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CREEP AND CONSOLIDATION AROUND CIRCULAR OPENINGS IN INFINITE MEDIA

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Abstract—The time dependent response of saturated soil, following the creation of a long, deep cylindrical opening has been investigated. The time dependence occurs because of consolidation of the saturated medium and creep of its solid skeleton. A method of analysis for this problem has been developed and solutions for the displacement and changes in pore pressure and stress in the ground around the opening have been presented. Several different constitutive models for the creep of the soil skeleton have been studied. It has been concluded that the response of the soil skeleton in shear is of major importance in determining the behaviour of the ground around the opening.

1. INTRODUCTION

Engineers concerned with the design of tunnel linings or tunnel support systems are often called upon to make predictions of the movements and stress changes occurring in the ground surrounding the opening. For certain simple cases analytic solutions have been found on the assumption that the surrounding material is an isotropic elastic solid (e.g.[1–3]). For tunnels in saturated clay these solutions have relevance only to the short term (or undrained) conditions and the long term (or fully drained) condition.

For some time it has been recognised that the behaviour of the ground around openings through saturated clays is time dependent. It has been common for some tunnel engineers to attribute this time dependence solely to the creep behaviour of the soil medium. Other geotechnical engineers have tried to associate these time dependent phenomena with primary consolidation (e.g.[4]). Until recently, solutions have not been available for either of the problems of creep or consolidation within a soil following the removal of materials to form an opening, and thus it has been diffcult to make a decision about the relative importance of these time dependent processes. Solutions for the consolidation in an elastic soil surrounding a long and deep circular tunnel have recently been developed[5]. In this present paper a method of analysis is proposed for the simultaneous consolidation and creep of a visco-elastic soil due to the cutting of such a tunnel. The method involves the use of Laplace transforms of the governing equations of visco-elastic deformation. Closed form solutions have been found for the Laplace transforms of the various field quantities, i.e. displacement, pore pressure and stress, and these have then been inverted. In some cases it has not been possible to perform the inversion explicitly and it has been necessary to perform numerical integration in the complex plane.

2. PROBLEM DESCRIPTION

It is assumed that the circular opening is created at a depth which is large when compared with its radius. To sufficient accuracy the *in situ* stress state in the ground prior to removal of material may be idealised as being uniform with a total normal stress σ_v, which acts in the vertical direction and a total normal stress σ_H, which acts in all horizontal directions. The horizontal total stress may be expressed as a proportion of the total vertical stress, thus

$$\sigma_H = N\sigma_v. \tag{1}$$

The principle of effective stress gives

$$\sigma_v = \sigma_v' + p_0$$

$$\sigma_H = \sigma_H' + p_0$$

where σ'_v and σ'_H are vertical and horizontal effective stress components, respectively, and p_o is the *in situ* pore water pressure. Compressive stress is considered to be positive. The coefficient of earth pressure at rest K_0, is defined by

$$\sigma'_H = K_0 \sigma'_v. \tag{2}$$

For a soil deposit with the water table at the surface, and a bulk unit weight γ, then

$$N = K_0 - \frac{\gamma_w}{\gamma}(K_0 - 1) \tag{3}$$

where γ_w is the unit weight of water.

For the circular opening it is more convenient to express the *in situ* stress state in terms of polar coordinates (see Fig. 1). It follows that the total stresses acting on a circular boundary are given by

$$\sigma_{rr} = \sigma_m + \sigma_d \cos 2\theta \tag{4a}$$

$$\sigma_{\theta\theta} = \sigma_m - \sigma_d \cos 2\theta \tag{4b}$$

$$\sigma_{r\theta} = -\sigma_d \sin 2\theta \tag{4c}$$

where

$$\sigma_m = \frac{1}{2}(\sigma_v + \sigma_H)$$

$$\sigma_d = \frac{1}{2}(\sigma_v - \sigma_H).$$

Equations (4) give the stresses which act at the circular periphery before the opening is made. After boring, the normal total stress σ_{rr} and the shear stress $\sigma_{r\theta}$ are both removed from this boundary. Hence the stress boundary conditions for this problem may be idealised as

$$\Delta\sigma_{rr}(r_0, t) = -(\sigma_m + \sigma_d \cos 2\theta) \tag{5a}$$

$$\Delta\sigma_{r\theta}(r_0, t) = \sigma_d \sin 2\theta \tag{5b}$$

where

$$\sigma_m = \sigma_m(r_0, t)$$

$$\sigma_d = \sigma_d(r_0, t)$$

where the symbol Δ indicates a change in the appropriate quantity.

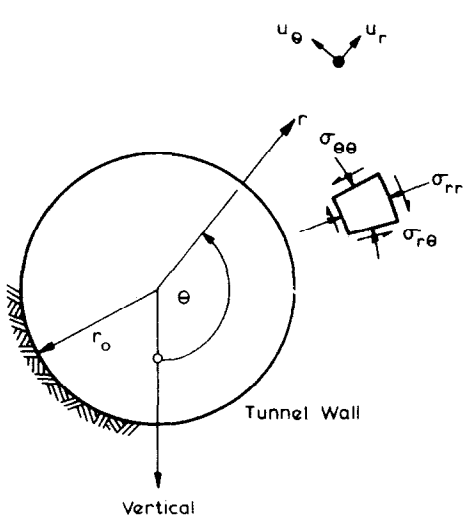


Fig. 1. Problem description.

It is also necessary to consider the hydraulic conditions at the inner boundary, since the soil is saturated. There are two important cases which may be identified. These are:

(a) a permeable boundary; and

(b) an impermeable boundary.

The corresponding conditions imposed on the pore pressure p are

(a) $$p = 0 \text{ at } r = r_0 \tag{6a}$$

(b) $$\frac{\partial p}{\partial r} = 0 \text{ at } r = r_0. \tag{6b}$$

In this paper only the case of an impermeable inner boundary is considered. This is probably the most realistic condition, since for most circular tunnels in saturated clay, some form of impermeable lining is placed around the boundary soon after the soil is removed.

Equations (5) and (6) describe the complete set of stress conditions which are imposed at the inner boundary when material is removed to form a circular opening in saturated clay. In a linear material it is possible to employ the principle of superposition in order to simplify the analysis. The overall problem may be divided conceniently into two quite separate components. The two sets of boundary conditions are then:

(a) *Removal of the* in situ *mean total stress*

$$\left.\begin{aligned} \Delta\sigma_{rr} &= -\sigma_m \\ \Delta\sigma_{r\theta} &= 0 \\ \frac{\partial p}{\partial r} &= 0. \end{aligned}\right\} \text{ at } r = r_0$$

(b) *Removal of the* in situ *deviator stress*

$$\left.\begin{aligned} \Delta\sigma_{rr} &= -\sigma_d \cos 2\theta \\ \Delta\sigma_{r\theta} &= \sigma_d \sin 2\theta \\ \frac{\partial p}{\partial r} &= 0. \end{aligned}\right\} \text{ at } r = r_0$$

3. GOVERNING EQUATIONS

The equations of consolidation of a saturated soil with a visco-elastic skeleton under conditions of plane strain are:

(1) *The equilibrium equations*

$$\begin{aligned} \frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} &= 0 \\ \frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{2\sigma_{r\theta}}{r} &= 0. \end{aligned} \tag{7}$$

(2) *The effective stress strain relation*

$$\begin{aligned} \bar{\sigma}_{rr} - \bar{p} &= \bar{\lambda}\bar{\epsilon}_V + 2\bar{G}\bar{\epsilon}_{rr} \\ \bar{\sigma}_{\theta\theta} - \bar{p} &= \bar{\lambda}\bar{\epsilon}_V + 2\bar{G}\bar{\epsilon}_{\theta\theta} \\ \bar{\sigma}_{r\theta} &= \bar{G}\bar{\gamma}_{r\theta} \end{aligned} \tag{8}$$

where

$$\begin{aligned} \epsilon_{rr} &= -\frac{\partial u_r}{\partial r} \\ \epsilon_{\theta\theta} &= -\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}\right) \\ \gamma_{r\theta} &= -\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right) \end{aligned} \tag{9}$$

u_r, u_θ are the displacement components and the bar denotes a Laplace transform, viz.

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)\,dt \tag{10}$$

where the symbol t is used to represent time. The transformed moduli $\bar{\lambda}$, $\bar{G}$ are related to the volumetric and deviatoric creep functions $J_V(t)$, $J_D(t)$ as follows:

$$\begin{aligned} \bar{G} &= 1/(s\bar{J}_D) \\ \bar{\lambda} + \frac{2}{3}\bar{G} &= 1/(s\bar{J}_V). \end{aligned} \tag{11}$$

(3) *The volume constraint condition*

This condition follows from the assumed incompressibility of the pore water fluid and the skeletal material so that

$$\nabla^T \cdot \mathbf{v} = \frac{\partial \epsilon_V}{\partial t} \tag{12}$$

and

$$\epsilon_V = 0 \text{ at } t = 0+$$

where $\mathbf{v}$ is the superficial velocity of the pore water and ϵ_V is the volume strain.

On applying a Laplace transform to these equations we find

$$\nabla^T \cdot \bar{\mathbf{v}} = s\bar{\epsilon}_V. \tag{13}$$

(4) *Darcy's law*

$$\mathbf{v} = -\frac{\mathbf{k}}{\gamma_w}\nabla p \tag{14}$$

where k is the soil permeability and γ_w is the unit weight of the pore water.

If σ_{rr}, $\sigma_{r\theta}$, $\mathbf{v}$ are eliminated from these equations it is found that:

$$\begin{aligned} \frac{1}{r}\frac{\partial \bar{\omega}}{\partial \theta} &= -\frac{\partial \bar{\phi}}{\partial r} \\ \frac{\partial \bar{\omega}}{\partial r} &= +\frac{1}{r}\frac{\partial \bar{\phi}}{\partial \theta} \\ \nabla^2 \bar{\epsilon}_V &= \frac{s}{\bar{c}}\bar{\epsilon}_V \end{aligned} \tag{15}$$

where

$$\begin{aligned} \bar{p} &= \bar{G}\bar{\phi} + (\bar{\lambda} + 2\bar{G})\bar{\epsilon}_V \\ \omega &= \frac{1}{r}\frac{\partial}{\partial r}(ru_\theta) - \frac{1}{r}\frac{\partial u_r}{\partial \theta} \\ \epsilon_V &= \frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} \end{aligned} \tag{16}$$

and

$$\bar{c} = \frac{k}{\gamma_w}(\bar{\lambda} + 2\bar{G}).$$

All the solutions sought may be expressed as the sum of components having the form

$$\begin{aligned} u_r &= U_r(r, t)\cos(n\theta + \epsilon) \\ u_\theta &= U_\theta(r, t)\sin(n\theta + \epsilon) \\ p &= P(r, t)\cos(n\theta + \epsilon). \end{aligned} \tag{17}$$

In this case eqs (15) reduce to the ordinary differential equations

$$n\bar{\Omega} = -\frac{d\bar{\phi}}{dr}$$

$$\frac{d\bar{\Omega}}{dr} = -n\bar{\phi}$$

$$\frac{d^2E_V}{dr^2} + \frac{1}{r}\frac{dE_V}{dr} = \left(n^2 + \frac{s}{\bar{c}}\right)\bar{E}_V \tag{18}$$

where

$$\Omega = \frac{1}{r}\left[\frac{d}{dr}(rU_\theta) + nU_r\right]$$

$$E_V = \frac{1}{r}\left[\frac{d}{dr}(rU_r) + nU_\theta\right]$$

$$\bar{p} = \bar{G}\bar{\phi} + (\bar{\lambda} + 2\bar{G})\bar{E}_V.$$

4. SOLUTIONS FOR THE LAPLACE TRANSFORMS

In a previous investigation[5], it was shown that the general solution to eqs (18) may be written in terms of six, independent components, i.e.

$$\bar{\mathbf{U}} = M\mathbf{A} \tag{19}$$

where

$$\bar{\mathbf{U}} = \left\{\bar{U}_r, \bar{U}_\theta, \frac{\bar{P}}{2\bar{G}}, \frac{\bar{S}_{rr}}{2\bar{G}}, \frac{\bar{S}_{\theta\theta}}{2\bar{G}}, \frac{\bar{S}_{r\theta}}{2\bar{G}}\right\}^T$$

$$\mathbf{A} = \{A_1, A_2, A_3, A_4, A_5, A_6\}^T$$

and the matrix M is set out below

$$M = \begin{bmatrix} \frac{\bar{c}}{s}\psi_1 & \frac{\bar{c}}{s}\psi_2 & \frac{1}{4}\left(\frac{n}{n-1}\right)r^{1-n} & \frac{1}{4}\left(\frac{-n}{n+1}\right)r^{n+1} & r^{-n-1} & r^{n-1} \\ -\frac{\bar{c}}{s}\frac{n}{r}\phi_1 & -\frac{\bar{c}}{s}\frac{n}{r}\phi_2 & \frac{1}{4}\left(\frac{n-2}{n-1}\right)r^{1-n} & \frac{1}{4}\left(\frac{n+2}{n+1}\right)r^{n+1} & r^{-n-1} & -r^{n-1} \\ \left(\frac{\bar{\lambda}+2\bar{G}}{2\bar{G}}\right)\phi_1 & \left(\frac{\bar{\lambda}+2\bar{G}}{2\bar{G}}\right)\phi_2 & \frac{1}{2}r^{-n} & -\frac{1}{2}r^{n} & 0 & 0 \\ -\frac{\bar{c}}{s}\left(\frac{n^2}{r^2}\phi_1 - \frac{1}{r}\psi_1\right) & -\frac{\bar{c}}{s}\left(\frac{n^2}{r^2}\phi_2 - \frac{1}{r}\psi_2\right) & \left(\frac{n}{4}+\frac{1}{2}\right)r^{-n} & \left(\frac{n}{4}-\frac{1}{2}\right)r^{n} & (n+1)r^{-n-2} & -(n-1)r^{n-2} \\ \left(1+\frac{n^2}{r^2}\frac{\bar{c}}{s}\right)\phi_1 - \frac{\bar{c}}{s}\frac{1}{r}\psi_1 & \left(1+\frac{n^2}{r^2}\frac{\bar{c}}{s}\right)\phi_2 - \frac{\bar{c}}{s}\frac{1}{r}\psi_2 & \left(-\frac{n}{4}+\frac{1}{2}\right)r^{-n} & -\left(\frac{n}{4}+\frac{1}{2}\right)r^{n} & -(n+1)r^{-n-2} & (n-1)r^{n-2} \\ -\frac{\bar{c}}{s}\left(\frac{n}{r^2}\phi_1 - \frac{n}{r}\psi_1\right) & -\frac{\bar{c}}{s}\left(\frac{n}{r^2}\phi_2 - \frac{n}{r}\psi_2\right) & \left(\frac{n}{4}\right)r^{-n} & -\left(\frac{n}{4}\right)r^{n} & (n+1)r^{-n-2} & (n-1)r^{n-2} \end{bmatrix}$$

where

$$\phi_1 = K_n\left(\sqrt{\frac{s}{\bar{c}}}\,r\right),\ \phi_2 = I_n\left(\sqrt{\frac{s}{\bar{c}}}\,r\right),\ \psi_1 = \sqrt{\frac{s}{\bar{c}}}\,K_n'\left(\sqrt{\frac{s}{\bar{c}}}\,r\right),\ \psi_2 = \sqrt{\frac{s}{\bar{c}}}\,I_n'\left(\sqrt{\frac{s}{\bar{c}}}\,r\right)$$

and I_n and K_n are the modified Bessel functions of order n. The constants $\mathbf{A}$ are determined from the boundary conditions.

Case (a). Removal of the in situ *mean total stress.* In this case it is obvious that since the quantities of interest are independent of θ then $n = 0$ and $\epsilon = 0$ and thus U_θ and $S_{r\theta}$ vanish identically. The solution which remains bounded as $r \to \infty$ consists of only two parts; only the

constants A_1 and A_5 are of importance. They have the values

$$A = 0$$

$$A = -\frac{\sigma_m r_0}{2\bar{G}}.$$

Case (b). Removal of the in situ *deviator stress.* In this case, in view of the symmetry of the problem about the vertical and horizontal directions through the centre of the opening, it is clear that $n = 2$ and $\epsilon = 0$ and the solution which remains bounded as $r \to \infty$ is composed of three parts, requiring values for A_1, A_3 and A_5. These are

$$A_1 = \frac{1}{\chi} \cdot 2 \cdot \left(\frac{\sigma_d}{2\bar{G}s}\right)$$

$$A_3 = \frac{1}{\chi} \cdot 2 \cdot \left(\frac{\sigma_d}{2\bar{G}s}\right) \cdot r_0^3 \cdot \left(\frac{\bar{\lambda} + 2\bar{G}}{2\bar{G}}\right) \cdot \psi_1(r_0)$$

$$A_5 = \frac{1}{\chi} \cdot \left(\frac{\sigma_d}{2\bar{G}s}\right) \cdot r_0^4 \left\{\left(\frac{\bar{c}}{sr_0^2}\right) \cdot [2\phi_1(r_0) - r_0 \cdot \psi_1(r_0)] - \frac{1}{2}\left(\frac{\bar{\lambda} + 2\bar{G}}{2\bar{G}}\right) \cdot r_0 \psi_1(r_0)\right\}$$

where

$$\chi = \left(\frac{\bar{c}}{sr_0^2}\right) \cdot [2\phi_1(r_0) + r_0 \cdot \psi_1(r_0)] - \frac{1}{2}\left(\frac{\bar{\lambda} + 2\bar{G}}{2\bar{G}}\right) \cdot r_0 \cdot \psi_1(r_0).$$

5. NUMERICAL INVERSION OF THE LAPLACE TRANSFORMS

Equation (19) together with the constants set out in the last section provide the solution to the tunnel problem in Laplace transform space. In order to recover the coefficients U_r, etc. these transforms must be inverted. This can be achieved by an application of the Complex Inversion Theorem, i.e. for the general function $f(t)$ the inversion is given by

$$f(t) = \frac{1}{2\pi i} \int_C \bar{f}(s)\, e^{st}\, ds$$

where s is a complex variable and C is any contour in the complex plane such that all singularities of $\bar{f}(s)$ lie to the left of C. For the results presented in this paper the integration was performed numerically using the contour and the efficient numerical scheme developed by Talbot[6].

6. THE CONSTITUTIVE MODELS

One of the aims of the present work is to estimate to what extent consolidation of the soil and creep of its solid skeleton determine the deformation around a circular opening. Both phenomena will, of course, contribute to the time dependence of these deformations.

Five different models for the stress–strain behaviour of the soil skeleton have been selected for examination. Four of them exhibit some time dependence (creep), the fifth assumes an isotropic elastic skeleton. Details of the models are set out in Table 1. Figure 2 shows the rheological models used to describe the form of time dependence of either the volumetric or the deviatoric behaviour.

The response functions for these creep models are as follows:

$$\text{Volumetric creep: } J_V(t) = A_V - B_V \exp\{-\alpha_V t\}$$

$$\text{Deviatoric creep: } J_D(t) = A_D - B_D \exp\{-\alpha_D t\}.$$

Table 1. Soil models

Model No.	Description	Volumetric Behaviour	Deviatoric Behaviour
1	Pure volumetric creep	As Fig. 2(a)	G = constant
2	Pure shear creep	K = constant	As Fig. 2(b)
3	Linked volumetric and shear creep	$K = \frac{2(1+\nu)G}{3(1-2\nu)}$ (ν = constant)	As Fig. 2(b)
4	Independent volumetric and shear creep	As Fig. 2(a)	As Fig. 2(b)
5	Isotropic elastic	K = constant	G = constant

K = bulk modulus

G = shear modulus

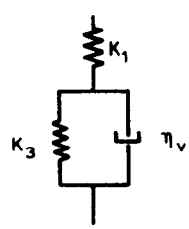
ν = Poisson's ratio

If the soil undergoes volumetric creep, and if it is subjected to a constant increment of bulk stress σ_0, and there are no consolidation effects, it will exhibit a volume strain ϵ_v given by

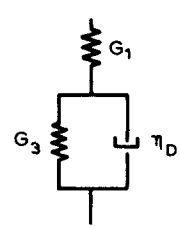
$$\epsilon_v = \sigma_0\left[\frac{1}{K_0}\exp\{-T_v\} + \frac{1}{K_\infty}(1-\exp\{-T_v\})\right]$$

where

$$K_0 = K_1 = \frac{1}{A_V - B_V} = \text{Bulk "modulus" at } t = 0$$



(a) Time Dependent Volumetric Behaviour



(b) Time Dependent Deviatoric Behaviour

Fig. 2. Soil skeleton models.

$$K_\infty = \frac{K_1 K_3}{K_1 + K_3} = \frac{1}{A_V} = \text{Bulk "molulus" at } t = \infty$$

$$T_V = \frac{K_3 t}{\eta_V} = \alpha_V t = \begin{array}{c}\text{Viscoelastic time factor for}\\ \text{volume strain,}\end{array}$$

and will thus creep from an initial strain

$$\epsilon_{v0} = \frac{\sigma_0}{K_0} \text{ to a final strain } \epsilon_{v\infty} = \frac{\sigma_0}{K_\infty}$$

Similarly, if the soil undergoes deviatoric creep, and if it is subjected to a constant increment in shear stress τ_0, and there are no consolidation effects, it will exhibit a shear strain γ given by

$$\gamma = \tau_0\left[\frac{1}{G_0}\exp\{-T_D\} + \frac{1}{G_\infty}(1 - \exp\{-T_D\})\right]$$

where

$$G_0 = G_1 = \frac{1}{A_D - B_D} = \text{Shear "modulus" at } t = 0$$

$$G_\infty = \frac{G_1 G_3}{G_1 + G_3} = \frac{1}{A_D} = \text{Shear "modulus" at } t = \infty$$

$$T_D = \frac{G_3 t}{\eta_D} = \alpha_D t = \begin{array}{c}\text{Viscoelastic time factor for}\\ \text{shear strain,}\end{array}$$

and thus will creep from an initial strain $\gamma_0 = \frac{\tau_0}{G_0}$ to a final strain $\gamma_\infty = \frac{\tau_0}{G_\infty}$.

In presenting the results it will be useful to adopt the following notation, which is based on that suggested by Gibson and Lo[7]:

$$T = 2G_0\left(\frac{1-\nu_0}{1-2\nu_0}\right)\frac{k}{\gamma_w}\frac{t}{r_0^2} = \text{Time factor for primary consolidation}$$

$$\nu_0 = \text{"Poisson's ratio" of soil skeleton at } t = 0$$

$$M_V = \frac{K_0}{K_\infty} = \text{Volumetric compressibility factor}$$

$$M_D = \frac{G_0}{G_\infty} = \text{Deviatoric compressibility factor}$$

$$N_V = 2\left(\frac{1-\nu_0}{1-2\nu_0}\right)\frac{T_V}{T}$$

$$N_D = 2\left(\frac{1-\nu_0}{1-2\nu_0}\right)\frac{T_D}{T}$$

where N_V, N_D are the time influence factors relating the volumetric, deviatoric and consolidation time scales.

Values for all of these parameters have been selected so that, when each soil is subjected to the same loading, it will exhibit the same initial response, i.e. at time $t = 0$. In all cases the initial

Table 2. Summary of parameter values

Soil Model	M_V	N_V	M_D	N_D
1. Pure volumetric creep	2	0.1, 1, 10	1	-
2. Pure shear creep	1	-	2	0.1, 1. 10
3. Linked volumetric and shear creep	$\nu = 0.3$		2	0.1, 1, 10
4. Independent volumetric and shear creep	2	0.1, 1, 10	2	0.1, 1, 10
5. Isotropic elastic	1	-	1	-

"Poisson's ratio" of the skeleton is assumed to be 0.3. In model 3, for which the shear and volumetric creep are linked, the value of Poisson's ratio remains constant for all time, and further, because of the manner in which values have been selected, models 3 and 4 will respond identically for all times. It should be emphasised, however, that in general this will not always be the case. The numerical values assumed for each constitutive model in the present study have been summarised in Table 2.

7. RESULTS

Some solutions for the relevant field quantities are now presented.

The numerical study has been divided into two parts. In the first part, the time scales for both creep and consolidation are similar, i.e. $N_V = 1$ or $N_D = 1$ as appropriate, while in the second, the effect of a variation in the time scales has been studied.

7.1 *Similar time scales for consolidation and creep*

Case (a). Removal of the in situ *mean total stress.* The solutions in this case are independent of the coordinate θ, but depend on the radial position r. All Laplace transforms may be inverted analytically and explicit expressions for the field quantities are given below.

$$u_r = -\frac{1}{2}\sigma_m \frac{r_0^2}{r}(A_D - B_D \exp\{-\alpha_D t\})$$

$$u_\theta = 0$$

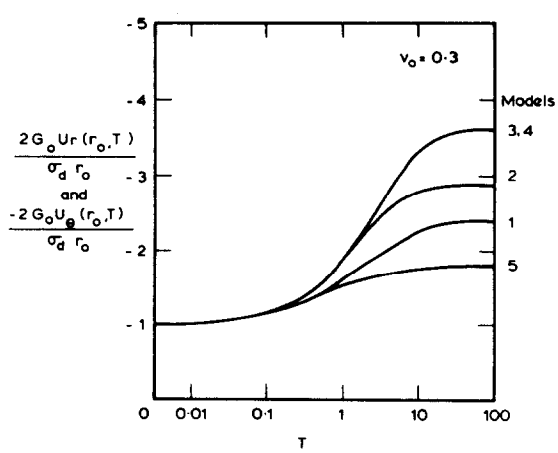


Fig. 3. Radial and circumferential displacements at tunnel wall.

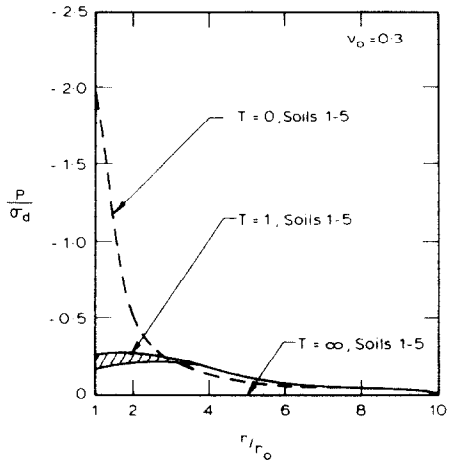


Fig. 4. Coefficient of excess pore pressure for removal of *in-situ* deviator stress.

$$p = 0$$

$$\sigma_{rr} = -\sigma_m\left(\frac{r_0}{r}\right)^2$$

$$\sigma_{\theta\theta} = +\sigma_m\left(\frac{r_0}{r}\right)^2$$

$$\sigma_{r\theta} = 0.$$

Only the radial displacement u_r varies with time, while the changes in radial and circumferential stress are independent of time.

Case (b). Revoval of the in situ *deviator stress.* For this problem it has not been possible to invert all Laplace transforms analytically. Numerical inversion was required and selected results for the Fourier coefficients of the field quantities, i.e. U_r, U_θ and P are given in Figs. 3–6. In general, the field quantities depend on θ as well as on r and t and may be recovered from the Fourier coefficients using eqs (17). On all figures, results have been indicated for each of the five soil models, to enable comparisons to be made.

Figure 3 shows the variation with time of the movement at the tunnel wall ($r = r_0$). These results have been plotted against the non-dimensional time T. Parameters were selected so that

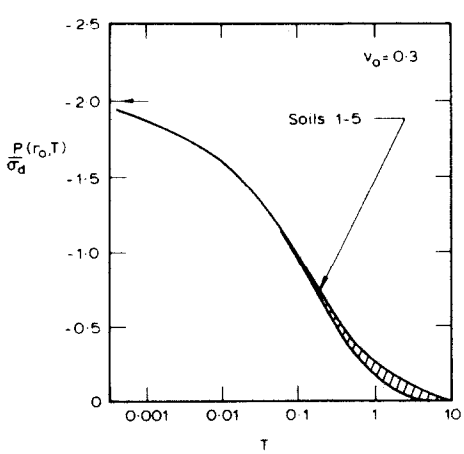


Fig. 5. Excess pore pressure at tunnel wall.

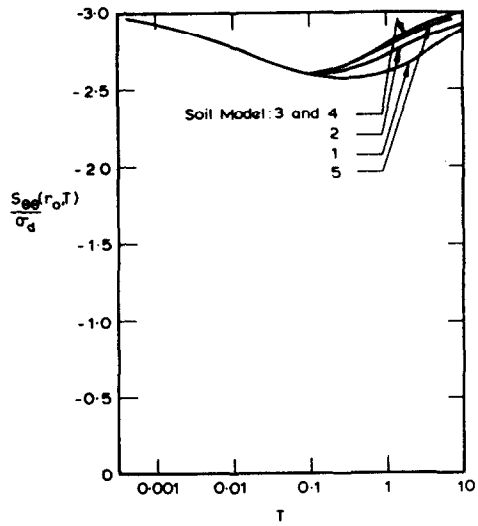


Fig. 6. Coefficient of circumferential total stress at tunnel wall.

the initial response was the same for all soil models. At very large times the models 3 and 4 predict the greatest displacements, followed by models 2, 1 and 5 in that order. The soil which experiences linked creep is, ultimately, the least stiff. Shear stiffness is more important than the volumetric stiffness is determining deformation magnitudes in this class of problem, and, of course, the perfectly elastic soil, which can only consolidate, exhibits the stiffest final response of all.

Figure 4 shows the distribution of excess pore pressure within the various soils at selected times, viz. $T = 0$, 1 and ∞, and Fig. 5 is a plot of the variation with time of the pore pressure at the inner boundary ($r = r_0$). When creep and consolidation have similar time scales, the choice of soil model has little effect on the rate of consolidation.

For all of the soils investigated the changes in total stress show little variation with time. The solutions at $T = 0$ and $T = \infty$ are identical, and the expressions for the Fourier coefficients are

$$S_{rr} = \sigma_d\left[-4\left(\frac{r_0}{r}\right)^2 + 3\left(\frac{r_0}{r}\right)^4\right]$$

$$S_{\theta\theta} = \sigma_d\left[-3\left(\frac{r_0}{r}\right)^4\right]$$

$$S_{r\theta} = \sigma_d\left[-2\left(\frac{r_0}{r}\right)^2 + 3\left(\frac{r_0}{r}\right)^4\right].$$

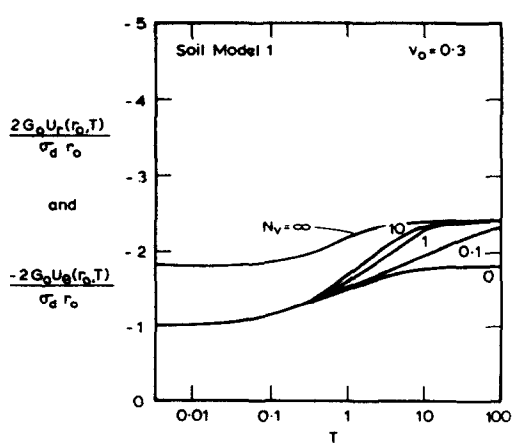


Fig. 7. Radial and circumferential displacements at tunnel wall.

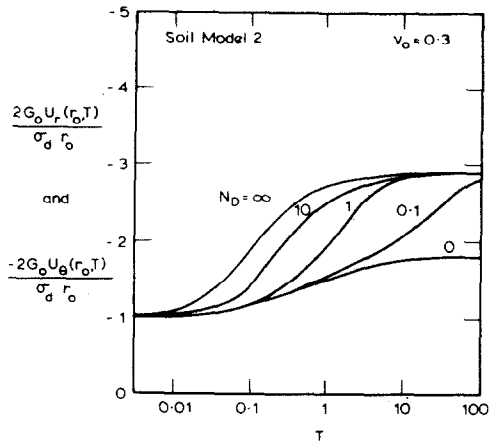


Fig. 8. Radial and circumferential displacements at tunnel wall.

The above results are generally known as the Kirsch solution. Some indication of the time dependence of the circumferential stress at the boundary ($r = r_0$) is shown in Fig. 6. Both time and the type of soil have only a small effect on the stress change.

7.2 *Variation in time scales for consolidation and creep*

The behaviour of a soil in which the consolidation time scale may be different from the creep time scale is now studied.

For Case (a), removal of the *in situ* mean total stress, there are no consolidation effects whatsoever. The stress changes are completely independent of time, while the radial displacement is dependent on the deviatoric behavior of the soil skeleton. Closed form expressions for these quantities have been presented above.

For Case (b), removal of the *in situ* deviator stress, some results are presented in Figs. 7–9. Only the displacement components at the tunnel wall are shown, and for each type of soil a number of values of N_V and N_D have been investigated. On each figure solutions have also been plotted for a soil which under goes no creep only elastic consolidation ($N_V = N_D = 0$, i.e. model 5), and for a soil which undergoes no consolidation only creep (N_V or $N_D = \infty$, as appropriate). In all soils the time dependent tunnel displacements are a function of either N_V or N_D. The greatest difference in response occurs for those soils exhibiting some shear creep.

8. CONCLUSIONS

A method of analysis has been presented for the consolidation and creep around a circular opening in an infinite medium and solutions have been given for the time dependent displace-

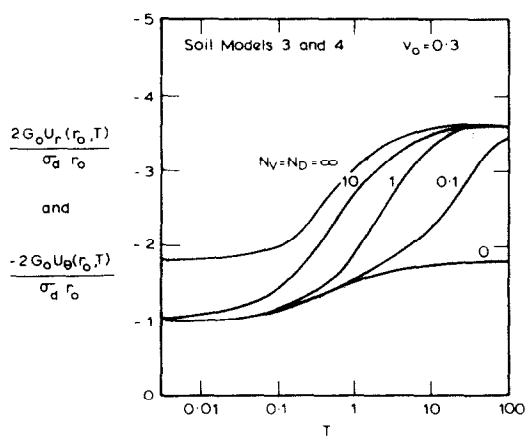


Fig. 9. Radial and circumferential displacements at tunnel wall.

ments and stress changes. These results indicate that the shear behaviour of the soil skeleton is more important than its volumetric behaviour in determining the magnitude of these changes and the rate at which they occur.

No consideration has been given here to plastic flow within the soil; only elastic and viscoelastic soil models have been investigated. Nevertheless, the method and the results may be useful in the understanding of the time dependent behaviour around tunnels through clay.

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