated soil can be sufficiently defined by a single effective stress.

4 IMPLICATIONS OF USING A BISHOP EFFECTIVE STRESS

An important issue about constitutive modelling of unsaturated soils is the choice of the stress space where the constitutive model is built. Early models were usually established in the net stress and suction space (e.g. Alonso *et al.* 1990), while the second generation models have mostly used Bishop effective stress and suction (e.g., Wheeler *et al.* 2003; Sheng *et al.* 2004; Tamagnini 2004). The more recent SFG model (Sheng *et al.* 2008) has reverted back to the net stress and suction space.

Net stress and suction are both independent and controllable variables in laboratory tests. Therefore, it is straight forward to represent laboratory tests using models built in such a space. The definitions of net stress and suction are independent of material states and hence the stress space remains fixed. This is a significant advantage of the independent stress variables. As pointed out by Morgenstern (1979), we normally link equilibrium considerations to deformations through constitutive behaviour and should not introduce constitutive behaviour into the stress state variables. However, models based on net stress and suction usually have difficulties in (1) handling the transition between saturated and unsaturated states, (2) the shear strength variation with suction, (3) the effects of degree of saturation and the coupling with hydraulic hysteresis. One exception is perhaps the more recent SFG model (Sheng *et al.* 2008), which seems to have overcome most of the difficulties. These difficulties are indeed the main motivation for the shift to the Bishop effective stress in the second generation models.

The Bishop effective stress is not a controllable variable in laboratory tests. The effective stress definition usually involves material state variables such as the degree of saturation or air entry value, and hence the stress space changes as the material state changes. The constitutive behaviour of the material is embedded both in the constitutive equation and the effective stress definition, leading to less explicit physical meaning of the constitutive law. Nevertheless, models built in the Bishop effective stress space have many advantages. For example, the smooth transition between saturated and unsaturated states,



(a) Drying an unconsolidated slurry soil under zero effective mean stress (stress path AB).

(b) Shift of normal compression lines with suction?

Figure 7. Inconsistency of a varying N in the Bishop effective stress approach.

coherent relationship between volume change, shear strength and yield surface, easy incorporation of the effects of hydraulic hysteresis and the degree of saturation, a straightforward finite element implementation.

The above-mentioned advantages and shortcomings of the two categories of models are relatively well known in the literature. However, there is one specific issue of using the Bishop effective stress, which is related to the volume change equation and is less known in the literature, as described below.

A general form of the effective stress can be

$$p' = \bar{p} + \chi(s, S_r)s \qquad (2)$$

where $\bar{p}$ is the net mean stress, p' is the effective mean stress, χ is the Bishop effective stress parameter and is treated either as a function of suction or as a function of suction and degree of saturation. Obviously such a definition of effective stress is very general and covers all existing definitions we use in the literature (perhaps not Vlahinić *et al.* 2011).

With such an effective stress, the volume change equation for normally consolidated soils is usually assumed to take the following form:

$$v = N(s) - \lambda(s)\ln p' \qquad (3)$$

where v is the specific volume of the soil, λ is the compression index, and N is the specific volume when $\ln p' = 0$ or the specific volume for a soil slurry (at the liquid limit). If the effective stress is indeed effective in controlling volume change, the specific volume (v) should remain constant under constant p', and hence N and λ should be independent of suction. However, this is seldom true in reality. In the literature, N and λ are usually assumed to be function of s. We will show here that a varying N can lead to inconsistency with the yield surface evolution and can not be recommended.

In all models that are based on effective stress, the apparent tensile strength surface or the zero shear strength surface is assumed to be the vertical line that goes through zero effective mean stress, i.e.,

$$p' = 0 \qquad (4)$$

In other words, the soil is assumed to have zero shear strength as long as the effective mean stress remains zero, or the soil has no apparent tensile strength in the effective stress space. This line must also represent the yield surface for a slurry soil that has never been consolidated. For a saturated soil that has a zero yield stress, i.e. a slurry, the size of the elastic zone remains zero as long as the effective mean stress remains zero, irrespective of the pore water pressure. When the soil becomes unsaturated, the yield surface will continue along the zero shear strength line, if the effective mean stress remains zero. To keep the effective mean stress zero, a tensile net mean stress has to be applied to balance out the suction increase. As a consequence, the size of the elastic zone remains zero, which also reflects the effective stress principle. Furthermore, if the yield surface does not collapse to the zero shear strength line when p'_{c0}=0 (the dashed LC curve in Figure 7a), there would be plastic deformation for loading along the yield surface, and the shear strength as well as the yield stress would become non-uniquely defined.

Therefore, drying a slurry soil under zero effective mean stress is a neutral loading process, meaning that the stress point is always on the yield surface (path AB in Figure 7a). The elastic zone does not expand, which is different from the case of drying a slurry soil under zero net mean stress (Figure 5b). Such a loading process is also logical if there exists an effective stress. To keep the effective mean stress zero as the suction is increased, a tensile net mean stress has to be applied. This tensile net mean stress

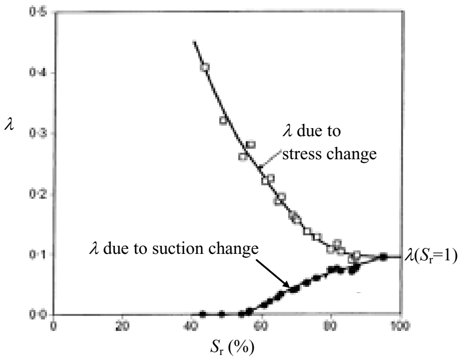


Figure 8. Variation of λ with degree of saturation (after Toll 1990).

balances out the suction increment; the essence of the effective mean stress principle.

As a consequence, drying a slurry soil under zero effective mean stress implies that the soil is always on the normal compression line (NCL). This is only possible:

1. if N remains constant or decreases with increasing suction (the red dashed line in Figure 7b), or
2. if N increases with increasing s (the red dotted line in Figure 7b).

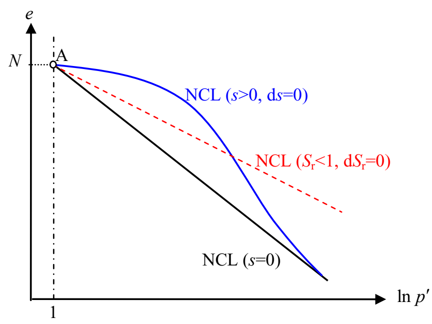
Case one would lead to the constraint that λ must decrease with increasing suction, which suggests that the collapse strain increases with increasing stress level and the soil compressibility decreases with increasing suction. Such behaviour is not always supported by experimental data. Experimental data by, for example, Toll (1990), Sharma (1998), Sivakumar & Wheeler (2000) and Toll & Ong (2003) all indicate that the soil compressibility can increase with increasing suction (Figure 8). The data by, for example, Vilar & Davies (2002), Sun et al. (2007a) and Vilar & Rodrigues (2011) show that the collapse potential reaches a maximum at intermediate stress levels. Case two suggests that the soil volume increases when drying under constant effective mean stress, which is not supported by any experimental data.

A possible strategy to overcome the above limitation of the Bishop effective stress approach is to adopt a volume change equation of the following form:

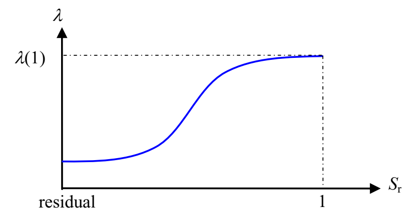
$$v = N - \lambda(S_r)\ln p' \qquad (5)$$

where N is a constant, and λ is a function of degree of saturation (S_r).

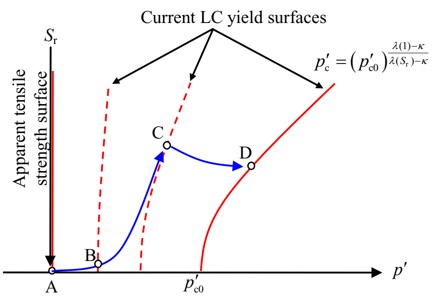
Because the compression index (λ) is a function of S_r, the normal compression line under a constant suction would not be a straight line unless the suction is zero (Figure 9a). Compressing an unsaturated soil under constant suction will generally lead to an increase in degree of saturation. If the compression index (λ) is assumed to vary with degree of saturation according to Figure 9b, the normal compression line (NCL) for constant suction would take the form of Figure 9a, which mimics the oedometer curves (Figure 10) for a silty sand tested by Jennings and Burland (1962).



(a) λ expressed as a function of degree of saturation (S_r)



(b) variation of soil compressibility with S_r



(c) Evolution of the yield surface for a slurry soil (ABC: constant net mean stress, CD: constant suction).

Figure 9. Saturation-dependent compression index ($\lambda(S_r)$) and its implications.

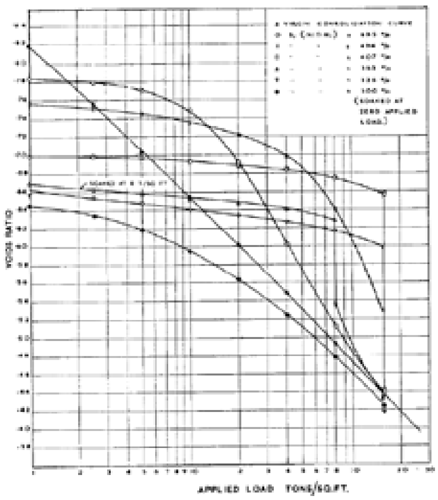


Figure 10. Oedometer curve for a silty sand at various initial degrees of saturation (Jennings & Burland 1962).

The implication of using a saturation-dependent compression index is that the yield surface will also depends on S_r. In fact, the LC yield surface can be derived from equation (5):

$$p'_c = \left(p'_{c0}\right)^{\frac{\lambda(1)-\kappa}{\lambda(S_r)-\kappa}} \qquad (6)$$

where p'_c is the yield stress, p'_{c0} is the yield stress at saturation (S_r=1), and κ is the elastic compression index. Because the non-unique relationship between S_r and s due to hydraulic hysteresis, the yield surface can no longer be plotted in the space of effective mean stress versus suction. Instead, we have to use the degree of saturation as the additional axis of the stress space, and plot the yield surface in the space of effective mean stress versus degree of saturation (Figure 9c). Because neither the degree of saturation nor the effective stress is a controllable variable in laboratory tests, it will be difficult to represent simple stress paths like constant net mean stress (ABC in Figure 9c) or constant suction (CD in Figure 8c) in the $p' - S_r$ space. Nevertheless, there are a few advantages on this approach:

1. The constraints on the compression index due to the use of the Bishop effect stress is avoided;
2. It is a natural outcome of equation (5) that the collapsible volume reaches a maximum value at an intermediate stress level (Figure 9a);
3. According to such a model, it is possible to compress an unsaturated soil to full saturation even if the suction is kept constant.

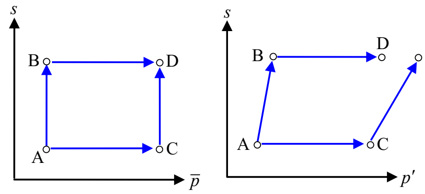


Figure 11. A closed loop in the net stress space and its corresponding open loop in the effective stress space

The above approach was initially suggested by Sheng (2011) and a complete model has recently been developed by Zhou *et al.* (2011). The model by Zhou *et al.* (2011) shows very promising features in capturing unsaturated soil behaviour. The challenge of using this approach is that simple stress paths in laboratory tests become very complex and a complete mathematical model is always needed to interpret laboratory tests, in addition to the other implications of the Bishop effective stress mentioned above.

Another issue related to the Bishop effective stress is that a closed loop of net stress and suction changes do not necessarily lead to a closed loop of effective stress changes (Figure 11), because of the material state dependency of the Bishop effective stress. For example, the effective mean stress change along path AB is usually different from that along path CD, because the degree of saturation has changed from A to C, even though the net mean stress is kept constant during both paths (Figure 11). Such an unclosed loop means that the model is stress-path dependent, even if the stress changes are inside the elastic zone. In a recent discussion by Zhang & Lytton (2008), it was found that models based on net stress are usually stress-path dependent in the elastic zone, but the discussion did not realise that models based on effective stress are also stress-path dependent. A model that exhibits stress-path dependent elastic behaviour is at variance with classical elastoplasticity theory and thermodynamics and should generally be avoided. However, an unsaturated soil model that exhibits stress-path independent elastic behaviour does not currently exist in the literature and is a remaining challenge.

5 EFFECTS OF DEFORMATION ON WATER RETENTION BEHAVIOUR

In constitutive models, the influences of mechanical properties on the hydraulic behaviour are usually modelled via the dependency of the SWCC on soil density. Gallipoli *et al.* (2003) suggested including a function of specific volume (v) in the SWCC equation of van Genuchten (1980). It is also common to express the SWCC equation in incremental forms. For example, Sun *et al.* (2007b) proposed a hydraulic model in the following form:

$$\mathrm{d}S_\mathrm{r} = \lambda_\mathrm{se}\mathrm{d}e - \lambda_\mathrm{ss}\,\mathrm{d}s/s \tag{7}$$

where λ_ss is the slope of main drying or wetting curve, and λ_se the slope of degree of saturation versus void ratio curve under constant suction. In theory, λ_ss in the equation above can only be determined from constant-volume tests (de=0). However, such tests are not common. Mašín (2010) used a similar equation as (7). In his model both air entry value (s_ae) and the slope of main drying curve (λ_ss) vary with void ratio. Nuth & Laloui (2008) provided an alternative approach of modelling SWCC for a deforming soil. They assume there is an intrinsic SWCC for a non-deforming soil and deformation of the soil can shift this intrinsic SWCC along the suction axis. The shift is governed by an air entry value that depends on the volumetric strain.

The models by Sun *et al.* (2007b), Nuth & Laloui (2008), Mašín (2010) and many others (e.g. Sheng *et al.* 2004, 2008; Nuth & Laloui 2008; Zhou 2009) essentially all adopt a water retention equation in the following form:

$$\mathrm{d}S_\mathrm{r} = (C)\,\mathrm{d}s + (D)\,\mathrm{d}\varepsilon_\mathrm{v} \tag{8}$$

where C and D are two general functions. This equation is not wrong, but the embedded $S_\mathrm{r} - s$ relationship is for constant volume ($\mathrm{d}\varepsilon_\mathrm{v} = 0$). Therefore, the function C is not equivalent to the gradient of conventional SWCC equations, which are usually used for water retention behaviour under constant stress. The volume change along a conventional SWCC can be significant, particularly at low suctions. For expansive soils, it is also common to study the water retention behaviour under constant volume (Romero *et al.* 1999; Lloret *et al.* 2003; Romero *et al.* 2003). On the other hand, if one specific $S_\mathrm{r} - s$ relation is valid for constant stress, the $S_\mathrm{r} - s$ relation for constant volume would inevitably involve soil compressibility and hence stress (suction) level and stress (suction) history. Therefore, it is unlikely that the same $S_\mathrm{r} - s$ relation can be calibrated for water retention behaviour both under constant stress and constant volume. Furthermore, neglecting the volume change along a SWCC can lead to inconsistent prediction of the degree of saturation during undrained compression, as pointed out by Zhang & Lytton (2008).

Sheng & Zhou (2011) proposed a new method for coupling hydraulic with mechanical behaviour. This new method is based on the fact that SWCCs are obtained under constant stress. The volume change behaviour and the water retention behaviour under isotropic stress states are represented by the following incremental equations, respectively:

$$\mathrm{d}\varepsilon_\mathrm{v} = A\,\mathrm{d}\bar{p} + B\,\mathrm{d}s \tag{9}$$

$$\begin{aligned}\mathrm{d}S_\mathrm{r} &= E\,\mathrm{d}s + \frac{S_\mathrm{r}}{n}(1-S_\mathrm{r})^{\zeta} A\,\mathrm{d}\bar{p} \\ &= \left(E - B\frac{S_\mathrm{r}}{n}(1-S_\mathrm{r})^{\zeta}\right)\mathrm{d}s - \frac{S_\mathrm{r}}{e}(1-S_\mathrm{r})^{\zeta}\,\mathrm{d}e\end{aligned} \tag{10}$$

where variables A and B are used to define the soil compressibility under stress and suction changes, respectively, variable E is the gradient of the conventional SWCC, e is the void ratio, n is the porosity, and ζ is a fitting parameter. Variable A and B depend on the specific volume change equation adopted. If the SFG model is used to describe the volume change, A and B would then take the form:

$$A = \frac{\lambda_\mathrm{vp}}{\bar{p} + f(s)}, \qquad B = \frac{\lambda_\mathrm{vs}}{\bar{p} + f(s)} \tag{11}$$

where λ_vp is the compression index due to stress change, λ_vs is the compression index due to suction change, and $f(s)$ is a function of suction and was set to s in Sheng *et al.* (2008).

The $S_\mathrm{r} - s$ relationship is defined for constant stress ($\mathrm{d}\bar{p}$=0) in equation (10) and hence parameter E refers to the gradient of the conventional SWCC:

$$E = \frac{\mathrm{d}\left(S_\mathrm{r}^\mathrm{SWCC}(s)\right)}{\mathrm{d}s} \tag{12}$$

where $S_\mathrm{r}^\mathrm{SWCC}$ represents the conventional SWCC equation. The void ratio in equation (10) refers to the value at the current stress and suction.

It is clear from equation (10) that the $S_\mathrm{r} - s$ relationship for constant volume (de=0) is more complex than the conventional SWCC equation ($\mathrm{d}S_\mathrm{r} = E\,\mathrm{d}s$). Equation (10) was proposed based on experimental observation as well as the intrinsic phase relationship for undrained condition:

$$\mathrm{d}S_\mathrm{r} = \frac{S_\mathrm{r}}{n}\mathrm{d}\varepsilon_\mathrm{v}, \qquad \mathrm{d}w = 0 \tag{13}$$

where w is the gravimetric water content. Equation (13) actually imposes a constraint on suction change under undrained compression. This constraint is obtained by substituting equation (13) into (10):

$$\left(S_\mathrm{r} - S_\mathrm{r}(1-S_\mathrm{r})^{\zeta}\right)A\,\mathrm{d}\bar{p} = (nE - B)\,\mathrm{d}s, \quad \mathrm{d}w = 0 \tag{14}$$

Equation (13) implies that the degree of saturation (S_r) remains constant as long as $\varepsilon_\mathrm{v} = 0$ and dw=0, irrespective of suction change. Equation (13) is satisfied as long as the suction changes according to Equation (14). The synchronised evolution of the

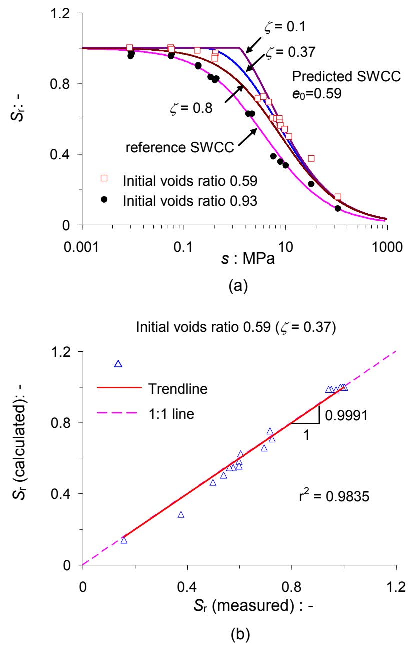


Figure 12. Calibration of reference SWCC and parameter ζ according to equation (15): (a) calibration curves with different ζ; (b) calculated S_r vs. measured S_r ($\zeta = 0.37$). Van Genuchten fitting parameters: a = 4MPa, m = 0.7, n = 1.0, and S_r^{res} =0.0. Data after Romero (1999).

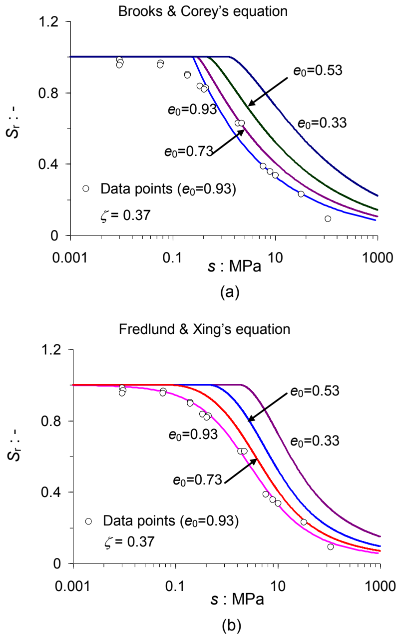


Figure 13. Calculated SWCCs for different initial densities according equation (15): (a) Brooks & Corey's equation (s_{ae} = 0.25 MPa, λ = 0.3); (b) Fredlund & Xing's equation (α = 2.0 MPa, $\beta = 0.8$, $\gamma = 1.8$, $s_{re} = 10^5$ kPa). Data after Romero (1999).

LC, SI and suction-decrease (SD) yield surfaces is not necessary for consistent predictions of saturation and volume changes. Indeed, the suction path can be 'elastoplastic' even though the stress path is elastic. The following equation can be derived from Equation (10):

$$\frac{\partial S_r}{\partial e} = -\frac{S_r(1-S_r)^{\zeta}}{e}, \qquad ds=0 \tag{15}$$

The void ratio (e) in equation (15) refers to the void ratio at the current stress and its variation is purely due to stress change. This void ratio can also be interpreted as the initial void ratio at the start of the SWCC tests where the stress is kept constant. Equation (15) shows that the SWCC for a soil shifts with its initial void ratio. This is similar to the approach by Gallipoli *et al.* (2003) where the van Genuchten equation was modified to include the initial void ratio. Their SWCC equation can be re-written as:

$$\frac{\partial S_r}{\partial e} = -mn\psi\frac{S_r(1-S_r^{1/m})}{e} \tag{16}$$

where m and n are two fitting parameters in the original van Genuchten equation, and ψ is another parameter introduced by Gallipoli *et al.* (2003). If the product ($mn\psi$) was set to 1, equation (16) would be equivalent to Equation (15). The intrinsic phase relationship requires (Sheng & Zhou, 2011):

$$\frac{1-S_r}{e} \geq \frac{\partial S_r}{\partial e} \geq \frac{-S_r}{e} \tag{17}$$

The above constraint is satisfied if $mn\psi$=1 in equation (16). It is also interesting to note that all the numerical examples in Gallipoli *et al.* (2003) used a value of 1.10 for $mn\psi$.

Equation (15) can be integrated either analytically for certain ζ values or numerically in more general cases. Because equation (15) is in an incremental form, integration of the equation requires one specif-

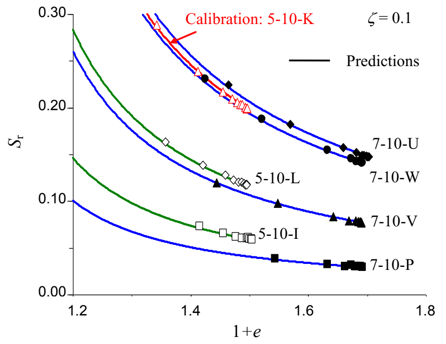


Figure 14. Validation of equation (15) using suction-controlled oedometer compression data of a compacted soil (data after Jotisankasa, 2005).

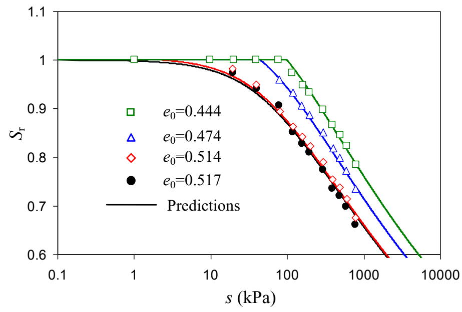


Figure 15. SWCCs for specimens compacted at optimum water content (data after Vanapalli *et al.* 1999).

ic SWCC equation that corresponds to a reference initial void ratio. In other words, the conventional SWCC equation is only used for the reference initial void ratio and the new SWCC for a new initial void ratio is obtained by integration of equation (15). The SWCC at the reference void ratio is named as the reference SWCC in Figures 12 and 13.

One advantage of Equation (15) is that it can be used in conjunction with any existing SWCC equations, including those for unimodal pore size distribution (e.g. van Genuchten 1980; Fredlund & Xing 1994) and those for bimodal pore size distribution (e.g. Burger & Shackelford 2001; Gitirana & Fredlund 2004), as well as those advanced models that adopt hysteretic scanning loops within the main drying-wetting loop (Li 2005; Pedroso *et al.* 2008; Pedroso & Williams 2010). For example, Figure 12 shows the calibration of equation (15) against the van Genuchten's equation, using the data of Romero (1999). Figure 13 shows the predicted SWCCs based on Brooks and Corey's equation and Fredlund and Xing's equation, for different initial voids ratios (e_i = 0.93 0.73, 0.53 and 0.33). In all these cases, the proposed equation is able to capture the effect of initial voids ratio on the SWCC. It can also be seen from Figure 13 that the air entry value is a very inaccurate measure of the experimental suction threshold where the soil becomes unsaturated. Fredlund and Xing's equation and van Genuchten's equation do not use this threshold suction and generally better represent the experimental data.

The model by Sheng & Zhou (2011) is validated against a variety of data sets. In Figure 14, the results from suction-controlled oedometer compression tests by Jotisankasa (2005) are used to validate equation (15). Since the suction remains constant for each curve in Figure 14, no SWCC equation is required here and the only parameter required is ζ. Equation (15) can directly be used to predict the variation of degree of saturation due the variation of stress-induced volume change. In this case, the voids ratio (e) in equation (15) is the current voids ratio and it changes due to stress variation in the oedometer tests. Figure 14 clearly shows that equation (15) predicts very well the relationship between the degree of saturation and the voids ratio when the suction is kept constant. Indeed, the experimental data are almost exactly on the predicted $S_r - e$ curves.

In Figure 15, the data by Vanapalli *et al.* (1999) on a compacted till were used. The SWCC for e_0=0.517 was fitted by the van Genuchten equation, while the other SWCCs are predicted by equation (15) with ζ=0.03. The fitting parameters for van Genuchten's equation were calibrated as follows: a = 65 kPa, m = 1.0, n = 0.15, and S_r^{res} = 0.0. Figure 15 shows that both the slope and the air entry value of the SWCCs change with the initial void ratio. In the two cases studied, the model by Sheng & Zhou (2011) seems to be able to capture the effect of initial void ratio on the soil water retention behaviour.

6 CONCLUSIONS

In an attempt to answer those questions posed in the Introduction, the following remarks can be made:

1. It seems possible to use the same theoretical framework to model reconstituted soils and compacted soils. The pore size distribution evolves with stress and suction paths and can be modelled by the evolution of the loading collapse yield surface.
2. The volume change, yield stress and shear strength behaviour of an unsaturated soil are co-related to each other and it is not recommended to define these functions separately.
3. The loading-collapse yield surface should recov-

er the apparent tensile strength surface when the preconsolidation stress at zero suction is set to zero, to avoid non-uniqueness of the yield surface. The suction-increase yield surface used to capture the possible plastic volume change associated with drying is not truly needed, if the loading-collapse yield surface is properly defined.

4. A common perception is that the shear strength of an unsaturated soil can be defined uniquely by a single effective stress. However, this perception is only true when the friction angle of the soil does not change with suction.
5. There are implications associated with using the Bishop effective stress. Because there is only one compression index associated with both stress and suction changes in the volume change equation, this compression index is constrained to decrease with increasing suction. Such a constraint is not always supported by experimental data. A possible solution is to adopt a saturation-dependent compression index and to form the constitutive equations in the stress – saturation space.
6. It is still a challenging task to develop an elasto-plastic model where the elastic behaviour is stress-path independent.
7. When coupling the hydraulic behaviour with the mechanical behaviour, it is recommended to take into account the volume change along soil-water characteristic curves. Neglecting this volume change can lead to inconsistent prediction of volume and saturation changes.
8. Conventional SWCC equations can not be used to calibrate experimental data both from constant stress and constant volume. If one specific $S_r - s$ equation is valid for constant stress, the $S_r - s$ relation for constant volume would inevitably involve soil compressibility and hence stress (suction) level and stress (suction) history.

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