MATHS AND STATS

Indices and Logarithms I

The parts of an exponential term are labelled below

Index /power/exponent → 3; Base → 2; Number → 8

$$2^3 = 8$$

just means $2 \times 2 \times 2 = 8$

Find

$4^3 =$

$5^4 =$

$2^6 =$

$3^4 =$

$10^5 =$

$2^4 =$

In algebra, with pronumerals (also called letters or variables) it works the same way so that the algebraic expression can be written in a compact form.

$$x \times x \times x = x^3$$

or

$$x \times x \times y \times x \times y = x^3 y^2$$

So the index is just a counter for the number of letters multiplied together.

Examples and the 6 index rules

1. Add indices when multiplying terms with the same base

$$x^3 \times x^4 = (x \times x \times x) \times (x \times x \times x \times x)$$
$$= x^7 \text{ or } x^{3+4}$$

Note - the base doesn't change - the index is just a counter

2. Subtract indices when dividing terms with the same base

$$x^6 \div x^2 = \frac{x^6}{x^2} \text{ or } \frac{x \times x \times x \times x \times x \times x}{x \times x}$$
$$= x^4 \text{ or } x^{6-2}$$

Note – cross off (ie divide) each of the two bottom *x's* into two *x's* on the top line

3. When raising to a power, multiply the indices

$$(x^2)^4 \text{ or } x^2 \times x^2 \times x^2 \times x^2$$
$$= x^8 \text{ or } x^{2\times 4}$$

Note - the base doesn't change - the index is just a counter

4. A term raised to the power zero is equal to one

$$x^0 = 1$$
$$(3x)^0 = 1$$
$$4x^0 = 4 \times 1 = 4$$

Do you see why? What term is raised to index 0?

The rules are a convenient shortcut but make sure you understand each process.

The last two rules introduce important alternative notations.

5. The denominator in the index means a root

$\sqrt[5]{x} = x^{\frac{1}{5}}$ — 5 => or 5th root of x

$\sqrt[3]{8} = 8^{\frac{1}{3}} = 2$ — 3 => or cube root of 8

6. A term to a negative index means the term can change lines to form a positive index

ie top ⟷ bottom *or* $\frac{top}{bottom}$

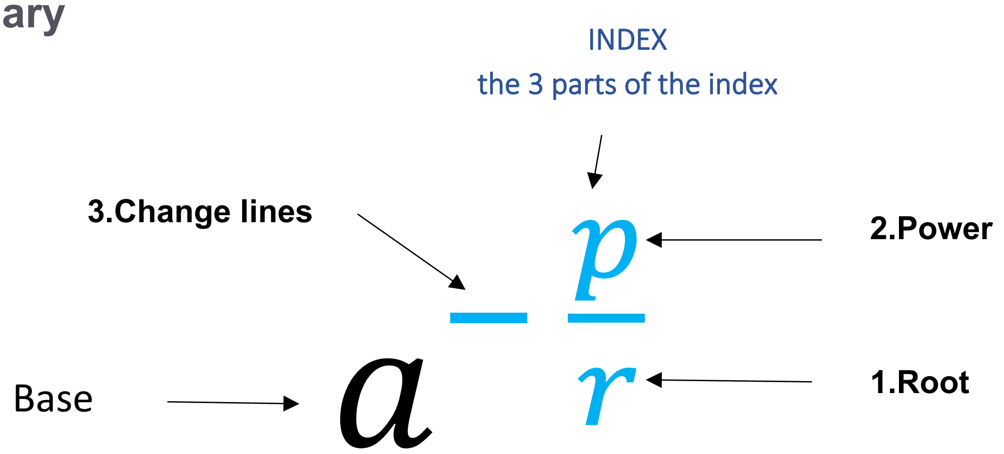
$7x^{-5} = \frac{7}{x^5}$ — only x is raised to the negative power

$\frac{a^2x^{-6}}{2y^{-4}} = \frac{a^2y^4}{2x^6}$ — only x and y are raised to the negative power

$(3x)^{-2} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$ — all $3x$ is raised to the negative power

or $\frac{5}{3x^{-4}} = \frac{5x^4}{3}$ — Focus on moving only the term(s) with a negative index

Summary



Example

Simplify $8^{-\frac{2}{3}}$.

- Always **start with the root** – cube root 8 is 2
- That leaves only the square and the minus – the square of 2 or 2^2 is 4
- Now we just have the minus – so change the 4 from the top to the bottom line and we get $\frac{1}{4}$

Warmup exercises

Common powers and roots– get familiar with these numbers so you can recognise when numbers have exact roots or not.

1. **Squares and roots**. Fill in the missing.

Squares – a number times itself … and roots with the 2 notations (meaning the same thing)

$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$1^{\frac{1}{2}} = 1$
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$4^{\frac{1}{2}} = 2$
$3^2 =$	$\sqrt{9} = 3$	$9^{\frac{1}{2}} =$
$4^2 = 4 \times 4 = 16$	$\sqrt{16} = 4$	$16^{\frac{1}{2}} = 4$
$5^2 = 5 \times 5 = 25$	$\sqrt{25} = 5$	$25^{\frac{1}{2}} = 5$
$6^2 = 6 \times 6 = 36$	$\sqrt{36} =$	$36^{\frac{1}{2}} = 6$
$7^2 =$	$\sqrt{49} =$	$49^{\frac{1}{2}} =$
$8^2 =$	$\sqrt{64} =$	$64^{\frac{1}{2}} =$
$9^2 =$	$\sqrt{81} =$	$81^{\frac{1}{2}} =$
$10^2 = 10 \times 10 = 100$	$\sqrt{100} = 10$	$100^{\frac{1}{2}} =$
$11^2 =$	$\sqrt{121} = 11$	$121^{\frac{1}{2}} =$
$12^2 =$	$\sqrt{144} =$	$144^{\frac{1}{2}} = 12$
$13^2 =$	$\sqrt{169} =$	$169^{\frac{1}{2}} =$
$14^2 =$	$\sqrt{196} =$	$196^{\frac{1}{2}} =$
$15^2 =$	$\sqrt{225} =$	$225^{\frac{1}{2}} = 15$

2. **Cubes and cube roots.** Fill in the missing.

$1^3 = 1 \times 1 \times 1 = 1$	$\sqrt[3]{1} = 1$	$1^{\frac{1}{3}} = 1$
$2^3 = 2 \times 2 \times 2 = 8$	$\sqrt[3]{8} = 2$	$8^{\frac{1}{3}} =$
$3^3 = 3 \times 3 \times 3 =$	$\sqrt[3]{27} =$	$27^{\frac{1}{3}} = 3$
$4^3 = 4 \times 4 \times 4 =$	$\sqrt[3]{64} = 4$	$64^{\frac{1}{3}} =$
$5^3 = 5 \times 5 \times 5 = 125$	$\sqrt[3]{125} =$	$125^{\frac{1}{3}} = 5$

3. **Various powers and roots of 2 and 3.** Fill in the missing.

$2^1 = 2$		
$2^2 = 4$	$\sqrt{4} = 2$	$4^{\frac{1}{2}} = 2$
$2^3 =$	$\sqrt[3]{8} = 2$	$8^{\frac{1}{3}} =$
$2^4 = 16$	$\sqrt[4]{16} = 2$	$16^{\frac{1}{4}} = 2$
$2^5 =$	$\sqrt[5]{32} =$	$32^{\frac{1}{5}} =$
$2^6 = 64$	$\sqrt[6]{64} = 2$	$64^{\frac{1}{6}} = 2$
$2^7 =$	$\sqrt[7]{128} =$	$128^{\frac{1}{7}} =$
$2^8 =$	$\sqrt[8]{256} = 2$	$256^{\frac{1}{8}} = 2$
$2^9 =$	$\sqrt[9]{512} =$	$512^{\frac{1}{9}} =$
$2^{10} =$	$\sqrt[10]{1024} = 2$	$1024^{\frac{1}{10}} = 2$

$3^1 = 3$	$\sqrt{3} = \sqrt{3}$	$3^{\frac{1}{2}} = \sqrt{3}$
$3^2 = 9$	$\sqrt{9} = 3$	$9^{\frac{1}{2}} = 3$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$27^{\frac{1}{3}} =$
$3^4 = 81$	$\sqrt[4]{81} =$	$81^{\frac{1}{4}} = 3$
$3^5 = 243$	$\sqrt[5]{243} = 3$	$243^{\frac{1}{5}} =$

Exercises

4. Simply each of the following

a) $100^{\frac{1}{2}} =$

b) $125^{\frac{1}{3}} =$

c) $2^{-3} =$

d) $27^{\frac{2}{3}} =$

e) $5^{-1} =$

f) $16^{-\frac{3}{4}} =$

g) $8^{\frac{4}{3}} =$

h) $5^{-2} =$

i) $64^{\frac{5}{6}} =$

j) $\frac{1}{6^{-2}} =$

5. Simplify the following in index form

a) $2^3 \times 2^5 =$

b) $x^4 \times x^3 =$

c) $x^6 \div x^4 =$

d) $(x^4)^5 =$

e) $a^6 \div a^3 =$

f) $(x^3)^{\frac{3}{2}} =$

g) $a^{-3} \times a^7 =$

h) $y^{-2} \times x^4 \times y^{-3} \times x^{-5} =$

6. Simplify the following

a) $\sqrt{64x^8}$

b) $\sqrt{81x^{18}}$

c) $(8x^{12})^{\frac{1}{3}}$

d) $\sqrt{49a^{16}}$

e) $\sqrt[3]{125y^{12}}$

f) $(16x^{24})^{\frac{1}{4}}$

Answers

4.

a) $100^{\frac{1}{2}} = 10$

b) $125^{\frac{1}{3}} = 5$

c) $2^{-3} = \frac{1}{8}$

d) $27^{\frac{2}{3}} = 9$

e) $5^{-1} = \frac{1}{5}$

f) $16^{-\frac{3}{4}} = \frac{1}{8}$

g) $8^{\frac{4}{3}} = 16$

h) $5^{-2} = \frac{1}{25}$

i) $64^{\frac{5}{6}} = 32$

j) $\frac{1}{6^{-2}} = 36$

5.

a) $2^3 \times 2^5 = 2^8$

b) $x^4 \times x^3 = x^7$

c) $x^6 \div x^4 = x^2$

d) $(x^4)^5 = x^{20}$

e) $a^6 \div a^3 = a^3$

f) $(x^3)^{\frac{3}{2}} = x^{9/2} \ or \ x^{4 1/2}$

g) $a^{-3} \times a^7 = a^4$

h) $y^{-2} \times x^4 \times y^{-3} \times x^{-5} = x^{-1}y^{-5}$

6.

a) $8x^4$

b) $9x^9$

c) $2x^4$

d) $7a^8$

e) $5y^4$

f) $2x^6$