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# On the use of dimensional analysis to predict swelling strain

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ABSTRACT

Soil swelling is a complex phenomenon, the magnitude of which is not trivial to predict. It is affected by many parameters including the initial hydration state of the soil, its initial level of compaction (or density) and the level of confinement. It is hereby proposed to extend a preliminary research which dealt with using the dimensional analysis to predict the swelling strain of different soils. Dimensional analysis is a common technique in Fluid Mechanics but it is not used that often in Geotechnical Engineering despite its versatility. The present work presents the derivation of dimensionless numbers for three swelling configurations: under oedometric conditions (one dimensional) until full saturation, under isotropic confinement (three dimensional) until full saturation and one dimensional with controlled suction. Some extensive experimental testing has been conducted in order to provide a database for validation of the first two dimensionless models. Data from the literature have been used to validate the approach on partial swelling under controlled suction. Results of calibration–prediction exercises are provided, which suggest that dimensional analysis is a simple, efficient and accurate tool to predict swelling strain, at least for the three configurations tested. The paper also provides a discussion on some limitations of the model and on the minimum number of tests required to calibrate the models.



## 1. Introduction

Studies of soil swelling date back to the 1950s (e.g. Aitchinson and Holmes, 1953; Norrish, 1954). Volume change due to a reduction of the effective stress and often referred to as elastic rebound, is usually not critical because of limited magnitude. On the other hand, osmotic swelling, which is driven by electro chemical effects, results in much larger volume changes (Norrish, 1954). This latter phenomenon has early been identified as relatively complex. It results from water adsorption on the clay platelets, which finds its origin in different mechanisms (Sposito and Prost, 1982). Several studies on soil swelling have dealt with the influence of parameters measurable at the macro scale such as the initial hydration condition of the soil, its initial level of compaction (or density) and the confinement (Noble, 1966; Yevnin and Zaslavsky, 1970; Brackley, 1973; Komine and Ogata, 1996). However, the list of factors affecting swelling is much longer and includes variables like soil structure, temperature, chemistry of pore water, to name a few (Holtz and Gibbs, 1956; Ladd and Lambe, 1961; Parcher and Liu, 1965; Popescu, 1980). The complexity lying within soil swelling also comes from its dependence on boundary conditions, stress path and stress history (Uppal and Palit, 1969; Schreiner and Burland, 1987; Dif and Bluemel, 1991).

Some engineering applications make specifically use of compacted expansive soils (e.g. nuclear waste storage) or sometimes use them due to a lack of available non expansive material (e.g. road sub grade). In these cases, it is useful to assess the swelling potential of a given soil with respect to its conditions of placement. This is not an easy task. Models capable of handling expansive soils are mainly the privilege of academia; they are not trivial to implement and they require long lasting and sophisticated calibration procedures (Dormieux et al., 1995; Alonso et al., 1999; Oka et al., 2009). Above all, there are not many such models. Another possible approach is to conduct extensive testing in the laboratory and to use multi variable regressions to predict the swelling potential for any combination of the parameters tested (Erguler and Ulusay, 2003). This option is not always simple to implement (Cohen et al., 2003).

In view of the need for an efficient and simple tool to predict soil swelling, Buzzi et al. (in press) have proposed to use dimensional analysis for this problem. One advantage of dimensional analysis is that it allows intelligent experimentation i.e. a reduction of the number of tests to be performed to characterize a physical phenomenon. This is possible through the use of dimensionless parameters, the number and form of which can be derived from the Buckingham Pi theorem (Buckingham, 1914). In particular, the theorem states that an initial equation involving N independent parameters and P dimensions can be reduced to a dimensionless equation involving only N–P dimensionless parameters. Dimensional analysis is a very common approach in Fluid Mechanics but it remains an exception more than a rule in Geotechnical Engineering (Butterfield, 1999).

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Buzzi et al. (in press) have derived a dimensionless number called $DSP_w$ which is based on initial water content, initial dry unit weight and vertical stress. Despite the approach and the resulting dimensionless model were validated, some questions about the limitations of the model and the correlation between soil properties and model parameters remained unanswered. Also, issues arose from the presence of the initial height of the specimen in the dimensionless number $DSP_w$. Indeed, a strain is supposedly independent from the initial height.

The present study aims to improve the dimensionless model originally proposed by Buzzi et al. (in press). A new dimensionless number, based on initial suction rather than initial water content has been formulated. Extensive testing has been carried out to provide a database allowing for the validation of the associated dimensionless model for swelling in oedometric conditions (one dimensional) and swelling under isotropic confinement (three dimensional). These tests were conducted until full saturation (zero suction) but the application of dimensional analysis to partial swelling was nonetheless assessed using the data after Escario and Saez (1973). Finally, this study provides a discussion about the inability of the dimensionless model to capture the collapse of unsaturated soils upon wetting and includes an investigation of the minimum number of tests required for an accurate calibration of the model.

## 2. Dimensional analysis applied to soil swelling

As discussed by Buzzi et al. (in press) and several other authors (e.g.Parcher and Liu, 1965; Yevnin and Zaslavsky, 1970; Brackley, 1973, to name a few), soil swelling is a very complex phenomenon influenced by procedural, environmental and structural factors (Morgenstern and Balasubramonian, 1980). Note that it is out of the scope of this paper to provide a discussion of the factors influencing soil swelling or of the mechanisms of soil swelling. As for experimental testing or modeling of soil swelling, some simplifications are required to apply dimensional analysis to this phenomenon. Indeed, not all influencing factors can be accounted for. A significant number of studies undertaken on swelling have focused on the effect of initial level of compaction, initial state of hydration and confinement. This is particularly relevant for practical engineering applications (e.g. engineered barrier for nuclear waste storage or earthworks). Buzzi et al. (in press) have recently validated the use of dimensional analysis to describe soil swelling by deriving a dimensionless number, namely $DSP_w$, based on initial water content, initial dry unit weight and vertical stress. The present study aims to improve the dimensionless model originally presented by Buzzi et al. (in press):

- The initial height of the specimen $h_o$ figures in the dimensionless parameter $DSP_w$, which is not appropriate. Indeed, the purpose of a strain is to be independent from the initial height. Consequently, a new dimensionless parameter based on initial suction rather than initial water content, referred to as $DSP_s$, has been defined for swelling under oedometric conditions. 36 swelling tests under oedometric conditions were performed to validate the new dimensionless number. The resulting dimensionless model is referred to as dimensionless model for 1-D swelling.
- The dimensional analysis has then been validated for swelling under isotropic confinement (referred to as dimensionless model for 3-D swelling). To do so, 24 swelling tests under isotropic confinement were performed.
- Finally, the dimensional analysis has been applied to swelling tests under controlled suction (but with constant void ratio) (Escario and Saez, 1973). A dimensionless number, namely $DSP_{\Delta s}$, has been derived to suit the new set of influencing parameters. Validation of the dimensionless model (referred to as dimensionless model for partial swelling) has been provided.

As discussed in Buzzi et al. (in press), the application of Buckingham Pi theorem (Buckingham, 1914), yields a dimensionless number which incorporates several exponents. The value of these latter is determined by a calibration procedure where the exponents are incremented until a satisfactory coefficient of determination is found between the swelling strain and the dimensionless number. The calibration process is achieved using Matlab: a subroutine increments the exponents and maps the corresponding value of the coefficient of determination $R^2$. The calibrated values of the exponents are determined on the basis of the mapping. The next sub-sections describe the derivation of the dimensionless number for each testing configuration.

### 2.1. Dimensionless model for 1-D swelling

As discussed by Buzzi et al. (in press), different parameters can be used to quantify the initial level of compaction and the initial state of hydration. The combination of initial dry unit weight and initial water content led to some issues related to the presence of the initial height of the specimen in $DSP_w$. It is now proposed to combine the initial suction and initial void ratio. The 1-D swelling problem can be expressed as:

$$f(\Delta h, h_o, V_{vo}, V_s, s_o, \sigma_v) = 0 \tag{1}$$

where $\Delta h$ is the change in height of the specimen, $h_o$ the initial height, $V_{vo}$ the initial void volume, $V_s$ the volume of solid particles, $s_o$ the initial suction and $\sigma_v$ the confining stress. Eq. (1) involves 6 independent variables and 3 dimensions (length, time and mass). The Buckingham Pi theorem (Buckingham, 1914) states that Eq. (1) could be re-written using 6–3 = 3 dimensionless parameters. It comes quite naturally to define these as the swelling strain $\varepsilon_{sw}$, the void ratio $e_o$ and the ratio of initial suction over vertical stress $s_o/\sigma_v$. Eq. (1) can then be re-written:

$$g\left(\varepsilon_{sw}, e_o, \frac{s_o}{\sigma_v}\right) = 0 \tag{2}$$

In the first rigorous application of dimensional analysis by Buzzi et al. (in press), the three influencing parameters (initial dry unit weight, initial water content and vertical stress) were incorporated in one unique dimensionless number. This proved to be convenient as the initial equation describing the soil swelling was reduced to a relationship between only two entities. On the basis of this existing empirical evidence, it is herein proposed to modify Eq. (2) to the following:

$$h(\varepsilon_{sw}, DSP_s) = 0 \tag{3}$$

where $DSP_s$ is defined as:

$$DSP_s = \left(\frac{1}{e_o}\right)^a \cdot \left(\frac{s_o}{\sigma_v}\right)^b \tag{4}$$

$a$ and $b$ are the parameters of the model to be calibrated in order to obtain a satisfactory correlation between the swelling strain and $DSP_s$. The validation of the model is presented in Section 4.1.

### 2.2. Dimensionless model for 3-D swelling

There are little changes when extending the dimensionless model from 1-D to 3-D. The swelling strain has to reflect a volume change and becomes:

$$\varepsilon_v = \frac{\Delta V}{V_o} \tag{5}$$

and the dimensionless number $DPS_s$ is now defined as:

$$DSP_s = \left(\frac{1}{e_o}\right)^a \cdot \left(\frac{s_o}{p}\right)^b \quad (6)$$

where $\Delta V$ is the volume change of the soil specimen, $V_o$ the initial volume and $p$ is the isotropic confining pressure. The general form of the dimensionless model (Eq. (3)) still applies. The validation of the model will be presented in Section 4.2.

### 2.3. Dimensionless model for partial swelling

In order to assess the applicability of dimensional analysis to even more complex swelling tests, the results obtained by Escario and Saez (1973) have been utilized. The authors performed one dimensional swelling test under controlled suction and highlighted the non linearity of the swelling behaviour. Most of the swelling occurs at the low end of the suction values. In their study, the initial void ratio and the initial suction have been kept constant and should consequently not appear in the swelling equation the dimensional analysis is based on. However, in order to subject the dimensional analysis to a more complex problem, relative swelling strains between any intermediate suction values have also been considered. Doing so, more data can be used but they do not necessarily represent the swelling behaviour of the soil properly. In fact, it is an approximation.

The influencing parameters of the problem are the initial height $h_o$, the change in height $\Delta h$, the initial suction $s_o$, the change in suction $\Delta s$ and the vertical stress $\sigma_v$. With 5 parameters and 3 dimensions, the Buckingham Pi theorem allows to combine the parameters into two dimensionless numbers; one being the swelling strain $\varepsilon_{sw} = \Delta h / h_o$. The second dimensionless number, called $DSP_{\Delta s}$, is defined as:

$$DSP_{\Delta s} = \frac{(\Delta s)^{(a+b)}}{(s_o)^a \cdot (\sigma_v)^b} \quad (7)$$

with $a$ and $b$ positive. $DSP_{\Delta s}$ is built so that when it increases, the swelling strain increases. The tests performed by Escario and Saez (1973) showed that, for the same suction change, more volume change is produced at the low end of suction values. Consequently, $s_o$ is placed at the denominator with a positive exponent. On the other hand, a larger suction change leads to a larger volume change suggesting that the initial suction should go to the numerator with a positive exponent. As for the stress, it goes to the denominator with a positive exponent like in $DSP_w$ and $DSP_s$. Validation of this model will be presented in Section 4.3.

## 3. Experimental testing

### 3.1. Testing program and methods

Two series of swelling tests were conducted to validate the new formulation of the dimensionless model proposed in this paper. First, one dimensional swelling tests were performed using standard oedometers (specimen diameter 45 mm, height 19 mm). The specimens were loaded under a given vertical stress and simultaneously flooded with tap water. 36 tests were conducted, the results of which are available in Appendix A. Then, a series of swelling tests under isotropic confinement were carried out in isotropic cells. The confining pressure was applied using a GDS pressure controller of maximum capacity 2 MPa. The specimens (diameter 38 mm, height 38 mm) were placed into a rubber membrane and connected to tap water at top and bottom. Note that the difference in specimen dimensions and shape is not an issue as the results of the two test series are not compared. The volume change of the specimen during the test was inferred by measuring the variation of water volume in the cell. Errors in volume measurement were minimized by using stiff cells made of stainless steel with thick walls (10 mm) and short connecting tubes. Pressure and volume changes were logged using a GDS software. 24 tests were performed, the results of which are available in Appendix B. The objective of this study was not to check the well known influence of the initial void ratio, initial suction and confinement on swelling. Therefore, the testing program was not conducted by varying one parameter at a time and keeping the other two constant. On the contrary, the test parameters were varied in order to obtain a large number of combinations of initial suction, initial void ratio and confinement.

### 3.2. Materials and specimen preparation

An expansive clay, namely the Maryland clay, coming from the experimental field site of the University of Newcastle and located in Maryland (NSW, Australia) has been used in this study. It contains around 45% in mass of smectite and has a liquid limit around 75% and a plasticity index around 50%. More details about mineralogy, engineering properties and composition of the Maryland clay can be found in Fityus and Smith (2004). Remolded material was used to get homogeneous samples and good test repeatability. The wet soil was manually cleared from the biggest inclusions (roots, stones), compacted using a proctor compacting machine and cut in dices. This process was repeated several times until a homogeneous soil color was reached and no more inclusions were found. Finally, the compacted specimen was grated to produce flakes in view to accelerate the hydraulic equilibration to follow. The flakes of initial water content of around 30% were air-dried until different target water contents and placed in an air tight container to equilibrate for a minimum of one week. The initial total suction was then measured using the chilled mirror WP4 potentiameter from Decagon. Before testing, the flakes were statically compacted in a stiff steel ring to produce unsaturated specimens of expansive clay of known suction and void ratio.

## 4. Results

### 4.1. Validation of the 1-D dimensionless model

All the test results are provided in Appendix A. Fig. 1(a) shows the experimental values of swelling strain plotted as a function of the sole void ratio. A significant scattering can be observed, which is due to the fact that the influencing parameters were varied in a broad range. Consequently, it is barely possible to define a clear trend when plotting the swelling strain as a function of only one influencing parameter. The purpose of this figure is also to demonstrate the efficiency in gathering all the data in a clear trend when using the dimensional analysis. There is a clear contrast between the high scattering of Fig. 1(a) and the very limited scattering of Fig. 1(b). This latter figure shows the same experimental results expressed via the new dimensionless parameter $DSP_s$ derived from the dimensional analysis and calculated with $a = 3$ and $b = 1$. A good correlation is obtained ($R^2 = 0.95$ with 36 tests) with much limited scattering. Using $DSP_s$ allows accounting for the three influencing parameters via only one dimensionless parameter.

The map of $R^2$ values produced by the calibration is shown in Fig. 2. It shows that $a = 3$ and $b = 1$, but not only, are possible candidates for an accurate calibration. For example, $a = 4$ and $b = 1$ would also have given a satisfactory correlation. Obviously, changing $a$ and $b$ affects the equation of the model but as long as the coefficient of determination is high, this is not detrimental to the accuracy of the model.

Buzzi et al. (in press) have tried to correlate the exponent $b$ of the dimensionless parameter $DSP_w$ to the plasticity index of the soil without being in a position to draw clear conclusions due to the limited amount of data. Since the possible values to obtain a high coefficient of determination are not unique (see map of $R^2$ in Fig. 2), it seems inappropriate to predict the model parameters i.e. $a$ and $b$ from a soil descriptor. Indeed, this raises the question of which values of $a$

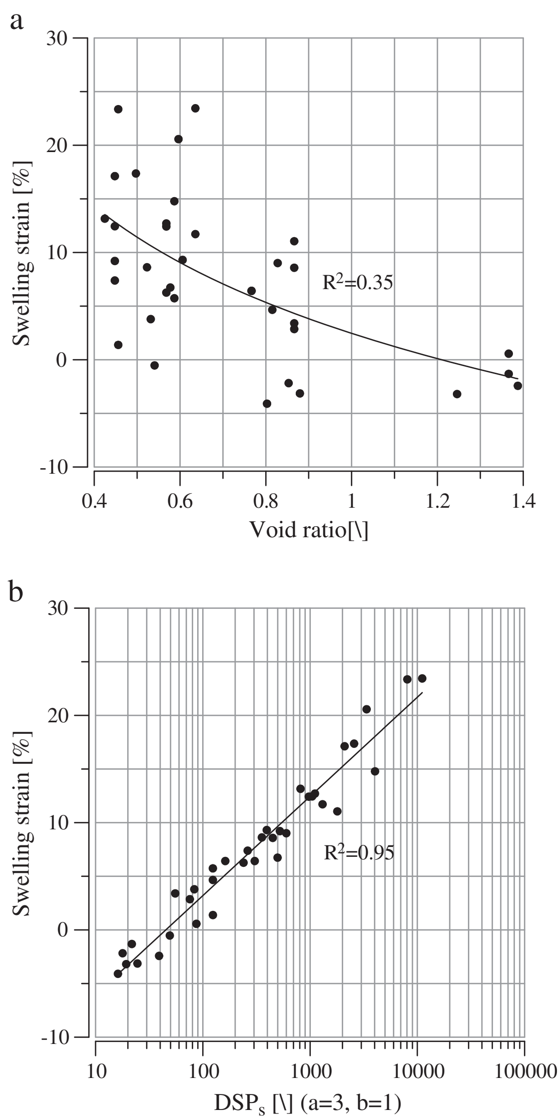


**Fig. 1.** One dimensional swelling tests: (a) Experimental swelling strain vs. void ratio. (b) Experimental swelling strain vs. $DSP_s$ ($a=3$ and $b=1$).

and $b$ should be correlated to the soil descriptor. In conclusion, a calibration of the model appears to be the most adequate procedure with the final $a$ and $b$ values decided by the operator.

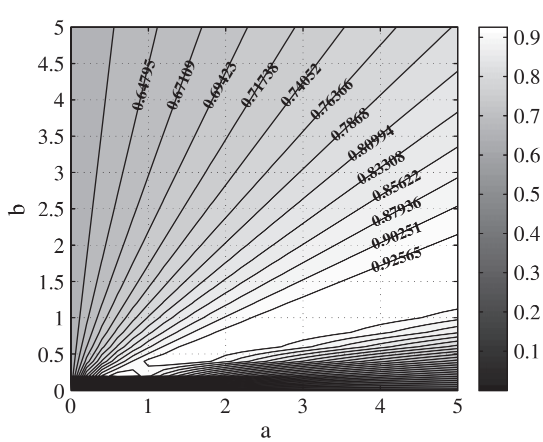


**Fig. 2.** Results of the calibration of $a$ and $b$ for the one dimensional swelling tests: map of $R^2$ (values on the color bar and indicated on the contours).

The predictive capability of the new dimensionless model for 1-D swelling has been assessed by randomly dividing the data in two sets: the first half has been used for calibration and the second half for prediction. $a=3$ and $b=1$ still prevail with a dimensionless model expressed as:

$$\varepsilon_{sw} = 3.90 \cdot ln\left(\left(\frac{1}{e_o}\right)^3 \cdot \left(\frac{s_o}{\sigma_v}\right)\right) - 15.15 \quad (8)$$

Eq. (8) has been used to predict the swelling strain for the second half of the data set. Results of this prediction exercise are shown in Fig. 3. It can reasonably be concluded that the model is capable of good predictions.

### 4.2. Validation of the 3-D dimensionless model

A similar study was conducted for swelling tests under isotropic confinement using the test results given in Appendix B. Again, a significant scattering is observed when plotting the swelling strain as a function of only one parameter (Fig. 4(a)) but this is clearly reduced when applying the dimensional analysis (Fig. 4(b)). This result tends to suggest that the dimensionless model for soil swelling can be extended to swelling under isotropic confinement. This time, the calibration resulted in $a=1$ and $b=2$, which shows that, for a similar soil, the boundary conditions clearly affect the calibration of the exponents $a$ and $b$. Indeed, $a=1$ and $b=2$ only lead to $R^2=0.70$ for the 1-D swelling (Fig. 2). This is not that surprising since soil swelling is known to depend on the boundary conditions.

The calibration–prediction exercise was also undertaken to assess the predictive capability of the dimensionless model for 3-D swelling. Similarly, the dataset was randomly split in two halves, one serving to the calibration, the other one to assess the accuracy of prediction. On the basis of the tests drawn for calibration, the dimensionless model for 3-D swelling is:

$$\varepsilon_v = 3.95 \cdot ln\left(\left(\frac{1}{e_o}\right) \cdot \left(\frac{s_o}{p}\right)^2\right) - 31.25 \quad (9)$$

The results, presented in Fig. 5, seem to be slightly less accurate than for the 1-D swelling. This is due to the fact that swelling tests under isotropic confinement are more delicate to carry out than one dimensional tests and results are also more variable (see Appendix B).

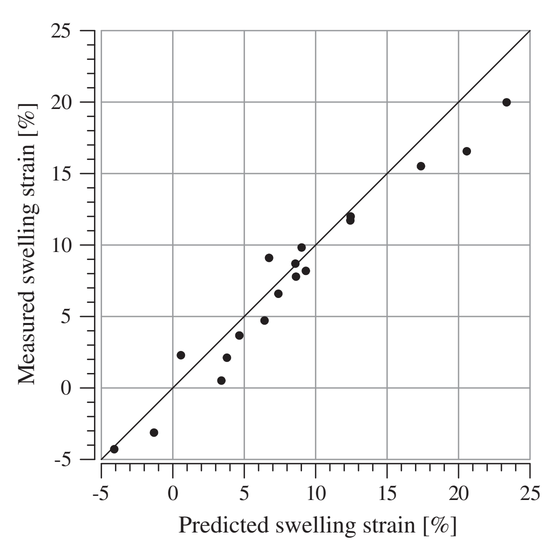


**Fig. 3.** Predictive capability of the dimensionless model for 1-D swelling: measured swelling strain vs. predicted swelling strain.

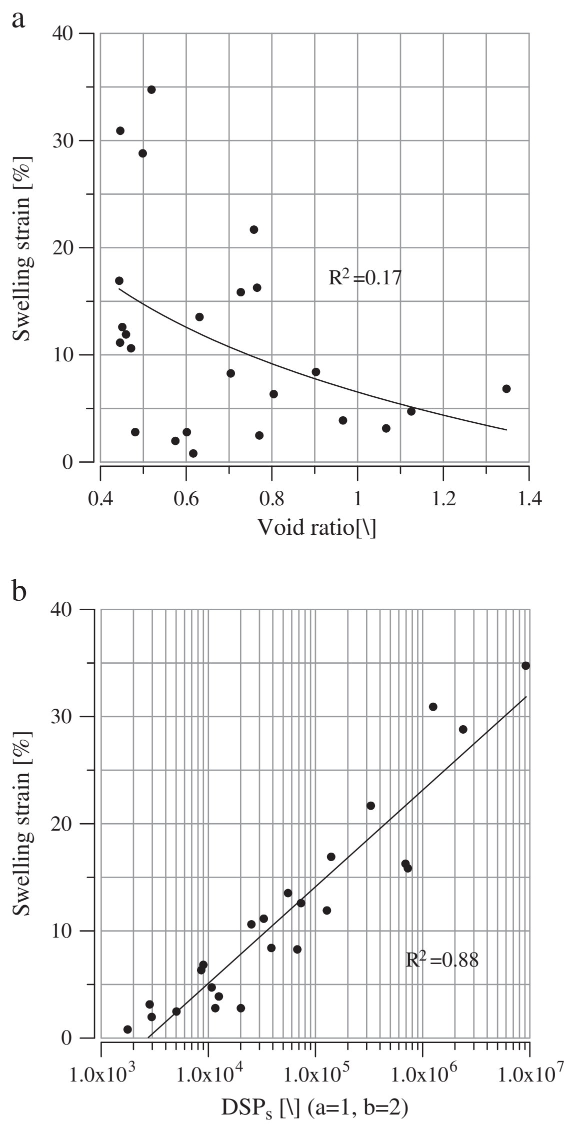


**Fig. 4.** Swelling tests under isotropic confinement. (a) Experimental swelling strain vs. void ratio. (b) Experimental swelling strain vs. $DSP_s$ ($a=1$ and $b=2$).

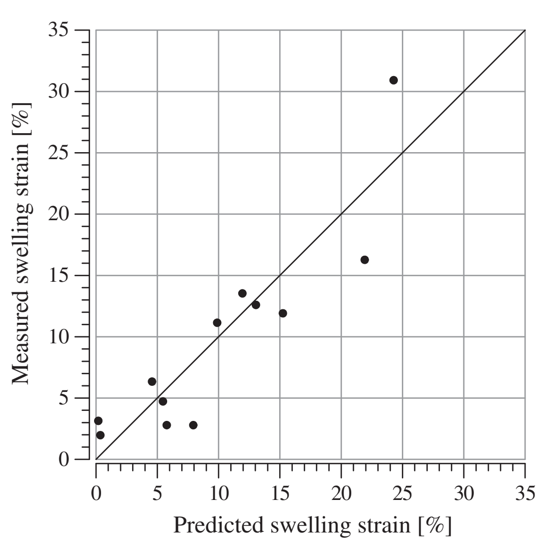


**Fig. 5.** Predictive capability of the dimensionless model for 3-D swelling: measured swelling strain vs. predicted swelling strain.

This reflects on the quality of the prediction. The fewer number of tests, compared to the first series, also tend to add to this impression but, overall, the dimensionless model for 3-D swelling can be validated.

### *4.3. Validation of the dimensionless model for partial swelling*

Once again, after a calibration process yielding $a=4$ and $b=1$, a good correlation can be found between the swelling strain and $DSP_{\Delta s}$. Like previously, the scattering of the tests shown in Fig. 6(a) contrasts neatly with the quality of the correlation displayed in Fig. 6(b). A power-type equation proves to be a better option than a logarithmic equation for this application. Note that the form of the curve fit does not affect the predictive capability of the model.

Like previously, the predictive capability of the model is assessed by partitioning the data in two random sets. After calibration ($a=4$ and $b=1$), the model for partial swelling can be expressed as:

$$\varepsilon_{sw} = 0.75 \cdot \left( \frac{\Delta s^5}{s_o^4 \cdot \sigma_v} \right)^{0.43} \tag{10}$$

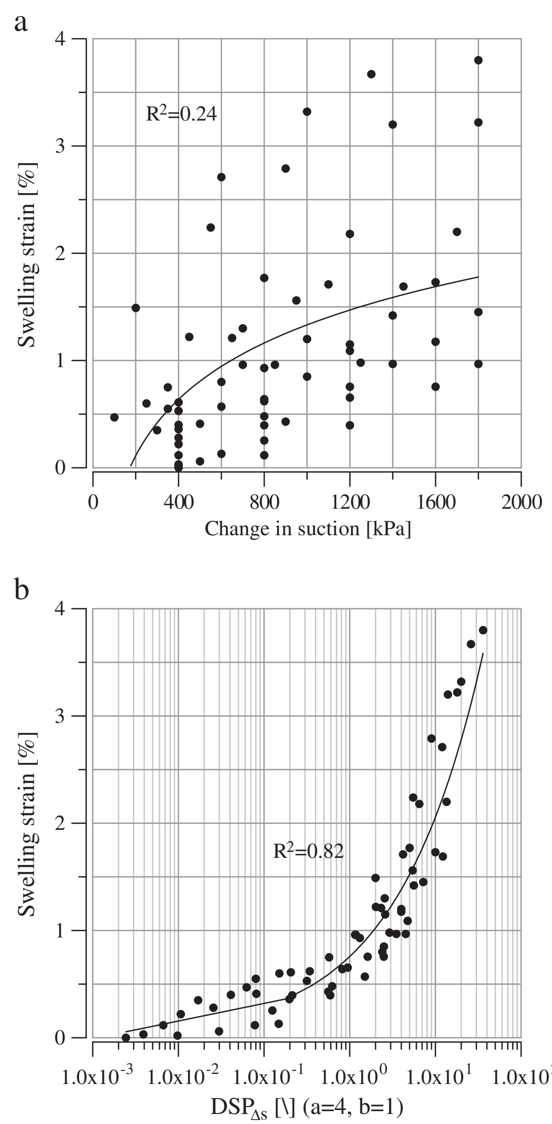


**Fig. 6.** Experimental results of swelling tests under controlled suction by (Escario and Saez, 1973). (a): Swelling strain vs. change in suction. (b) Swelling strain vs. $DSP_{\Delta s}$.

Again, despite a different set of influencing parameters and a different form of dimensionless number, the model derived from dimensional analysis achieves accurate predictions (Fig. 7). It has to be borne in mind that, at this stage, $DSP_{\Delta s}$ has been derived considering relative swelling strains whereas the tests were actually performed under constant initial suction and void ratio. The predictive capability of the dimensionless model was still demonstrated for a special case of partial swelling. However, another formulation would have to be proposed for $DSP_{\Delta s}$, should the initial suction and void ratio be variable.

### 4.4. Summary of the results

The outcomes of the three dimensionless models presented in the previous sections are summarized in Table 1.

The relative error is not really representative here since low values of experimental values yield very high values of error not reflecting the quality of the model. The 90% confidence interval has been used as an indicator of the quality of prediction. A value of 2.4 indicates that 90% of the predicted values are different from the measured values by less than 2.4 units of strain (here in %). Considering the range of swelling strain covered by the models, such narrow intervals indicate a good performance of the models.

## 5. Discussion

The dimensional analysis has proved to be a suitable and accurate method to predict soil swelling for different testing configurations. The evolution of dimensionless number from the work by Buzzi et al. (in press) has solved the issue of the dependence of the swelling strain on the initial height of the specimen ($h_o$ used to figure in $DSP_w$). Dimensional analysis is very versatile in nature: there is little limitation on the physical problems to describe, on the set of parameters to study or on the form of the dimensionless numbers. As for the model resulting from the dimensional analysis, any curve fit resulting in a high coefficient of determination could be used. However, some tests not reported here have shown the inability of the dimensionless model to capture the collapse upon wetting. This aspect will be discussed in this section, which also includes an investigation of the minimum number of tests required for an accurate calibration of the model.

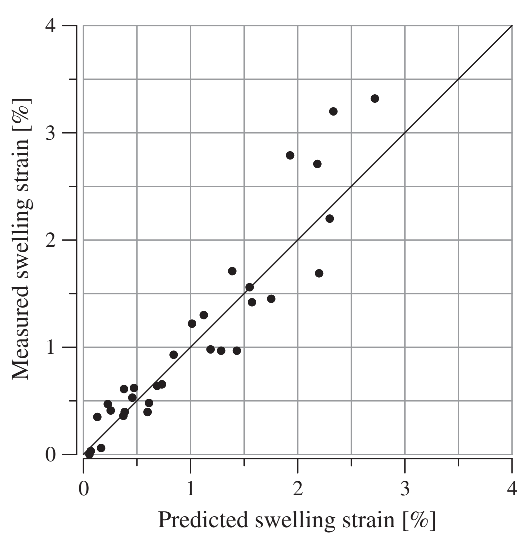


**Fig. 7.** Predictive capability of the dimensionless model for partial swelling: measured swelling strain vs. predicted swelling strain.

**Table 1**
Summary of the three dimensionless models. Each model was calibrated on half dataset resulting in the equation and coefficient of determination provided. 90% C.I: Confidence interval at 90%.

| Swelling test configuration | Equation of the model | $R^2$ | 90% C.I [%] | Range of strain [%] |
|---|---|---|---|---|
| 1-D | $\varepsilon_{sw} = 3.90 \cdot ln\left(\left(\frac{1}{e_o}\right)^3 \cdot \left(\frac{s_o}{\sigma_v}\right)\right) - 15.15$ | 0.95 | 2.4 | −5 to 25 |
| 3-D | $\varepsilon_v = 3.95 \cdot ln\left(\left(\frac{1}{e_o}\right) \cdot \left(\frac{s_o}{p}\right)^2\right) - 31.25$ | 0.92 | 5.1 | 0 to 35 |
| Partial | $\varepsilon_{sw} = 0.75 \cdot \left(\frac{\Delta s^5}{s_o^4 \cdot \sigma_v}\right)^{0.43}$ | 0.80 | 0.5 | 0 to 4 |

### 5.1. Issue with capturing the collapse upon wetting

Unsaturated soils having an open structure i.e. high initial void ratio can experience a collapse upon wetting (Escario and Saez, 1973). This phenomenon is well known and was already captured in one the very first constitutive models for unsaturated soils (Alonso et al., 1990). Some tests not reported here have shown that the dimensionless model cannot predict the collapse. This is due to the fact that $DSP_s$ is defined to obtain a monotonic relationship between this latter and the swelling strain: the higher $DSP_s$, the higher the swelling strain. However, as illustrated in Fig. 8, the collapse does not fit with this monotonic variation since low or negative values of strain are obtained for medium to high values of $DSP_s$. The model captures low or negative values of strain only for low values of $DSP_s$, usually corresponding to settlement under load. This limitation of the model is not necessarily problematic for engineering applications since proper compaction tends to limit the risk of collapse undergone by compacted materials.

The inability to capture the collapse potentially extends to any phenomenon for which a non-monotonic variation is expected. In that case, the scattered data cannot be gathered to form a clear trend since different values of swelling strain will be associated to a single value of dimensionless number.

### 5.2. Influence of the number of tests used for calibration

This section aims to provide some insight into the minimum number of tests to be carried out for an accurate calibration of the model. The results of the one dimensional swelling tests on remolded Maryland clay (Appendix A) have been used for this purpose. N test results (ranging from 4 to 36) have been drawn from the pool of test results and used to calibrate the dimensionless model for 1-D swelling. The highest value of $R^2$ has been determined. As the calibration depends on the results used, 10 sets of data were drawn, for each case, in order to provide some statistical description of the outcome. Fig. 9 shows the value of the coefficient of determination $R^2$ as a result of the calibration. The mean value is also plotted for each number of tests used for calibration.

The large scattering of $R^2$ for small numbers of tests shows that the calibration is highly sensitive to the tests drawn. The scattering tends to decrease with increasing number of tests After 12 tests, the mean value of $R^2$ is quasi constant and, after 16 tests, the range of $R^2$ is significantly narrowed. This observation suggests that 16 to 20 tests are required for the calibration. In order to confirm this point, the effect of the number of tests on the predictive capability of the model has also been assessed.

The model has been calibrated using one set of 6, 12 and 18 tests chosen randomly (if applicable) among the 18 tests used for the calibration–prediction exercise presented in Section 4.1 (left column of the table in Appendix A). The swelling strain has then been predicted for the other 18 tests (right column of the table in Appendix A). Fig. 10

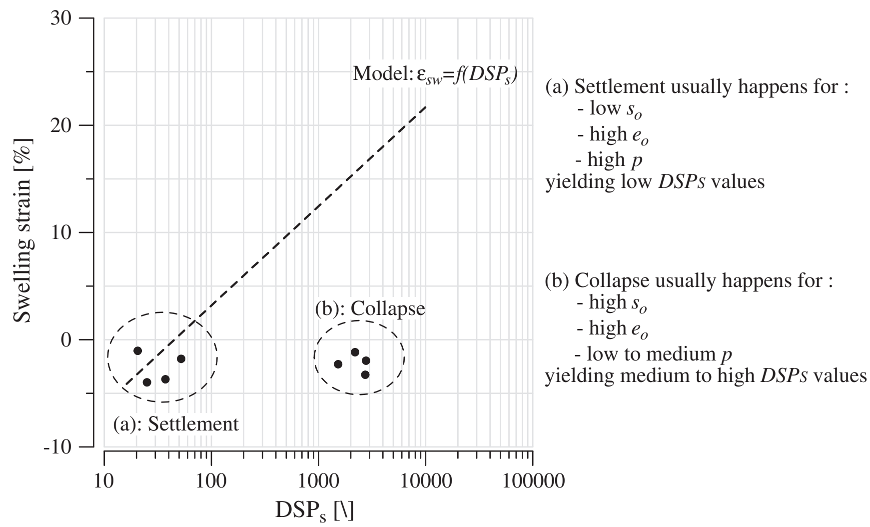


**Fig. 8.** Schematic explanation of the inability of the model to capture the collapse phenomenon. The dots are fictitious results to illustrate the explanation.

shows the performance of the model when using 6, 12 and 18 tests for the calibration. Note that the results for 18 tests are those presented in Fig. 3.

Quite logically, it is observed that the more tests used for the calibration, the more accurate the prediction. However, if 6 tests are clearly not enough, using 12 tests instead of 18 does not compromise that much the quality of the prediction. On the basis of Figs. 9 and 10, it is suggested that, for this specific set of results, 12 to 16 tests is a reasonable number for the calibration. Obviously, the quality of the results will affect the minimum number of tests required for calibration and the accuracy of the model. Also, more accurate predictions are expected when using the model for interpolations. It is important to bear this in mind when defining the calibration test program.

## 6. Conclusions

Buzzi et al. (in press) have proposed, for the first time, to predict the amount of soil swelling in oedometric conditions by the use of dimensional analysis. Despite the approach and the resulting dimensionless model were validated, some questions about the limitations of the model and the correlation between soil properties and model parameters remained unanswered. Also, issues arose from the presence of the initial height of the specimen in the dimensionless number $DSP_w$. The present work has confirmed that dimensional analysis can be used to predict soil swelling and it has even extended its use to swelling under isotropic confinement and to a special case of partial swelling. The limitations of the model and, in particular its inability to capture collapse upon wetting, have been discussed. Also, it appears that predicting the values of the exponents $a$ and $b$ (and other if any) from the soil properties is not appropriate since $a$ and $b$ are not unique.

The use of dimensional analysis to predict soil swelling is not only efficient but also very simple to implement. The Matlab routine, created for the calibration procedure is actually not a requirement, the calibration can be done manually (Buzzi et al., in press). Dimensional analysis is a very versatile approach that can be applied to a plethora of physical problems. If these latter are properly defined and described, there is little restriction on the set of influencing parameters to use, on the definition of the dimensionless parameters

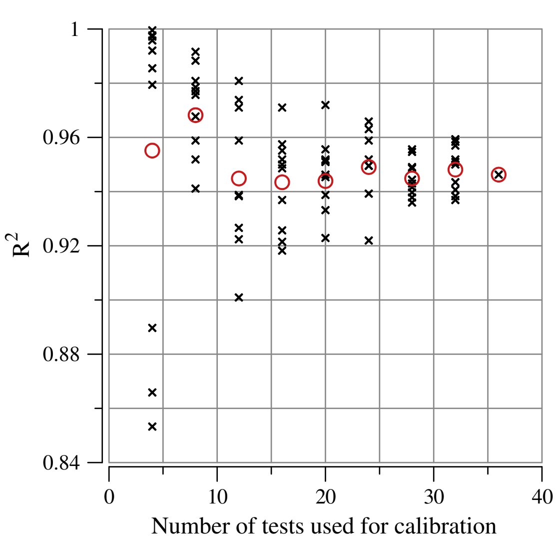


**Fig. 9.** Evolution of the coefficient of determination $R^2$ resulting from the model calibration as a function of the number of tests used for the calibration. For each number of tests, 10 calibrations were carried out, the mean value of which is represented by the red circle.

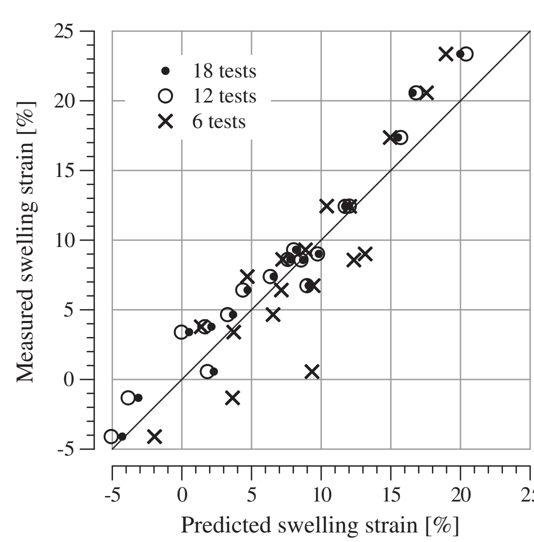


**Fig. 10.** Comparison between predicted swelling strain and measured swelling strain for a model calibrated using 6, 12 or 18 tests chosen randomly.

and, above all, on the equation of the curve fitting i.e. on the dimensionless model. In comparison to other data driven approaches such as neural networks or fuzzy inference systems, the theory and application of dimensional analysis is very simple. Considering the potential of this approach, here proved, one can think to apply it to other influencing parameters such as the amount of additives for stabilized expansive soils or temperature. One of the limitations is that only monotonic variations can be captured by the various dimensionless models.

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## Appendix A

Experimental results of one dimensional swelling tests on recompacted Maryland clay (oedometric conditions and vertical stress $\sigma_v$). The first half of the set (left column) has been used for calibration and the second half (right column) for prediction.

| $e_o$ | $s_o$ [MPa] | $\sigma_v$ [MPa] | $\varepsilon_{sw}$ [%] | $e_o$ | $s_o$ [MPa] | $\sigma_v$ [MPa] | $\varepsilon_{sw}$ [%] |
|---|---|---|---|---|---|---|---|
| 0.54 | 3.1 | 0.4 | −0.53 | 0.50 | 3.1 | 0.01 | 17.37 |
| 0.42 | 3.1 | 0.05 | 13.16 | 0.80 | 3.3 | 0.4 | −4.09 |
| 0.85 | 3.3 | 0.3 | −2.18 | 0.82 | 3.3 | 0.05 | 4.66 |
| 0.88 | 3.3 | 0.2 | −3.14 | 0.83 | 3.3 | 0.01 | 9.02 |
| 0.57 | 3.5 | 0.08 | 6.26 | 0.61 | 3.5 | 0.04 | 9.31 |
| 0.64 | 3.5 | 0.01 | 11.72 | 0.57 | 3.5 | 0.02 | 12.43 |
| 0.64 | 3.2 | 0.001 | 23.45 | 0.60 | 3.5 | 0.005 | 20.57 |
| 0.59 | 1 | 0.04 | 5.74 | 0.53 | 1 | 0.08 | 3.79 |
| 0.57 | 1 | 0.005 | 12.71 | 0.52 | 1 | 0.02 | 8.62 |
| 0.59 | 1 | 0.001 | 14.78 | 0.58 | 1 | 0.01 | 6.74 |
| 0.87 | 1.4 | 0.03 | 2.86 | 0.87 | 1.4 | 0.04 | 3.40 |
| 0.77 | 1.4 | 0.001 | 6.42 | 0.77 | 1.4 | 0.02 | 6.42 |
| 0.87 | 1.4 | 0.001 | 11.05 | 0.87 | 1.4 | 0.005 | 8.58 |
| 1.25 | 1.1 | 0.03 | −3.19 | 1.37 | 1.1 | 0.02 | −1.32 |
| 1.39 | 1.1 | 0.01 | −2.43 | 1.37 | 1.1 | 0.005 | 0.57 |
| 0.46 | 9.4 | 0.8 | 1.38 | 0.45 | 9.4 | 0.4 | 7.39 |
| 0.45 | 9.4 | 0.2 | 9.21 | 0.45 | 9.4 | 0.1 | 12.45 |
| 0.45 | 9.4 | 0.05 | 17.12 | 0.46 | 9.4 | 0.015 | 23.36 |

## Appendix B

Experimental results of swelling tests under isotropic confinement $p$ performed on recompacted Maryland clay. The first half of the set (left column) has been used for calibration and the second half (right column) for prediction.

| $e_o$ | $s_o$ [MPa] | $p$ [MPa] | $\varepsilon_v$ [%] | $e_o$ | $s_o$ [MPa] | $p$ [MPa] | $\varepsilon_v$ [%] |
|---|---|---|---|---|---|---|---|
| 0.50 | 21.8 | 0.02 | 28.81 | 0.45 | 21.8 | 0.12 | 12.60 |
| 0.73 | 21.8 | 0.03 | 15.85 | 0.46 | 21.8 | 0.09 | 11.91 |
| 0.70 | 21.8 | 0.1 | 8.28 | 0.77 | 21.8 | 0.03 | 16.27 |
| 0.52 | 21.8 | 0.01 | 34.75 | 0.45 | 21.8 | 0.18 | 11.14 |
| 0.47 | 21.8 | 0.2 | 10.62 | 0.48 | 7.5 | 0.1 | 2.79 |
| 0.44 | 7.5 | 0.03 | 16.92 | 0.63 | 7.5 | 0.04 | 13.53 |
| 0.77 | 7.5 | 0.12 | 2.48 | 0.80 | 7.5 | 0.09 | 6.34 |
| 0.90 | 7.5 | 0.04 | 8.41 | 0.45 | 7.5 | 0.01 | 30.92 |
| 0.76 | 7.5 | 0.015 | 21.69 | 0.60 | 3.3 | 0.03 | 2.79 |
| 0.62 | 3.3 | 0.1 | 0.8 | 1.07 | 3.3 | 0.06 | 3.14 |
| 1.35 | 3.3 | 0.03 | 6.84 | 0.57 | 3.3 | 0.08 | 1.97 |
| 0.97 | 3.3 | 0.03 | 3.88 | 1.13 | 3.3 | 0.03 | 4.73 |

## References


Aitchinson, G.D., Holmes, J.W., 1953. Aspects of swelling in the soil profile. Australian Journal of Applied Science 4, 244–259.

Alonso, E.E., Gens, A., Josa, A., 1990. A constitutive model for partially saturated soils. Geotechnique 40, 405–430.

Alonso, E.E., Vaunat, J., Gens, A., 1999. Modelling the mechanical behaviour of expansive clays. Engineering Geology 54, 173–183.

Brackley, I.J., 1973. Swell pressure and free swell in a compacted clay. Proceedings of the Third International Conference on Expansive Soils, vol. 1, pp. 169–176. Haifa, Israel.

Buckingham, E., 1914. On physically similar systems: illustrating the use of dimensional analysis. Physical Review 4, 345–376.

Butterfield, R., 1999. Dimensional analysis for geotechnical engineers. Geotechnique 49, 357–366.

Buzzi, O., Giacomini, A., Fityus, S., in press. Towards a dimensionless description of soil swelling behaviour. Géotechnique. doi:10.1680/geot.7.00194.

Cohen, J., Cohen, P., West, S., Aiken, L., 2003. Applied multiple regression/correlation analysis for the behavioral sciences, 2nd ed. Lawrence Erlbaum Associates, Hillsdale, NJ.

Dif, A., Bluemel, W., 1991. Expansive soils and cyclic drying and wetting. Geotechnical Testing Journal 14, 96–102.

Dormieux, L., Barboux, P., Coussy, O., Dangla, P., 1995. A macroscopic model of the swelling phenomenon of a saturated clay. European Journal of Mechanics—A/Solids 14, 981–1004.

Erguler, Z.A., Ulusay, R., 2003. A simple test and predictive models for assessing swell potential of Ankara (Turkey) Clay. Engineering Geology 67, 331–352.

Escario, V., Saez, J., 1973. Measurement of the properties of swelling and collapsing soils under controlled suction. Proceedings of the Third International Conference on Expansive Soils, vol. 1, pp. 195–200. Haifa, Israel.

Fityus, S., Smith, D., 2004. The development of a residual soil profile from a mudstone in a temperate climate. Engineering Geology 74, 39–56.

Holtz, W.G., Gibbs, H.J., 1956. Engineering properties of expansive clays. ASCE Transactions 121, 641–677.

Komine, H., Ogata, N., 1996. Prediction for swelling characteristics for compacted bentonite. Canadian Geotechnical Journal 33, 11–22.

Ladd, C., Lambe, T.W., 1961. The identification and behaviour of compacted expansive clays. Proceedings of the Fifth International Conference on Soil Mechanics and Foundation Engineering, pp. 201–205. Paris.

Morgenstern, N.M., Balasubramonian, B.I., 1980. Effects of pore fluid on the swelling of clay-shale. Proceedings of the Fourth International Conference on Expansive Soils. American Society of Civil Engineers, Denver, Colorado, pp. 190–205.

Noble, C.A., 1966. Swelling measurements and prediction of heave for a lacustrine clay. Canadian Geotechnical Journal 3, 32–41.

Norrish, K., 1954. Swelling of montmorillonite. Faraday Society Discussion 18, 120–134.

Oka, F., Kimoto, S., Higo, Y., Feng, H., 2009. Elasto-viscoplastic constitutive modeling of the swelling process of unsaturated clay. Proceedings of the Fourth Asia Pacific Conference on Unsaturated Soils, vol. 2. CRC Press, Newcastle, Australia, pp. 507–514.

Parcher, J.V., Liu, P., 1965. Some swelling characteristics of compacted clays. ASCE Journal of Soil Mechanics and Foundation Division 91, 1–17.

Popescu, M., 1980. Behavior of expansive soils with crumb structure. Proceedings of the Fourth International Conference on Expansive Soils, 2, pp. 158–171.

Schreiner, H., Burland, J., 1987. Stress paths during swelling of compacted soils under controlled suction. Proceedings of the Sixth International Conference on Expansive Soils, pp. 155–159.

Sposito, G., Prost, R., 1982. Structure of water adsorbed on smectites. Chemical Reviews 82, 554–573.

Uppal, H.L., Palit, P.M., 1969. Measurement of swelling pressure of expansive soils. Proceedings of the Second International Research and Engineering Conference on expansive clay soils, pp. 250–255.

Yevnin, A., Zaslavsky, D., 1970. Some factors affecting compacted clay swelling. Canadian Geotechnical Journal 7, 79–91.