

TECHNICAL NOTE

Towards a dimensionless description of soil swelling behaviour

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Soil swelling is a complex phenomenon resulting from adsorption of water onto the surface of clay platelets. It is influenced by procedural, environmental and structural factors. In particular, the initial hydration state of a soil (expressed in terms of saturation degree, water content or suction), its initial level of compaction (expressed in terms of void ratio or dry unit weight) and the level of confinement are among the influencing parameters most studied. Characterising the swelling potential of a soil in the laboratory is an extensive task and tools for prediction are somehow limited. This paper presents the first attempt to use dimensional analysis to predict the amount of soil swelling. This approach makes use of the Buckingham pi theorem to reduce an equation describing a physical phenomenon into an equation involving a reduced number of parameters, which are dimensionless. One of these numbers, called DSP_w , was specifically derived and validated using datasets from the literature. These data were obtained from monotonic one-dimensional swelling tests until full saturation. The results suggest that dimensional analysis and the resulting dimensionless model can be used to predict soil swelling with relatively good accuracy.

KEYWORDS: clays; expansive soils; laboratory tests; partial saturation

Le gonflement du sol est un phénomène complexe, résultant de l'adsorption d'eau sur la surface des feuillets d'argile. Il est influencé par des facteurs procéduraux, environnementaux et structurels. En particulier, l'hydratation initiale d'un sol (exprimée en terme de degré de saturation, de teneur en eau ou de succion), sa densité initiale (exprime en terme d'indice des vides ou de poids volumique sec), et le niveau de confinement, comptent parmi les paramètres déterminants les plus étudiés. La caractérisation du potentiel de gonflement d'un sol en laboratoire est un travail considérable, et les outils dont on dispose pour les prédictions sont plutôt limités. Cette communication présente une première tentative d'utilisation de l'analyse dimensionnelle pour prédire le degré de gonflement du sol. La méthode utilisée applique le théorème de Pi Buckingham pour réduire une équation décrivant un phénomène physique en une équation comportant un nombre réduit de paramètres, qui sont sans dimensions. Un de ces nombres, DSP_w , a été défini puis validé spécifiquement en utilisant des ensembles de données publiés dans la littérature et provenant d'essais de gonflement unidimensionnels monotones, jusqu'à saturation intégrale. Les résultats indiquent que l'on peut utiliser l'analyse dimensionnelle, et le modèle adimensionnel résultant, pour prédire, avec une précision relative bonne, le gonflement du sol.

INTRODUCTION

Soil swelling is a complex phenomenon with significant consequences for soil–structure interaction. Long-lasting experimental research has identified the influence of several factors on such deformation. For example, the influence of initial water content, initial dry unit weight or void ratio, confining pressure, pore water chemistry, previous mechanical history of the soil (e.g. soil preconsolidation pressure, drying–wetting history) have been demonstrated (Holtz & Gibbs, 1956; Seed *et al.*, 1961; Yevnin & Zaslavsky, 1970; Brackley, 1973; Morgenstern & Balasubramonian, 1980; Delage *et al.*, 1998; Alonso *et al.*, 2005; Monroy *et al.*, 2007). Many investigations have also recognised the influence of the amount and type of clay (Parcher & Liu 1965; Sivapullaiah *et al.* 1996) and of soil structure (Noble, 1966; Popescu, 1980; Salas & Serratosa, 1973; Ladd & Lambe, 1961) on the swelling process. In particular, partial or full shrinkage produce different structures (Al Homoud *et al.*, 1995; Basma *et al.*, 1996), which give rise to differences in swelling behaviour; so, for example, either amplification or reduction of swelling potential have been observed with increasing number of drying–wetting cycles (Chen, 1965;

Popescu, 1980; Pousada, 1984; Dif & Bluemel, 1991; Al Homoud *et al.*, 1995).

With such a large number of influencing factors, performing a comprehensive experimental characterisation of the swelling potential of a given soil has represented one of the main challenging tasks faced within the unsaturated soil mechanics research field in the last decades.

Given, however, the difficulties inherent to such challenging experimental investigations, dimensional analysis is proposed here as an alternative approach to the problem of defining a relation to predict the swelling response of clays. Dimensional analysis has been used in geotechnical engineering (Butterfield, 1999; Palmer, 2008), but it remains more an exception than a rule.

The objective of this paper is to show that dimensional analysis can be applied to predict the amount of swelling of compacted unsaturated clays. Swelling data from the literature have been used to validate a dimensionless model accounting for only few of the influencing factors discussed previously (initial water content, initial dry unit weight and vertical stress). The result is a dimensionless model capable of relatively accurate predictions of the volume change of compacted clays upon monotonic swelling. The limited number of factors taken into account in the analysis brings about the variability of some model parameters and the applicability of the model solely to some given swelling paths. The extension of the approach in order to account for other factors of the swelling process is thought to increase the predictive capacity of the modelling procedure being proposed.

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DIMENSIONAL ANALYSIS

Quite clearly, the derivation of a dimensional analysis to account for all of the factors governing the swelling phenomenon is a formidable task. However, the initial dry unit weight or void ratio, the initial suction, water content and confining pressure are here assumed to be primary factors (e.g. in Komine (2004)). Hence, in particular, the analysis developed in this work accounts only for the initial water content, the initial dry unit weight and the vertical stress.

Data from the literature resulting from one-dimensional swelling tests on compacted expansive clays, performed under a given vertical stress to a final condition of full saturation (i.e. zero matric suction), are used to validate the dimensional analysis. The initial mass of water M_{w_0} , the mass of solid particles M_s , the vertical stress σ_v and the initial dry unit weight γ_{d_0} can be used to describe the swelling phenomenon

$$f(\Delta h, h_0, \gamma_{d_0}, M_s, M_{w_0}, \sigma_v) = 0 \quad (1)$$

where h_0 is the initial height of the specimen and Δh is the change in height upon swelling.

Three dimensions (L , M and T) and five independent parameters can be identified in equation (1) (M_s is related to h_0 and to γ_{d_0}). According to the Buckingham pi theorem (Buckingham, 1914), the swelling phenomenon (equation (1)) can be reduced to a simpler equation involving $5 - 3 = 2$ dimensionless numbers, which depend on the combination of parameters present in the swelling equation (Buzzi *et al.*, 2008). Here, the first dimensionless number is the swelling strain ε_{sw}

$$\varepsilon_{sw} = \frac{\Delta h}{h_0} \quad (2)$$

Then, a second dimensionless number called the dimensionless swelling parameter (DSP_w) is defined

$$DSP_w = \left(\frac{\gamma_{d_0} \cdot h_0}{\sigma_v} \right)^a \left(\frac{M_s}{M_{w_0}} \right)^b = \left(\frac{\gamma_{d_0} \cdot h_0}{\sigma_v} \right)^a \left(\frac{1}{w_0} \right)^b \quad (3)$$

The dimensional analysis has been validated using some data sets from the literature in which the initial state of hydration of the soil is expressed in terms of water content. The use of initial water content leads to the presence of γ_{d_0} and h_0 in DSP_w . Indeed, the parameter describing the level of compaction (e_0 or γ_{d_0}) has to be combined with the vertical stress to form a dimensionless number. This is not possible using e_0 , so γ_{d_0} and h_0 are used. However, h_0 cannot be considered as an influencing factor in DSP_w , in accordance with the definition of the swelling strain that is not dependent on h_0 .

To avoid mathematical singularity, σ_v is considered to be at least equal to the average stress owing to the sample weight. For the sake of simplicity, a is set equal to 1 and b is chosen as a positive integer. These simplifications have not been found to be detrimental to the accuracy of the model predictions. The swelling equation (1) can now be formulated as a relationship involving two independent entities, ε_{sw} and DSP_w

$$\varepsilon_{sw} = f(DSP_w) \quad (4)$$

The validity of equation (4) will be presented in a following sections.

DATA USED FOR THE VALIDATION

Four data sets, from Noble (1966), Yevnin & Zaslavsky (1970), Brackley (1973) and Villar (2000), have been used to investigate the validity of the dimensional analysis (see Table 1). Data are all results of one-dimensional swelling tests on compacted montmorillonitic clays. Except for Noble's specimens, for which $0.9 \leq S_r < 1$, all other specimens were of low saturation degree ($0.4 \leq S_r < 0.9$). The dimensionless model being proposed does not necessarily apply only to partially saturated soils. When the saturation degree S_r equals unity, the water content depends on the void ratio and thus on the dry unit weight ($e_0 = w_0 G_s$). In that case, the dimensional analysis yields to one dimensionless parameter (incorporating γ_{d_0} , Δh , h_0 and σ_v). The solution is then said to be complete. This particular case was not validated in this paper owing to a lack of comprehensive experimental data.

VALIDATION

Figures 1(a)–1(c) show the typical scattering of the results obtained by Yevnin & Zaslavsky (1970) when swelling strain is expressed as a function of a single parameter: namely the vertical stress, the initial dry unit weight and the initial water content, respectively. When trying to fit these results with a linear relationship, low values of R^2 are obtained (from 0.16 to 0.55). Similarly poor single parameter correlations are achieved with the other data sets (not shown). On the contrary, plotting the swelling strain as a function of DSP_w leads to a satisfactory correlation (logarithmic trend in Fig. 1(d) with $R^2 = 0.86$ and a low scatter). Further, when plotting the swelling strain as a function of DSP_w for each of the other data sets, as shown in Fig. 2(a)–2(c), satisfactory correlation factors are obtained, with R^2 ranging from 0.86 to 0.90 (see correlation factors in Table 2). Here logarithmic correlations are chosen to fit the data, but this is not a limiting restriction: any other trend could be used if it leads to a higher correlation factor. A confidence interval at 90%, calculated as $1.64 \times$ the standard deviation of the residual values (Croucher & Oliver, 1986), has been estimated for each data set in Table 2. This confidence interval at 90% appears to be relatively narrow, with a maximum deviation of only 4.2%. This means that for 90% of the tests, the difference between predicted and measured values is less than 4.2%, confirming that the predictions are relatively accurate. The results suggest that the dimensional analysis is valid (at least for the swelling path of reference in the modelling).

When using a semi-logarithmic trend, the dimensionless model can be written as

Table 1. Summary of data used and range of variation of the parameters

	Number of results	Vertical stress: kPa	Initial dry unit weight: kN/m ³	Initial water content
Brackley (1973)	24	1	10.3–15.4	0.21–0.37
Yevnin & Zaslavsky (1970)	56	2.4–785	13.5–16.4	0.134–0.351
Noble (1966) (remoulded)	27	0.92–268	9.5–15.9	0.224–0.628
Villar (2000)	18	100–3000	14.9–17.2	0.125–0.184

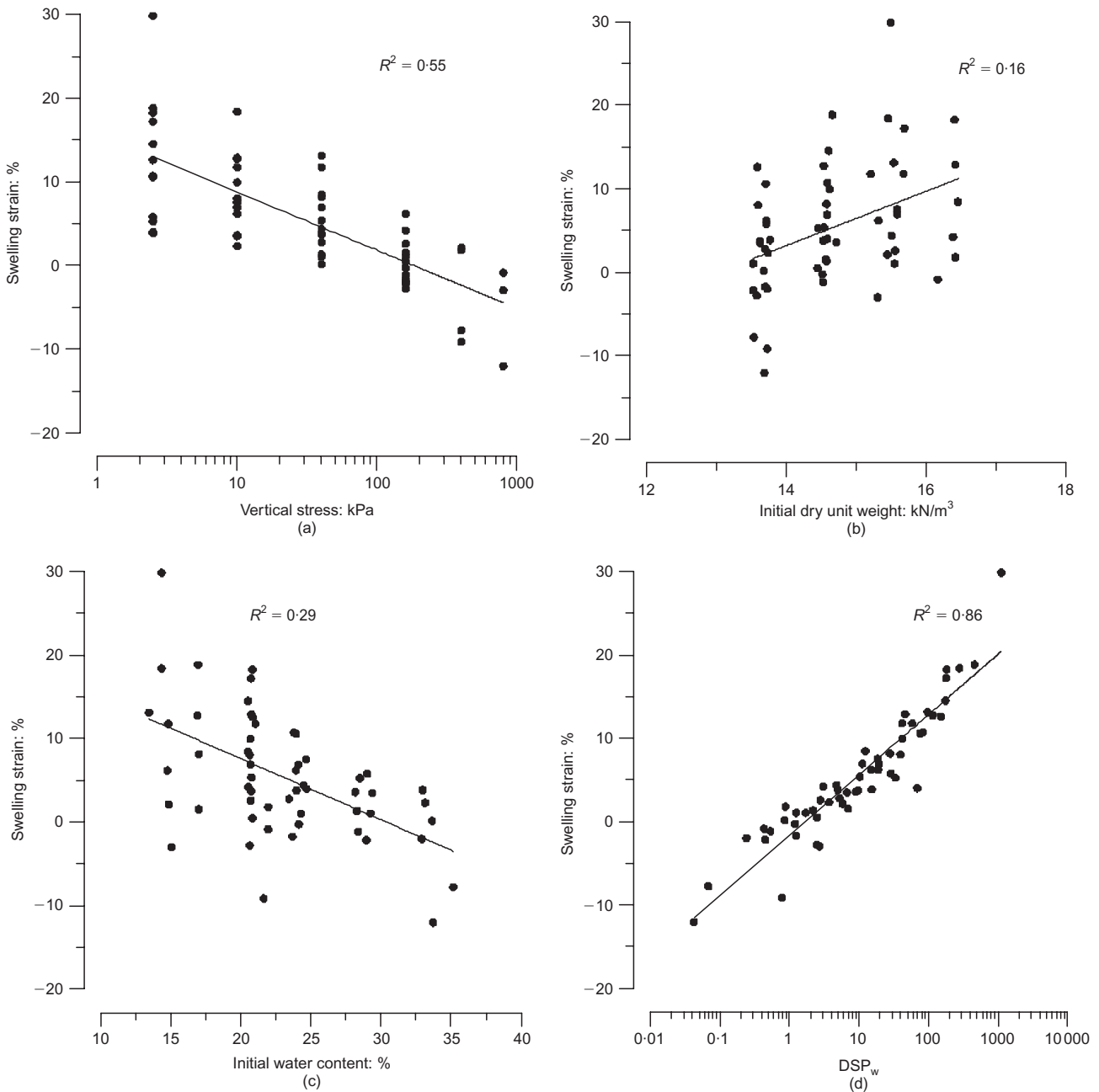


Fig. 1. Data after Yevnin & Zaslavsky (1970). (a)–(c) simple relationship between swelling strain and key parameters (vertical stress, initial dry unit weight and initial water content); (d) evolution of swelling strain plotted against DSP_w for $b = 5$

$$\varepsilon_{\text{sw}} = A_1 \cdot \ln \left[\left(\frac{\gamma_{\text{do}} \cdot h_0}{\sigma_v} \right) \left(\frac{1}{w_0} \right)^b \right] + A_2 \quad (5)$$

where A_1 , A_2 and b are the parameters of the model. As for any empirical model, a calibration of the parameters is required. This is achieved by choosing b in order to fit the data with an optimised correlation factor R^2 (Buzzi *et al.*, 2008). Effectively, b is the only parameter to calibrate, since A_1 and A_2 are the result of the curve fitting.

At this stage of the research, the relation of b with the other factors influencing the swelling process is unclear. It is believed that b could depend on soil structure, mechanical history and soil properties. In these studies, hardening was achieved from over-consolidation but it can also result from successive suction increases during wetting/drying cycles. Despite attempts to correlate b to the Atterberg limits (the

only soil property systematically reported in all four data sets), no useful relation could be derived; evidently, b depends also on additional factors other than solely the soil composition properties (Fig. 3). Identification of the factors that govern b could lead to a more general model and increase its accuracy. Comprehensive, systematic experimental testing has to be undertaken to better investigate this aspect of the dimensional analysis. Different materials should be tested to determine the influence of soil property or mineralogy. Then, for a given clayey material, differences in mechanical history or preparation processes leading to different structures should be taken into account. Also, the over-consolidation ratio could be changed to investigate its influence on b . This work is currently in progress. It should be noted that the phenomenon of collapse upon wetting (Alonso *et al.*, 1990), a phenomenon related to some

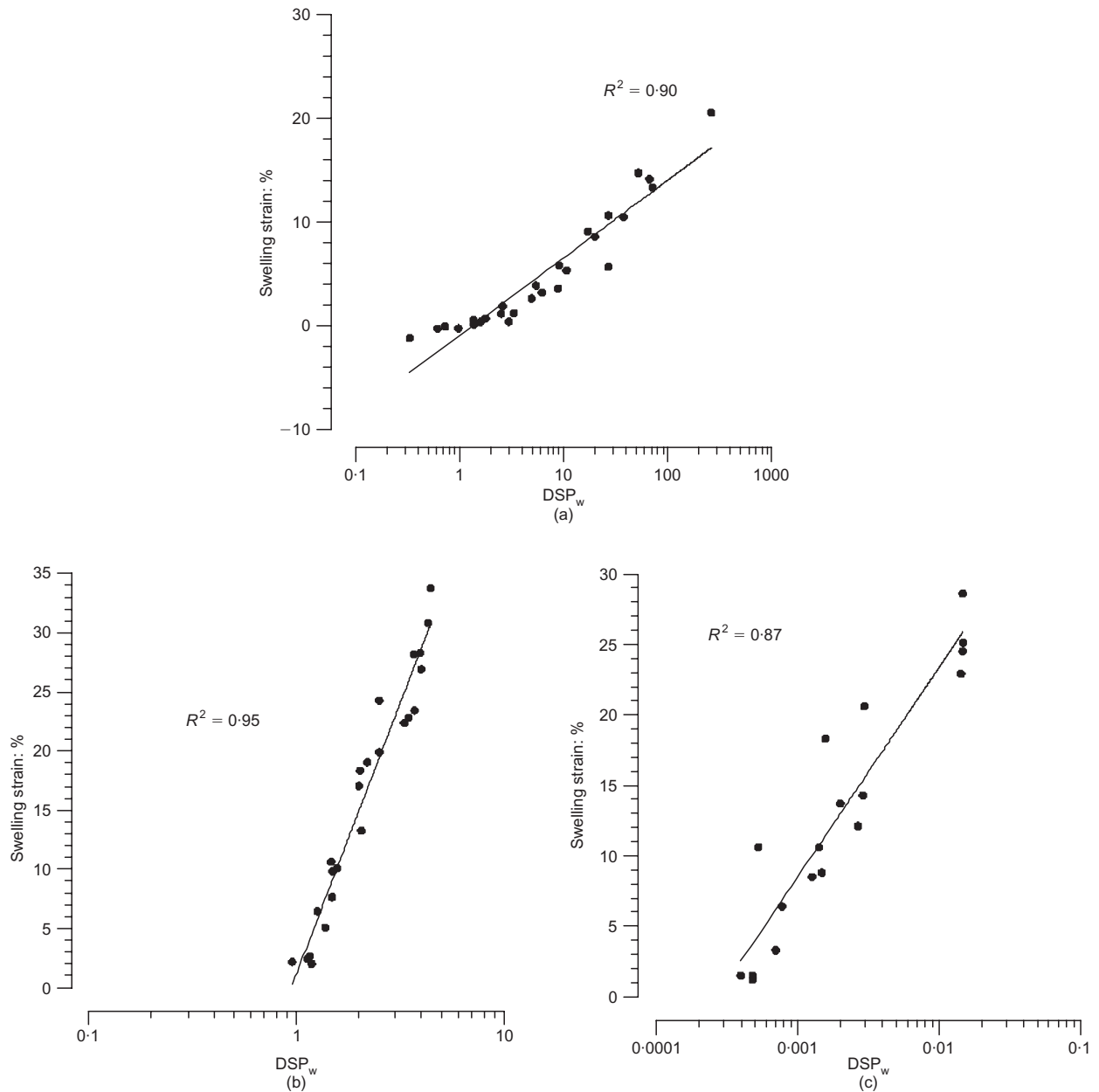


Fig. 2. Evolution of swelling strain plotted against DSP_w for different data sets: (a) data after Noble (1966), remoulded, $b = 6$; (b) data after Brackley (1973), $b = 2$; (c) data after Villar (2000), $b = 1$

Table 2. Values of factor b , confidence interval at 90% when using DSP_w and R^2 for the different data sets

Author	b	Confidence interval at 90%	R^2 for ε_{sw} as a function of only			
			DSP _w	σ_v	γ'_{do}	w_o
Brackley (1973)	2	$\Delta\varepsilon_{sw} = \pm 2\%$	0.95	NA	0.36	0.90
Yevnin & Zaslavsky (1970)	5	$\Delta\varepsilon_{sw} = \pm 3\%$	0.86	0.36	0.16	0.29
Noble (1966) (remoulded)	6	$\Delta\varepsilon_{sw} = \pm 2\%$	0.90	0.02	0.47	0.49
Villar (2000)	1	$\Delta\varepsilon_{sw} = \pm 4.2\%$	0.87	0.84	0.14	0.05

consolidation states of the clays, is not captured by the model.

EXAMPLE OF PREDICTIVE CAPABILITY

The data set of Yevnin & Zaslavsky (1970) includes 56 test results. In this example these are arbitrarily split into two subsets, with half used for model calibration and the

other half to check the prediction (Table 3). The calibration gives $b = 5$ and

$$\varepsilon_{sw} = 3.2 \ln(\text{DSP}_w) - 1.97 \quad (9)$$

with $R^2 = 0.89$ (Fig. 4(a)). The predicted and experimentally measured swelling strains for the remaining data are compared in Fig. 4(b) in order to assess the accuracy of the model. For each point, the difference between the measured

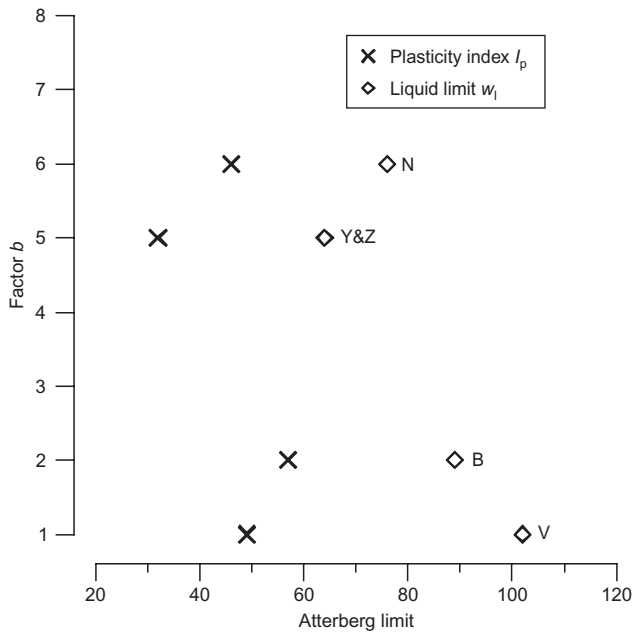


Fig. 3. Evolution of factor b with liquid limit and plasticity index of the remoulded soils coming from the data sets used. The initials of the authors are indicated beside each set

ε_{sw} value and its prediction is calculated and a histogram of the error is plotted in Fig. 4(c). The frequency distribution is a log-normal distribution with most of the errors concentrated in the lower classes. It can be seen that 53% of the predictions have an error less than 1.5%. Consequently, this dimensionless model can be used to obtain predictions for

one-dimensional swelling strain tests resulting in full saturation.

CONCLUSIONS

Swelling of clay soils is a complex phenomenon depending on numerous factors, mechanical history and confining pressure. This note presents an approach to quantify soil swelling based on dimensional analysis. In this work, the approach is applied to the common situation of compacted, unsaturated, montmorillonitic clays that are subjected to monotonic, one-dimensional swelling to a final condition of zero suction. This analysis is restricted to only three primary factors influencing swelling: the initial water content, the initial dry unit weight and the vertical stress. Four data sets from the literature have been used to assess the validity of the dimensionless description of soil swelling behaviour and its application as a predictive model. The outcome is a successful combination of the initial water content, the initial dry unit weight and the vertical stress into a single dimensionless number called DSP_w . Reasonably good correlations were found between the swelling strain and DSP_w , thus validating the application of dimensional analysis for a given testing configuration.

NOTATION

- A_1 first constant of the logarithmic swelling model
- A_2 second constant of the logarithmic swelling model
- a first exponent of the dimensionless model
- b second exponent of the dimensionless model
- DSP_w dimensionless swelling parameter based on water content
- e_0 initial void ratio
- G_s specific gravity

Table 3. Data after Yevnin & Zaslavsky (1970). Half of the results (28 tests) are used for the model calibration and the other half for the prediction

Calibration set				Prediction set			
γ_{do} : kN/m ³	σ_v : kPa	w_0 : %	ε_{sw} : %	γ_{do} : kN/m ³	σ_v : kPa	w_0 : %	ε_{sw} : %
15.53	31.17	13.40	13.11	13.72	391.68	21.60	-9.14
15.49	2.41	14.30	29.84	14.60	2.41	20.48	14.51
15.45	9.72	14.30	18.39	14.61	9.72	20.63	9.93
15.20	39.17	14.78	11.76	14.53	39.17	20.72	5.37
15.31	156.86	14.71	6.17	14.44	156.86	20.80	0.49
15.44	391.68	14.80	2.12	15.68	2.41	20.69	17.19
15.30	784.32	15.02	-2.98	15.67	9.72	21.00	11.77
14.65	2.41	16.91	18.82	15.58	39.17	20.65	6.93
14.53	9.72	16.86	12.72	15.55	156.86	20.67	2.57
14.57	39.17	16.97	8.17	16.40	2.41	20.79	18.24
14.56	156.86	16.95	1.53	16.41	9.72	20.72	12.86
13.58	2.41	20.80	12.60	16.45	39.17	20.47	8.44
13.59	9.72	20.60	8.03	16.38	156.86	20.50	4.20
13.62	39.17	20.70	3.73	16.41	391.68	21.92	1.80
13.57	156.86	20.60	-2.79	16.16	784.32	21.91	-0.86
13.52	156.86	28.94	-2.17	13.70	2.41	23.92	10.57
14.45	2.41	28.48	5.24	13.71	9.72	23.92	6.20
14.71	9.72	28.13	3.59	13.69	39.17	23.41	2.78
14.57	39.17	28.27	1.32	13.69	156.86	23.65	-1.72
14.52	156.86	28.36	-1.16	14.58	2.41	23.76	10.71
13.76	2.41	32.95	3.85	14.58	9.72	24.09	6.89
13.73	9.72	33.14	2.31	14.52	39.17	23.94	3.79
13.67	39.17	33.63	0.15	14.51	156.86	24.11	-0.27
13.72	156.86	32.90	-2.0	14.58	2.41	24.67	3.99
13.53	391.68	35.15	-7.77	15.58	9.72	24.63	7.52
13.68	784.32	33.73	-12.03	15.50	39.17	24.47	4.38
13.62	9.72	29.37	3.5	15.54	156.86	24.26	1.04
13.52	39.17	29.23	1.05	13.71	2.41	29.01	5.79

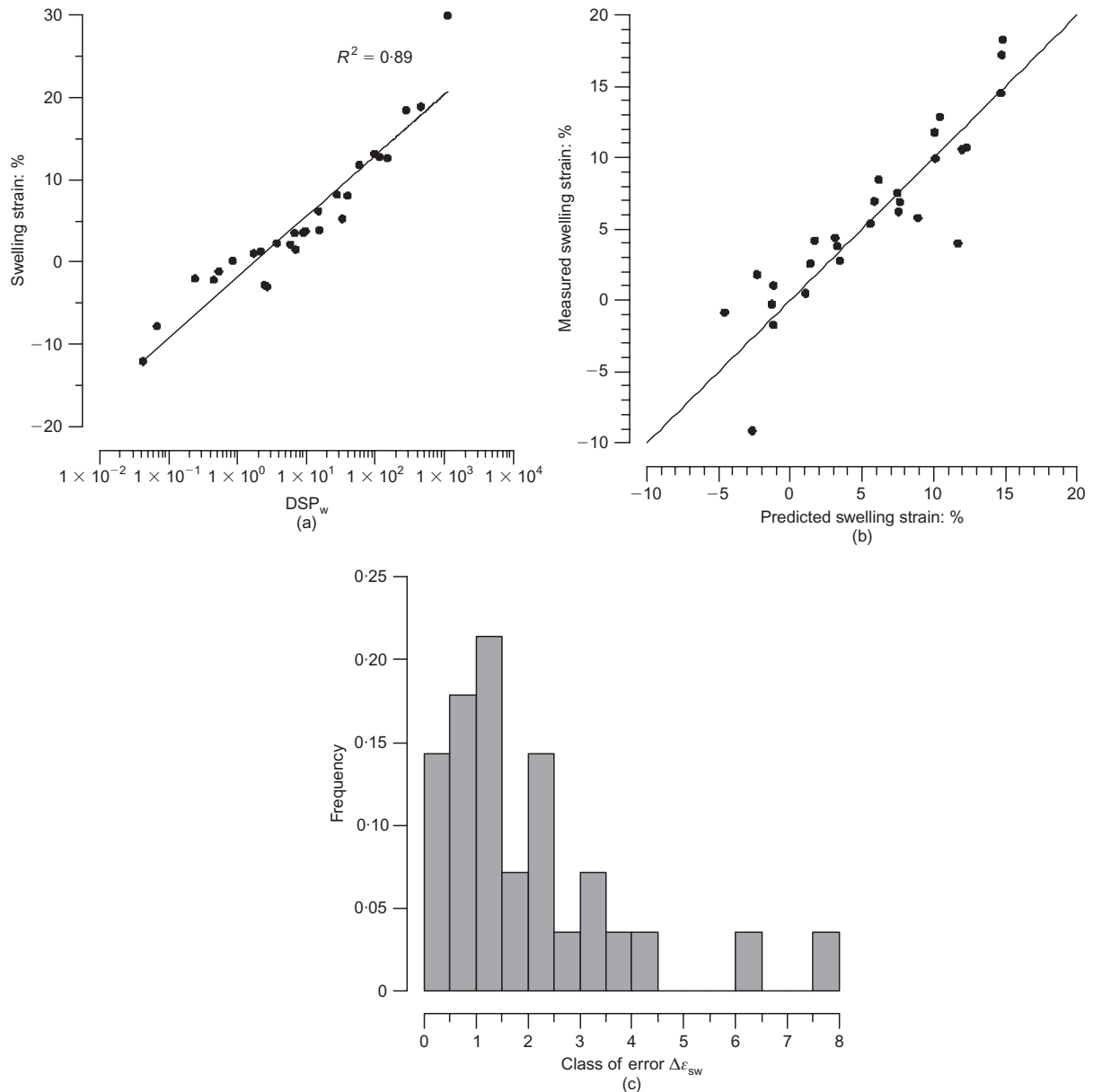


Fig. 4. (a) Experimental results after Yevnin & Zaslavsky (1970) used for the calibration of the model ($b = 5$, $R^2 = 0.89$). Half of the data set has been used. (b) Measured swelling strain against predicted swelling strain for the remaining half of the data. The line corresponds to 1:1 relationship. (c) Histogram of the absolute error between experimental values and predictions shown in (b)

Δh change in specimen height
 h_0 initial specimen height
 M_s mass of solid particles in the specimen
 M_{w0} initial mass of water in the specimen
 R^2 correlation factor
 S_r saturation degree
 w_0 initial water content
 γ_{d0} initial dry unit weight of the specimen
 ϵ_{sw} swelling strain
 σ_v vertical stress

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