MATHS AND STATS

Further trigonometry

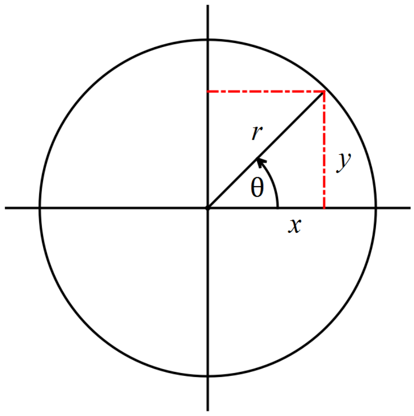
DEFINITIONS FROM COORDINATES

θ is measured anticlockwise (starting from "3 on a clock"). From the SOH CAH TOA you learnt in school you already know that :

$$\sin\theta = \frac{y}{r}$$

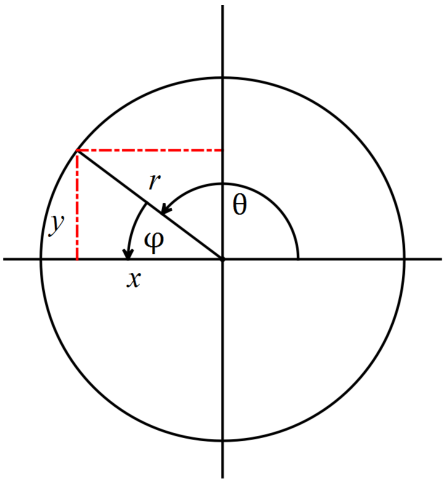
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



These definitions using coordinates can be used to extend the concept of trig ratios to angles that are bigger than 90˚. We will use the second quadrant as an example.

Let $\varphi = 180 - \theta$
Note that φ is an acute angle so it can be shown in a right-angled triangle.



Using the coordinate definition

$$\sin\theta = \frac{y}{r}$$

Using the acute triangle

$$\frac{y}{r} = \sin\varphi = \sin(180 - \theta)$$

So we have $\sin\theta = \sin(180 - \theta)$

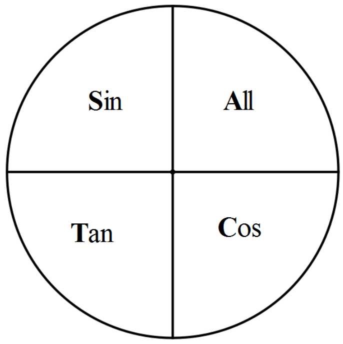
Similarly

$$\cos\theta = \frac{x}{r} = -\cos(180 - \theta)$$

$$\tan\theta = \frac{y}{x} = -\tan(180 - \theta)$$

Extending these definitions to cover the whole circle will reveal that for all angles the trig ratios can be related to a "working" angle that can be drawn in a right-angled triangle. **All working angles are measured back to thehorizontal line**. In some quadrants the x-coordinate or y-coordinate is negative so some trig ratios are negative.

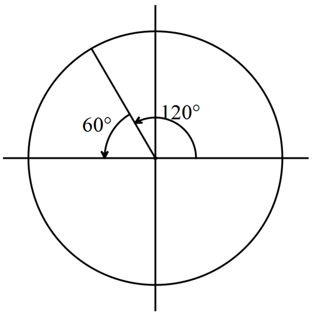
This diagram is a good way to remember which ratios are positive in which quadrants. Where the trig ratio named is positive and all other ratios are negative in that quadrant. A common mnemonic is '**A**ll **S**tations **T**o **C**entral'



Examples

***note the acute angle is ALWAYS made with the horizontal

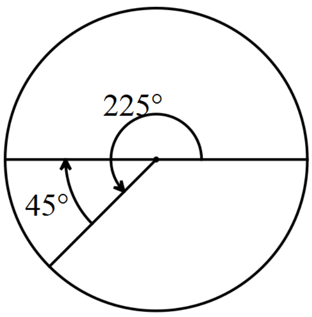
1) Find $\cos(120°)$



- Draw diagram to find "working" angle φ (working angle is measured back to the x-axis)
  $\theta = 120$ so $\varphi = 60$
- Decide if trig ratio is positive or negative.
- Calculate trig ratio of working angle (you may need your special triangles

$$\cos(120°) = -\cos(60°)$$
$$= -\frac{1}{2}$$

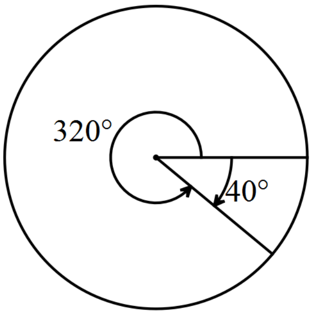
2) Find $tan(225°)$



- In the third quadrant the working angle is measured back to the horzintal.
- $\theta = 225$ so $\varphi = 45$
- tan is positive in this quadrant

$$\tan(225) = \tan(45)$$
$$= 1$$

3) Find tan $320°$



- In the fourth quadrant the working angle is measured up to the horizontal.
- $\theta = 320$ so $\varphi = 40$
- tan is negative in this quadrant

$$\tan(320) = -\tan(40) \quad \text{Use your calculator to find this}$$
$$= -0.8391\ (4d.p.)$$

Exercises

Find the following trigonometric ratios – note the acute angle is ALWAYS made with the horizontal

1. $\sin(45°)$
2. $\sin(300°)$
3. $\cos(210°)$
4. $\tan(315°)$
5. $\cos(115°)$
6. $\tan(240°)$

Answers

1. $\frac{1}{\sqrt{2}} \approx 0.707$
2. = - sin(60°) = $-\frac{\sqrt{3}}{2} \approx -0.866$
3. = - cos(30°) = $-\frac{\sqrt{3}}{2} \approx -0.866$
4. = - tan(45°) = -1
5. = - cos(65°) = -0.423
6. = tan(60°) = 1.732