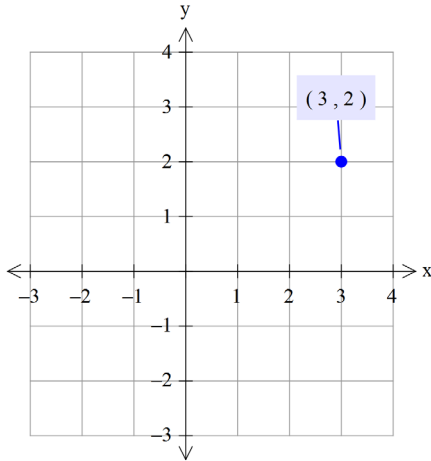


Coordinate geometry

In coordinate geometry a point is indicated by a set of coordinates (x, y) , where the x -value gives the location on the x -axis and the y -value the location with respect to the y -axis.



The two axes cross at $(0, 0)$, which is called the origin.

$(3, 2)$ refers to the point 3 units to the right of the origin and 2 units up from the origin.

Distance formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{It doesn't matter which point you use as } (x_1, y_1).$$

Midpoint formula

The midpoint of the line segment joining two points (x_1, y_1) and (x_2, y_2) , is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Perpendicular distance

The perpendicular distance between the line $Ax + By + C = 0$ and the point (x_1, y_1) is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

**Examples**

- 1) Find the distance between $(5, -1)$ and $(10, 4)$.

Label the points (x_1, y_1) $(5, -1)$ (x_2, y_2) $(10, 4)$ then substitute into the formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(10 - 5)^2 + (4 - (-1))^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \\ &\approx 7.07 \end{aligned}$$

- 2) Find the midpoint of the line segment joining $(5, -3)$ and $(6, -11)$.

Label the points (x_1, y_1) $(5, -3)$ (x_2, y_2) $(6, -11)$. then substitute into the formula.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{5 + 6}{2}, \frac{-3 + (-11)}{2} \right) \\ &= \left(\frac{11}{2}, -7 \right) \end{aligned}$$

- 3) Find the perpendicular distance between the line $2x - 3y + 4 = 0$ and the point $(7, 2)$.

$$\begin{aligned} d &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|2(7) - 3(2) + 4|}{\sqrt{2^2 + (-3)^2}} \\ &= \frac{12}{\sqrt{13}} \\ &\approx 3.3 \end{aligned}$$

Substituting the values

$$\begin{aligned} A &= 2 \\ B &= -3 \\ C &= 4 \end{aligned}$$

$$\begin{aligned} (x_1, y_1) \\ (7, 2) \end{aligned}$$

Exercises

- Find the distance between the points $(4, -2)$ and $(-3, 5)$.
- Find the midpoint of the line segment joining the points $(3, -1)$ and $(11, 7)$.
- Find the perpendicular distance between the point $(2, 8)$ and the line $6x + 2y - 4 = 0$.
- Find the coordinates of the centre of a square with vertices at coordinates $(-3, -1)$, $(5, -1)$, $(-3, 7)$ and $(5, 7)$.
(Hint: The diagonals of the square will cross at the centre. Graph the square.)

Answers 1) 9.9

2) $(7, 3)$

3) 3.8

4) $(1, 3)$