MATHS AND STATS

Solving Simultaneous Equations

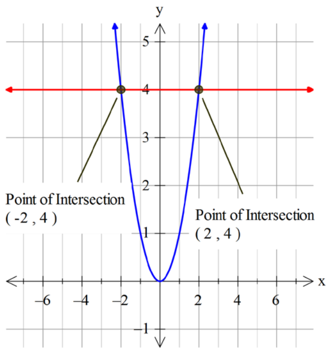
Simultaneous equations are where you have 2 equations relating the same 2 variables (or 3 equations and 3 variable, etc), and want to find a solution that works for both equations. This is the same as finding the co-ordinates at which the graphs of two equations intersect.

Graphing

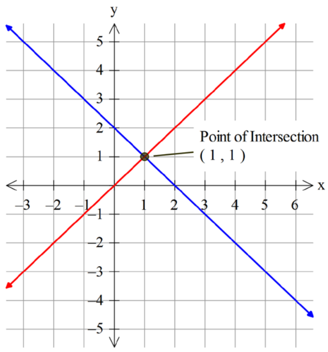
We can solve the following equations simultaneously by graphing them and looking for the coordinates of their intersection:

Examples

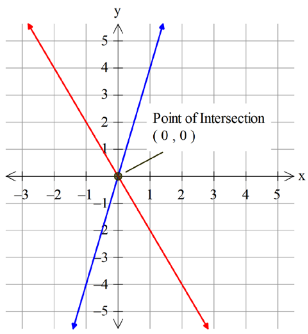
a) $y = 4$
$y = x^2$



b) $y = 2 - x$
$y = x$



c) $y = 4x$
$y = -2x$



The points at which these graphs intersect give an x and y pair of values which will make each of the equations true. These 3 could be found from the graph, but that is not always easy or accurate.

There are several algebraic methods that can be used. Most of these rely on finding the value for one variable and then using it to find the other.

Algebraic method 1

Make one variable the subject of both equations, and 'equate'.

Example

Solve $y = x - 2$ and $y = 4x + 1$.

If y is equal to both $x - 2$ and $4x + 1$, then we know that

$$\begin{aligned} x - 2 &= 4x + 1 \\ -3x &= 3 \\ x &= -1 \end{aligned}$$

Now substitute $x = -1$ into our first equation $y = x - 2$ giving, $y = -1 - 2 = -3$, so the point of intersection is $(-1, -3)$. We can check this also satisfies $y = 4x + 1$ by substituting these points in: $-3 = 4(-1) + 1$, which is true.

Algebraic method 2

This is a more general case of the equating method above. Make one variable the subject of one equation, and 'substitute' into the other equation.

Example

Solve $2y + 4x + 2 = 0$ and $3y + 3x + 9 = 0$.

Make y the subject of the first equation:

$$\begin{aligned} 2y &= -4x - 2 \\ y &= -2x - 1 \end{aligned}$$

Then use this y 'value' in the second equation:

$$\begin{aligned} 3(-2x - 1) + 3x + 9 &= 0 \\ -6x - 3 + 3x + 9 &= 0 \\ -3x + 6 &= 0 \\ -3x &= -6 \\ x &= 2 \end{aligned}$$

And substitute this back into your earlier equation: $y = -2(2) - 1 = -5$
So the solution is $(2, -5)$

Algebraic method 3 (Elimination method)

Add (or subtract) a multiple of the equations to 'eliminate' a variable.

This works on the principle that if $a = b$ and $x = y$, then $a + x = b + y$
So, if one equation has $2x$ on the left, and the other has $-2x$ on the left, if we add the equations together then the left side will have $2x - 2x$ which cancels and so we have eliminated a variable.

Example

Solve the following equations simultaneously

$$3x + 4y - 2 = 0 \qquad (1)$$
$$y - 3x = 7 \qquad (2)$$

If we add these equations together, we get:

$$3x + 4y - 2 + y - 3x = 0 + 7 \qquad (1) + (2)$$
$$5y - 2 = 7$$
$$5y = 9$$
$$y = \frac{9}{5} \quad or \quad 1.8$$

Then, once again, we use this value to find the other variable by using one of the original equations.

Substitute $y = \frac{9}{5}$ into $y - 3x = 7$

$$\frac{9}{5} - 3x = 7$$
$$-3x = 7 - \frac{9}{5}$$
$$-3x = \frac{26}{5}$$
$$x = -\frac{26}{15} \quad or \quad -1\frac{11}{15}$$

We would not have accurtely identified the point $\left(\frac{5}{9}, -\frac{26}{15}\right)$ by examining the graphs.

Example

Solve the following equations simultaneously

$$x + y = 4 \qquad (1)$$
$$3x - 2y = 5 \qquad (2)$$

Multipy equation (1) by 3

$$3x + 3y = 12 \qquad (3)$$

Now calculate (3) – (2)

$$3x - 3x + 3y - -2y = 12 - 5$$
$$5y = 7$$
$$y = \frac{7}{5}$$

Sub this back into (1)

$$x + \frac{7}{5} = 4$$
$$x = \frac{20}{5} - \frac{7}{5}$$
$$x = \frac{13}{5}$$

So the solution is $x = \frac{13}{5}$ and $y = \frac{7}{5}$ or $\left(\frac{13}{5}, \frac{7}{5}\right)$

It can take practice to tell which of these methods will be the quickest for a given question.

Exercises

a) $x = y + 6$
$y = 2x + 4$

b) $4x + y = 15$
$2x + y = 9$

c) $2x + 5y = 37$
$y = 11 - 2x$

d) $4x + 2y = 1$
$3x + 5y = 6$

e) $y = 4x - 3$
$y = x^2$

Challenge question

f) $x + y = -1$
$x + 2z = 4$
$z - 2y = 7$

Answers

a) $(-10, -16)$

b) $(3, 3)$

c) $\left(\frac{9}{4}, \frac{13}{2}\right)$

d) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

e) $(1,1)$ and $(3, 9)$

f) $x = 2$, $y = -3$, $z = 1$