Computers and Geotechnics 36 (2009) 31–40

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Parametric sensitivity analysis of coupled mechanical consolidation and contaminant transport through clay barriers

T.W. Lewis [a], P. Pivonka [b,*], S.G. Fityus [a], D.W. Smith [b]

[a] Discipline of Civil, Surveying and Environmental Engineering, School of Engineering, The University of Newcastle, University Drive, Callaghan, NSW 2308, Australia
[b] Department of Civil and Environmental Engineering, The University of Melbourne, VIC 3010, Australia

ARTICLE INFO

Article history:
Received 15 October 2007
Received in revised form 8 April 2008
Accepted 8 April 2008
Available online 29 May 2008

Keywords:
Geoenvironmental modeling
Contaminant transport
Consolidation
Large deformation models
Clay barriers

ABSTRACT

In this paper, an extensive parametric sensitivity analysis of coupled consolidation and solute transport in composite landfill liner systems has been undertaken. The analysis incorporates results of more than 3000 simulations for various combinations of barrier thickness, waste loading rate, initial void ratio, compression index, hydraulic conductivity and dispersion coefficient. However, it is noted that to limit the extent of the study a constant coefficient of consolidation is assumed in the analysis presented here, though this assumption is easily relaxed. Results of the parametric sensitivity analysis are succinctly presented using dimensionless plots, which allow the comparison of results for a large number of parameter values, and so the clear identification of the most important determinants on contaminant transport through the liner system. The dimensionless plots demonstrate a pessimum (for which the 'breakthrough time' is minimised). Numerical results reveal that in cases of extreme liner compressibility an order of magnitude reduction in contaminant transit time may arise due to coupling between solute transport and consolidation, while for barriers of low compressibility and porosity (such as well-engineered composite compacted clay landfill liners), it is found that the contaminant transit time may still be reduced by more than 30%. The numerical results suggest that the use of coupled consolidation–contaminant transport models are sometimes required for informed and conservative landfill liner design.

1. Introduction

Modern engineered waste-disposal facilities such as municipal landfills usually employ composite contaminant barrier systems (see Fig. 1). These typically consist of a low hydraulic conductivity clay layer (or equivalent) and an overlying geomembrane. A well-constructed composite barrier limits the migration of pollutants from the waste into surrounding groundwater largely by restricting the passage (leakage) of leachate. This can only occur through defects in the geomembrane and even here is restricted by the low hydraulic conductivity of the underlying clay. Low leachate leakage rates through well-constructed composite barrier systems mean that the advective transport of contaminants is kept to a minimum. As a consequence, diffusion is often considered to be the dominant mode of transport. Ionic contaminants are essentially incapable of diffusing through the organic polymer structure of most geomembrane materials (because of very low diffusion coefficients). However, the diffusion of small non-ionic molecules such as volatile organic compounds (VOCs) can be quite rapid. For this reason, it is the diffusion of small (and often toxic) VOCs that become the main focus of contaminant transport modelling in composite contaminant barriers [10].

Modelling of VOC transport through composite barriers is commonly based upon a relatively simple diffusion analysis. However, results from some field studies involving composite landfill liners have indicated that contaminant transit times may be significantly smaller than those expected from a "diffusion only" contaminant transport analysis. It has also been hypothesized that "consolidation water", expelled from a porous clay liner upon mechanical loading, may lead to advective transport through the clay liner, and higher than expected secondary leachate production beneath the liner. These observations have led to the hypothesis that "consolidation induced advection" may be the cause of the accelerated transit of contaminants [20].

Recently, a number of theoretical investigations of coupled consolidation and contaminant transport in composite barriers have been carried out [2,15,16,18,22]. These investigations have mainly focussed on the development and comparison of different model formulations and constitutive relations. Some of the investigations have incorporated case studies of landfill liners. These have shown that the coupling of consolidation and transport processes can be significant; resulting in contaminant transit times which are substantially lower than those predicted by a traditional "diffusion

* Corresponding author. Tel.: +61 3 8344 6932; fax: +61 3 8344 4616.
E-mail addresses: timothy.lewis@studentmail.newcastle.edu.au (T.W. Lewis), ppivonka@unimelb.edu.au (P. Pivonka), stephen.fityus@newcastle.edu.au (S.G. Fityus), David.Smith@unimelb.edu.au (D.W. Smith).

0266-352X/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compgeo.2008.04.003

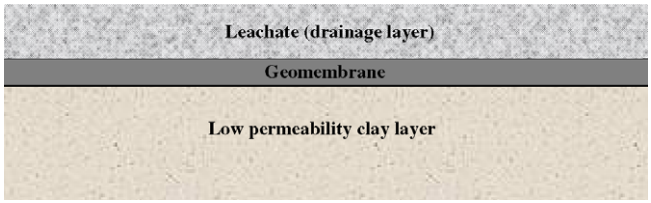

Fig. 1. Schematic diagram of a composite contaminant barrier.

only" analysis. However, since the coupled consolidation–contaminant transport models generally require knowledge of various soil and contaminant transport parameters that are either not available or not easy to measure under field conditions, there is some uncertainty regarding the relevance of the results obtained to practical circumstances. For this reason a more extensive investigation of the effects of consolidation on the transport of contaminants through composite barriers such as landfill liners is warranted.

The aim of this paper is to investigate the influence of the choice of model parameters and liner thickness on breakthrough times in composite liner systems. For this purpose an extensive parametric study consisting of over three thousand numerical simulations using the coupled, large-deformation consolidation–transport model of Lewis et al. [15] has been undertaken. Six key design variables are considered in the parametric study: the liner thickness, loading rate of waste in the landfill, and clay soils properties including the initial void ratio, the hydraulic conductivity, the compression index and the dispersion coefficient. Use of non-dimensional time variables allows representation of a vast amount of numerical data in concise form. These plots reveal the regions of the 'parameter space' for which the coupling is most influential in terms of reducing the transit (or breakthrough) time of contaminants across the barrier, so allowing 'worst case' or 'pessimum' scenarios to be identified. In addition, the investigation yields insight into the mechanisms through which consolidation affects contaminant transport in composite barrier systems. These insights contribute to a better understanding of composite liner system behaviour, and may help improve current engineering design.

2. Method

2.1. Barrier characteristics

The composite contaminant barrier system investigated here is typical of those employed in many large-scale solid waste landfills. The "single composite" barrier system is composed of a geomembrane and an underlying clay layer of low hydraulic conductivity. A schematic cross-section of the barrier is shown in Fig. 1.

During operation, leachate from the waste accumulates above the geomembrane in the overlying drainage layer. Removal of leachate is generally facilitated by a network of perforated leachate collection pipes incorporated within the drainage layer. It is assumed that leachate leakage through the geomembrane is negligible in terms of the contribution to contaminant transport across the barrier, that is, contaminant advection through the geomembrane is negligible compared to diffusion. It should be noted that this assumption does not preclude consolidation induced advection of contaminants within the clay layer, after they have diffused across the geomembrane. To both provide a conservative analysis and limit the complexity of the parametric investigation described here, a constant contaminant concentration in the leachate is assumed [12]. In other words, it is assumed that the leachate is 'well mixed' and contaminant decay over time is negligible (i.e., a constant contaminant concentration, c_0, is maintained in the leachate).

2.2. Consolidation–transport model

The theoretical investigation is based on a material coordinate (that is, Lagrangian coordinates: material displacement – a, time – t) large-deformation consolidation–transport model formulation. The model and its advantages over a variety of other model formulations have been described in detail previously [15] and so these arguments are not reproduced here. However, for reference, the governing equations, constitutive relations and boundary conditions are presented without derivation below together with relevant references. To keep the scope of the investigation manageable and provide conservative estimates of contaminant transit times, the effect of sorption on contaminant transport is neglected. Also, in considering the consolidation of the liner, the effect of the self-weight is neglected due to its relatively small magnitude for a thin clayey liner in comparison to the mechanical loading it is likely to carry. The model equations were solved using the multiphysics finite element software package FEMLAB 2.3 [8].

2.2.1. Governing equations – consolidation

The equations describing soil consolidation can be formulated as follows:

$$\frac{1}{1+e_0}\frac{\partial e}{\partial t} = \frac{\partial}{\partial a}\left(\frac{k(1+e_0)}{\rho_f g a_v(1+e)}\frac{\partial e}{\partial a}\right) \tag{1}$$

where e is the void ratio, e_0 the initial void ratio (constant, since self-weight is neglected), k the hydraulic conductivity, g the acceleration due to gravity, ρ_f the mass density of the fluid phase, $a_v(\equiv \frac{de}{d\sigma'})$ is the coefficient of compressibility, and σ' is the effective stress.

2.2.2. Governing equations – contaminant transport

Transport of contaminants through porous soil can be readily described as

$$\frac{e}{1+e_0}\frac{\partial c}{\partial t} = \frac{k(1+e_0)}{\rho_f g\, a_v(1+e)}\frac{\partial e}{\partial a}\frac{\partial c}{\partial a} + (1+e_0)\frac{\partial}{\partial a}\left(\frac{e}{(1+e)^2}D\frac{\partial c}{\partial a}\right) \tag{2}$$

where c is the solute concentration in the fluid phase and D is the coefficient of hydrodynamic dispersion. In Eq. (2) the first term on the right-hand side (rhs) describes transport due to advection caused by soil consolidation. The second term on the rhs is the contribution due to diffusion. The Darcy velocity is commonly introduced as $q = k(1+e_0)/(\rho_f g\ \ a_v)/(1+e)\partial e/\partial a$. Advective and diffusive flux can be expressed as $f_a = c \cdot q$ and $f_d = (1+e_0)e/(1+e)^2 D\,\partial c/\partial a$.

In order to describe the dependence of the compressibility coefficient, hydraulic conductivity, and dispersion coefficient on the void ratio the following constitutive relations have been used:

$$a_v = \frac{C_c}{\sigma'_p \cdot \ln 10}\exp\left(\ln 10 \cdot \left(\frac{e-e_p}{C_c}\right)\right),$$
$$k = k_p \exp\left(\ln 10 \cdot \left(\frac{e-e_p}{C_k}\right)\right), \quad D = D_e = \text{const} \tag{3}$$

where e_p and σ'_p are the void ratio and effective stress values, respectively, corresponding to the preconsolidation stress, C_c is the soil compression index (defined as absolute value of the slope of the idealised virgin compression line), k_p is the hydraulic conductivity of the soil corresponding to e_p and C_k is the hydraulic conductivity (or 'permeability') index. Note that in Eq. (2) we assumed that the coefficient of hydrodynamic dispersion (D) is equal to the effective diffusion coefficient (D_e) and constant. This assumption generally holds for fine-grained soils where the Darcy velocity is low.

2.2.3. Boundary conditions – consolidation

$$\frac{\partial e}{\partial a}(0,t) = 0; \quad e(L,t) = \begin{cases} e_p, & \sigma_a \leqslant \sigma'_p \\ e_p - C_c \log\left(\frac{\sigma_a}{\sigma'_p}\right), & \sigma_a > \sigma'_p \end{cases} \tag{4}$$

where σ_a is the applied (overburden) stress, which is assumed to take the form of a delayed linear ramp function with time:

$$\sigma_a(t) = \begin{cases} 0, & t < \Delta t_d \\ \frac{\sigma_{a,max}}{\Delta t_r}(t - \Delta t_d), & \Delta t_d < t < \Delta t_d + \Delta t_r \\ \sigma_{a,max}, & t > \Delta t_d + \Delta t_r \end{cases} \quad (5)$$

where $\sigma_{a,max}$ is the maximum applied stress, Δt_d is the time delay from the start of contaminant transport (exposure of the liner to leachate) to the start of load application and Δt_r is the duration of the ramp loading phase.

2.2.4. Boundary conditions – contaminant transport

$$\frac{\partial c}{\partial a}(0,t) = \frac{(1+e(0,t))^2}{e_0(0)(1+e_0(0))}\frac{P_G}{h_G D_e}(c(0,t) - c_0); \quad \frac{\partial c}{\partial a}(L,t) = 0 \quad (6)$$

where c_0 is the concentration of solute at the top surface of the geomembrane, h_G and P_G are the thickness of the geomembrane and the permeation coefficient for the solute in the geomembrane, respectively. P_G is defined as the product of the diffusion coefficient for the solute in the geomembrane (D_G) and the partitioning coefficient of solute between geomembrane and adjacent fluid (S_G) [21], i.e., $P_G = D_G \times S_G$. It is noted that depending on the type of contaminant the top boundary concentration may decay at different rates. We performed several analyses (results not shown here) with different half lives of the contaminant concentration at the top boundary. Given that for (dimensionless) breakthrough concentrations below 0.1 hardly any difference in breakthrough time has been observed we assume a constant top boundary concentration in all our analyses. Additionally, we assumed a zero concentration gradient at the bottom boundary. This condition does not imply a zero flux at this boundary as advective flux due to liner consolidation may still occur. This assumption may appear unrealistic since it implies that the diffusive flux is negligible. However, in some instances, such as when an unsaturated drainage layer exists beneath the clay layer, this may be a good approximation. It may also be noted that the above boundary condition will yield conservative breakthrough curves (plots of the concentration at the bottom boundary with time) because without diffusion across the boundary the concentration, $c(L^-, t)$, rises more quickly than with diffusion. In other circumstances, a semi-infinite domain approach, incorporating a fictitious, semi-infinite, non-consolidating secondary soil layer beneath the clay, may be more appropriate. Detailed discussion of boundary conditions for solute transport in porous media can be found in [18,19].

Simplified forms of the general large deformation (LD) model have also been employed to give additional insight into the mechanisms through which consolidation affects transport. For example, by employing a "no advection" form of the model (in which the advection term[1] is neglected) and comparing the solutions of the transport equation, the effect of advection on contaminant breakthrough can be revealed. Further considerable simplification of the model equations may also be made by assuming *instantaneous* consolidation. This removes the coupling and indeed the need to solve the consolidation equation. Accordingly, this model simply involves a traditional "diffusion only" analysis based on the final (consolidated) state. This model represents one limiting case where the rate of consolidation is very rapid compared to that of diffusion (timescale of consolidation negligible compared to timescale of diffusive transport).

It can be seen from the constitutive relations that the consolidation equation may be considerably simplified if the compression and hydraulic conductivity indices are equal, i.e., $C_c/C_k = 1$. In this case the changes in the compressibility and hydraulic conductivity of the clay liner have opposite effects. This implies that the coefficient of consolidation, c_v, which is defined as: $c_v = k(1 + e_0)/\rho_f g a_v$, is constant and so there is no material non-linearity. Under these circumstances the non-linearity of the consolidation equation is due only to geometric and void ratio variation.

As this investigation is focussed on the effect of consolidation upon the transit time of contaminants across the barrier – otherwise called the *breakthrough time* – this term must be clearly defined. A number of different definitions are commonly employed in the literature. Most commonly the breakthrough time is defined as the time taken for a prescribed concentration or flux of a specified contaminant to arise at the base of the contaminant barrier. In this investigation a prescribed concentration (rather than flux) criterion is employed in defining the breakthrough time. All concentrations referred to in the remainder of this paper are dimensionless, having been normalised with respect to the concentration of solute at the top of the geomembrane, c_0.

2.3. Parametric analysis

The parametric analysis investigates the sensitivity of the reduction in contaminant transit time (relative to a standard diffusion only analysis) to the variation of selected model parameters. However, in practice the variability of some model parameters is negligible, and so these are held constant throughout the investigation. For example, the acceleration due to gravity, $g = 9.81$ m s^{-2}, the density of the soil solid phase, $\rho_s = 2700$ kg m^{-3}, the density of the soil fluid phase, $\rho_f = 1000$ kg m^{-3} and the thickness of the geomembrane, $h_G = 1.5$ mm are all held constant. To keep the scope of the investigation within reasonable bounds, the analysis is limited to consideration of six factors – the liner thickness, L, loading rate, Δt_r, and soil layer/contaminant property parameters: e_0, k_p, C_c and D_e. Although other parameters – such as loading delay, Δt_d, geomembrane permeation coefficient, P_G, and material non-linearity of the consolidation ($C_c/C_k \neq 1$) – may also influence contaminant transport, these are considered likely to be of secondary importance and so consideration of their effects has been left for future investigation. The selection of parameter values used in the investigation is discussed below and a summary of the values employed is presented in Table 1.

2.3.1. Soil layer properties

Three types of soil component are most commonly used in composite barriers: natural in situ clay deposits, remoulded and compacted sand–clay (usually bentonite) mixtures, and remoulded compacted clay [11]. The salient properties of the soil layer employed may vary significantly depending upon factors such as the location of the waste impoundment and the type and quantity of soil used in construction. Generally though, the soils used must be of low hydraulic conductivity, have sufficient strength to resist

Table 1
Parameter values used in the numerical analyses

Parameter	Values			Units
L	0.6, 1.2, 1.8			m
Δt_r	0.5, 1, 2, 4, 8			years
e_0	Low: 0.3, 0.4, 0.5	Moderate: 0.6, 0.8, 1.0	High: 1.2, 1.6, 2.0	–
C_c	0.1, 0.2, 0.4	0.2, 0.4, 0.8	0.4, 0.8, 1.6	–
σ'_p	160	115	50	kPa
k_p	1, 3, 10 × 10^{-10}			m/s
D	2, 4, 8 × 10^{-10}			m^2/s
$\sigma_{a,max}$	500			kPa

L – liner thickness; Δt_r – duration of ramp loading; e_0 – initial void ratio; C_c – soil compression index; σ'_p – preconsolidation stress; k_p – hydraulic conductivity; D – dispersion coefficient; $\sigma_{a,max}$ – maximum applied stress.

[1] i.e. the first term on the right-hand side of the transport equation.

failure under the imposed (construction and operational) loads and possess a low potential for shrinkage and desiccation.

Data available in the research literature, though of limited scope indicates that for compacted clay barriers the porosity seldom exceeds 0.5 [5]. For natural, fine-grained soils the practical range of porosity is considered to range from 0.1 to 0.7, i.e., a void ratio of 0.11–2.3 [17]. At the upper end of this range the soils tend to have high natural water content and in most instances are unlikely to meet modern engineering requirements for use in a composite barrier system without moisture and porosity reduction. However, analysis of a fairly extensive void ratio range from 0.1 to 2 is considered reasonable for the current investigation given that we wish to gain insight into understanding coupling between consolidation of the liner system and contaminant transport through both engineered and natural liner systems.

The (initial) thickness, L, and (preconsolidation) hydraulic conductivity, k_p, of contaminant barriers employed in most modern landfills are carefully controlled in order to meet leakage rate criteria. Thicknesses between 0.6 and 1.8 m are commonly employed [3] and consequently, values of 0.6, 1.2 and 1.8 m have been analysed in the current investigation. The hydraulic conductivity of the soil component is assumed to range from 10^{-9} to 10^{-10} m/s, which is regarded as typical for well-constructed barriers [4].

The hydraulic conductivity index, C_k, of soils employed in composite barriers is seldom reported in the literature. However, Trast and Benson have presented data for 11 compacted clay test pads or landfill liners indicating that the hydraulic conductivity is reduced by increasing effective stress [24]. A comparison of the data with predictions from the constitutive models employed in the current investigation has shown them to be in good agreement when values of C_c/C_k between 0.2 and 1 are employed. The deviation from unity ($C_c/C_k \neq 1$) is indicative of a variable coefficient of consolidation. To limit the extent of the study, material non-linearity has been neglected in the current analysis and the value of C_k is set according to the relation $C_c/C_k = 1$.

Like the hydraulic conductivity index, reporting of compressibility data for contaminant barriers is particularly scarce in the literature. Acar and Haider investigated trichloroethylene transport through natural clays with a reported void ratio between 0.46 and 0.64 and compression index within the range 0.11–0.14 [1]. For bentonite-amended compacted clay Kumar and Yong reported void ratios between 0.55 and 0.7 and compression indices ranging from 0.12 to 0.16 [14]. Given the importance of the compressibility found from previous investigations, the scarcity of data and the desire to cover a relatively wide range of possible circumstances, the broader literature for clay soils generally was consulted. Examination of this literature indicates that the compressibility parameters are generally correlated with the porosity. As porosity increases soils tend to have a higher compression index, C_c, and a lower preconsolidation stress, ∂'_p. Consequently, in the current investigation the effect of variation of these parameters is investigated by considering three generic soil "types" corresponding to "low" (~0.4) "moderate" (~0.8) and "high" (~1.6) void ratio ranges with "high" (160 kPa), "moderate" (115 kPa) and "low" (50 kPa) preconsolidation stresses, respectively. A variety of data indicates that, while considerable variation exists in the relationship of the compression index to the initial void ratio, an extreme upper bound value of the compression index may be taken to be equal to the initial void ratio [13,9,6,7,23]. Two lower values of the compression index specified as being equal to half and one quarter of the initial void ratio, designated as "high" and "moderate" values, respectively, were also investigated.

2.3.2. Geomembrane properties

In addition to the data required for the soil component of the liner, data for the geomembrane is also required. In all cases the thickness of the geomembrane, h_G, is assumed to be 1.5 mm which is typical for many landfills. The value of the permeation coefficient of the geomembrane, P_G, is based on data for an HDPE geomembrane: a value of 3.9×10^{-12} m^2/s reported for dichloromethane [21] was used in the analysis.

2.3.3. Loading

The loading parameters ($\sigma_{a,max}$, Δt_d and Δt_r) are dependent upon the total amount and rate of waste emplacement and whether a significant delay in waste emplacement occurs. For large landfills the height of municipal solid waste overburden may be in the order of 50 m. Based upon a waste density of 1000 kg/m^3, a 500 kPa maximum applied stress may be expected and this value has been employed in the primary analysis. Loading of the liner due to waste emplacement is assumed to follow a linear rise (ramp) with time over a specified duration, Δt_r, up to the maximum applied stress, $\sigma_{a,max}$. However, to keep the scope of the investigation within reasonable bounds, consideration of the effects of loading delay was deferred to a future investigation.

3. Results and discussion

3.1. Contaminant breakthrough

In Fig. 2, a series of breakthrough curves (i.e., plots of the contaminant concentration at the bottom of the barrier versus time) corresponding to various loading durations and a fixed combination of other model parameters are displayed. From these curves it may be observed that the rate of loading has a significant effect upon the breakthrough time – here defined as the time taken for the 'breakthrough concentration' to be exceeded. To illustrate this the breakthrough times from the transport – consolidation model, t_b, and the diffusion only model, $t_{b,DO}$, for the 2 years loading duration case based on a breakthrough concentration of $c(L, t)/c_0 = 10^{-2}$ (i.e., 1%) are shown using dashed lines. For this breakthrough concentration it may be observed that the 2 years loading duration produces the greatest acceleration of contaminant breakthrough. This may be conveniently represented in terms of the breakthrough time ratio, BTR = t_b:$t_{b,DO}$, ≈0.4 which indicates that in this instance a 60% reduction in breakthrough time is brought about by consolidation of the contaminant barrier.

3.2. Representation of parametric analysis data

Given the extensive scope of the parametric study it is not feasible to present and compare all of the results in the form of breakthrough curves such as presented in Fig. 2. A preferred method of presentation is one that is concise yet easily interpreted and therefore useful to design engineers. For this reason a condensed presentation of the breakthrough time data is employed which focuses on the pessimum, or, worst case scenario, and the resultant minimum breakthrough time ratio (BTR) value. This is enabled by employing a uniform breakthrough concentration criterion of 10^{-2} for the BTR. The basis for this is the general observation that the minimum BTR value obtained from a series of load durations (with other parameters held constant) is relatively insensitive to changes in the breakthrough concentration criterion. For example, in the specific case presented in Fig. 2 it may be seen that if alternate breakthrough concentrations of 10^{-1} or 10^{-3} were employed, load durations of between 4 and 8 years or 1 and 2 years, respectively would yield a BTR ≈ 0.4 which is similar to that obtained from using the 10^{-2} criterion. Moreover, although it is not obvious from the limited data presented above, the insensitivity of the minimum BTR to the breakthrough concentration criterion was also observed to be generally true for breakthrough data obtained from other regions of the investigated parameter space. Consequently, it is as-

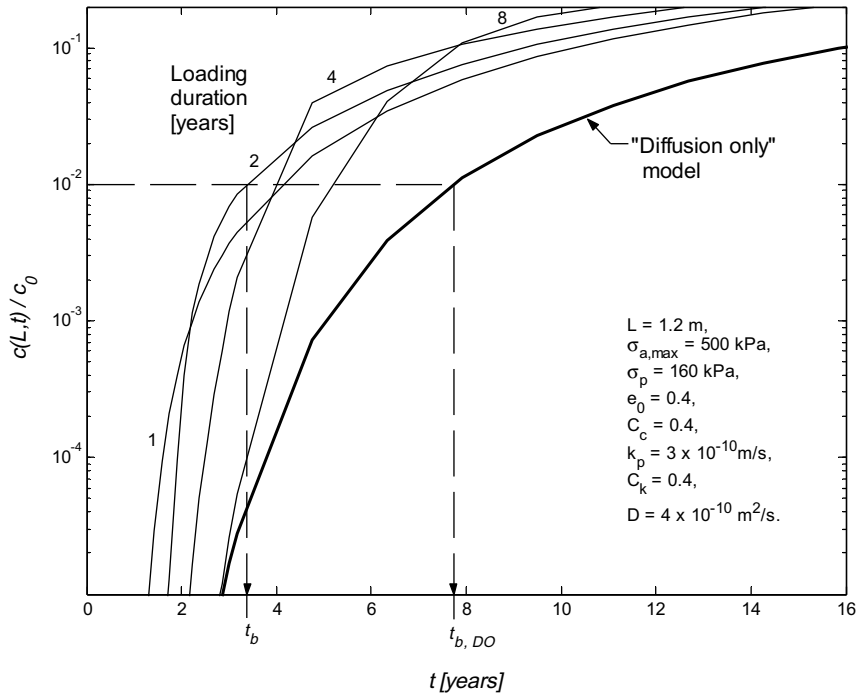

Fig. 2. Comparison of breakthrough curves for loading durations of 2, 4, and 8 years (characteristic breakthrough concentration 1%).

sumed that there is no significant loss of generality in presenting in the remainder of the paper minimum BTR values based on the chosen breakthrough concentration of 10^{-2}.

To further assist compact presentation of the large amount of breakthrough data and also simplify the identification of trends in the way the BTR is related to the various parameters, the parametric analysis results are presented in terms of a further dimensionless ratio – the process timescale ratio (PTR). Here the PTR is defined as being the ratio of the time required to achieve 90% of the theoretical maximum settlement, t_{90}, and the characteristic diffusion time, $t_D = L^2/D$. Thus the PTR = t_{90}/t_D. The PTR will obviously reflect changes in Δt_r, D and k_p, so the various values of these parameters must be differentiated through means other than the PTR coordinate of the BTR–PTR data (see below). It should be noted that, because the consolidation analysis incorporates a ramp loading regime and is also generally non-linear (c_v variable), the t_{90} value is employed in defining the PTR rather than a characteristic consolidation time (defined in an analogous fashion to the diffusion time but employing c_v in place of D).

3.3. Parametric analysis results

Results from the analysis for a 1.2 m thick liner are presented as a matrix of nine plots in Fig. 3. In order, from top to bottom, each row of plots corresponds to the low moderate and high void ratio ranges, respectively. Twenty-seven data series are presented in each plot except for the first two plots in the high void ratio range (see explanation below). Each data series consists of five data points corresponding to, in order of increasing PTR, loading durations of 0.5, 1, 2, 4 and 8 years which are joined by straight line segments. In the first two plots in the high void ratio range only eighteen data series are presented because in each plot nine data series representing parameter combinations which yield zero or negative final void ratios have been neglected. The 27 (or 18) data series correspond to the various combinations of each of the three (or two) C_c values, three k_p values and three D values investigated for a particular initial void ratio. Different C_c values are indicated by labelling and the use of different line styles to join the points of each data series (mixed length dashes for the moderate C_c, a solid line for the high C_c value, and a dashed line for the largest C_c value). Different k_p values are indicated by different marker styles as shown in the legend of each plot (i.e., triangle = low k_p, square = medium k_p, and inverted triangle = large k_p). Three different marker sizes are employed to distinguish the three different D values. The magnitude is inversely related to the marker size (i.e., large marker = low D, small marker = large D). Larger D values corresponds to a smaller t_D and, hence, also a larger PTR. Consequently, it may be observed that data series plotted for smaller D values (larger marker size) do not extend to such large PTR values as those for larger D values (smaller marker size). By noting the size, shape, and position of the markers in each data series, it is possible to discern the effects on the PTR and BTR of variations in each of the model parameters investigated. These are discussed below.

3.3.1. Effects of e_0 and C_c

From Fig. 3, it may be readily observed that there are relatively strong, monotonic trends of decreasing BTR with an increasing C_c and decreasing e_0. However, due to the correlation of the compressibility parameters, C_c and σ'_p, with e_0 assumed in the parameter selection, the lowest BTR for a designated compressibility level (i.e., extreme, high or moderate) is observed to occur in the high void ratio range. It should be noted that a direct comparison of data for a given C_c value and adjacent e_0 values from different void ratio ranges (which appears to show a decrease in BTR with an increase in void ratio) is invalid since a different preconsolidation stress is employed in each void ratio range.

3.3.2. Effects of Δt_r, D and k_p

The trends in the BTR resulting from changes in Δt_r, D and k_p, although more difficult to discern, may be observed to be non-monotonic. Despite this, the relationship between the BTR and the PTR is clear and, for a given combination of C_c and e_0, remarkably consistent (although obviously also non-monotonic). This indicates that beyond their influence on the PTR, Δt_r, D and k_p have only a slight effect on the BTR and also demonstrates the utility of

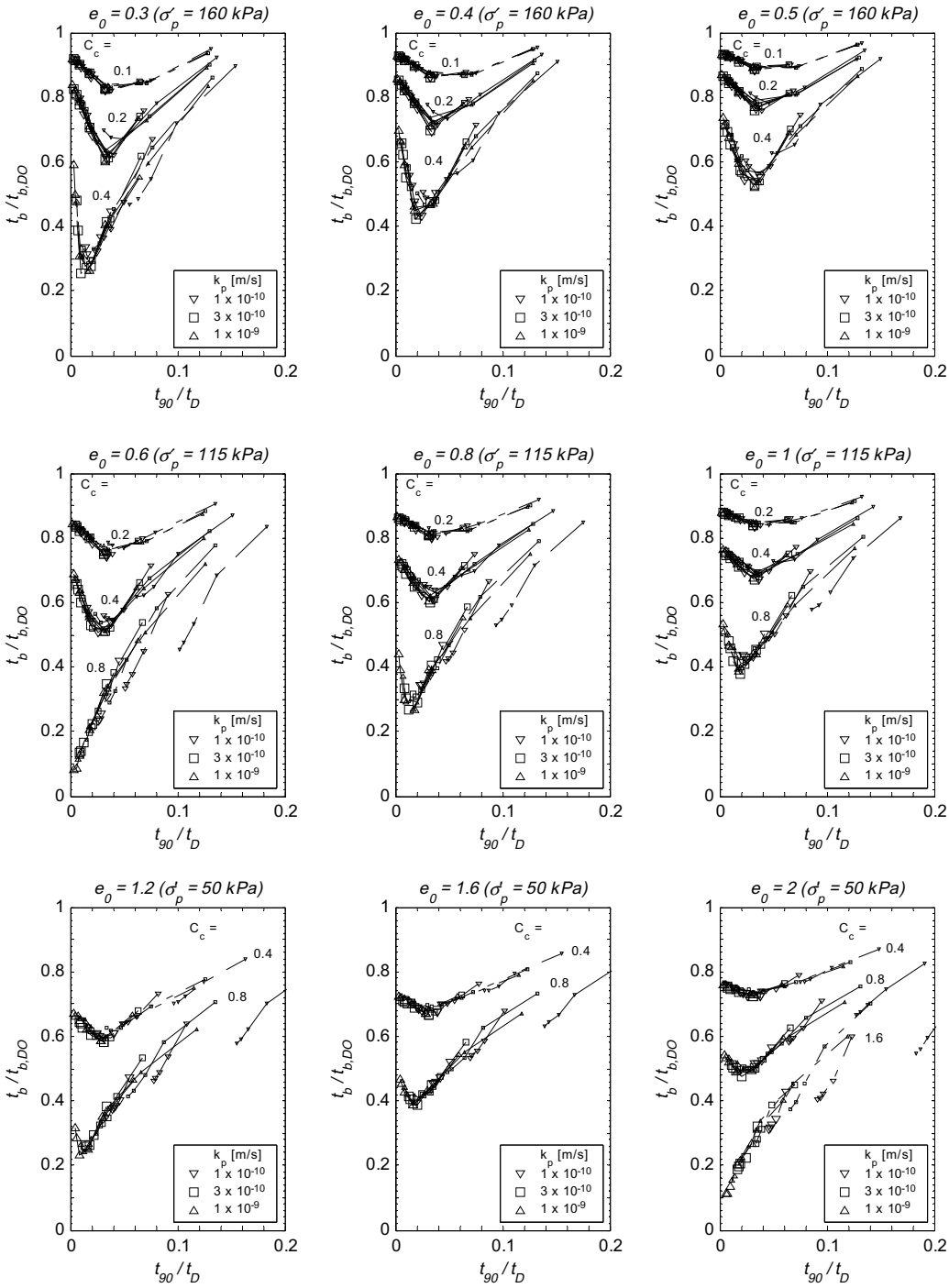

Fig. 3. Plots of breakthrough time ratio (BTR = $t_b/t_{b,DO}$) versus process timescale ratio (PTR = t_{90}/t_D) for a 1.2 m thick barrier at different initial void ratios e_0 (data curves are presented in each plot corresponding to combinations of C_c, k_p, and D each having three different values – see Table 1; C_c – different curves; k_p – different marker types (triangle, square, inverted triangle); D – different marker size (large marker = low D, small marker = large D)).

presenting the results in terms of the PTR. Furthermore, these parameters may be observed to be monotonically related to the PTR (the PTR increases as Δt_r or D increases, or as k_p decreases). As a consequence the trends in BTR resulting from variation of any of these parameters may be easily deduced from the strong BTR–PTR relation; the turning point minimum in this relation corresponds to a reversal in each trend.

Although the BTR–PTR relation is remarkably consistent considering the variation in Δt_r, D and k_p, there is some loss of consistency observed at low k_p. This is likely to be due to the increasing influence on the rate of consolidation (and hence the PTR) that k_p has as it decreases, compared to Δt_r. This leads to the observed increase in PTR for the same Δt_r and D. The low k_p also appears to affect the (turning point) minimum BTR value. The reason for this is discussed further below.

3.3.3. Effect of L

BTR–PTR plots for the complete primary analysis (i.e., combinations of L, Δt_r, e_0, C_c, k_p, and D which equal $3 \cdot 5 \cdot 9 \cdot 3 \cdot 3 = 3645$ cases from which negative or zero final void ratios have been discarded, giving altogether 3375 representative simulations), incorporating all liner thicknesses (i.e., $L = 0.6$, 1.2, 1.8 m, see also

Table 1), are presented in Fig. 4. Despite the extra degree of freedom involved in the investigation of the other liner thicknesses and the inability to discern individual data points, the same general consistency of the BTR–PTR relationship observed in Fig. 3 is still apparent and is thus essentially unaffected by the liner thickness variation.

3.3.4. Significance of minimum BTR

It may be observed from Fig. 4 that for cases of moderate compressibility and low to moderate void ratio the minimum value of the BTR does not generally fall below a value of 0.7. This is of considerable practical significance since many modern composite landfill liners utilise a compacted clay barrier, which is of only low compressibility and void ratio (i.e., less than unity). Consequently, for composite compacted clay liners a 30% reduction in contaminant transit time due to consolidation is a likely upper bound.

A consistent feature of the BTR–PTR data presented above is the existence of the turning point minimum. This indicates that there is an intermediate pessimum value of the PTR (i.e., the value that corresponds to the *undesirable* minimisation of the BTR). Similar behaviour [15] involving an observed pessimum hydraulic conductivity has also been noted previously for a hypothetical case

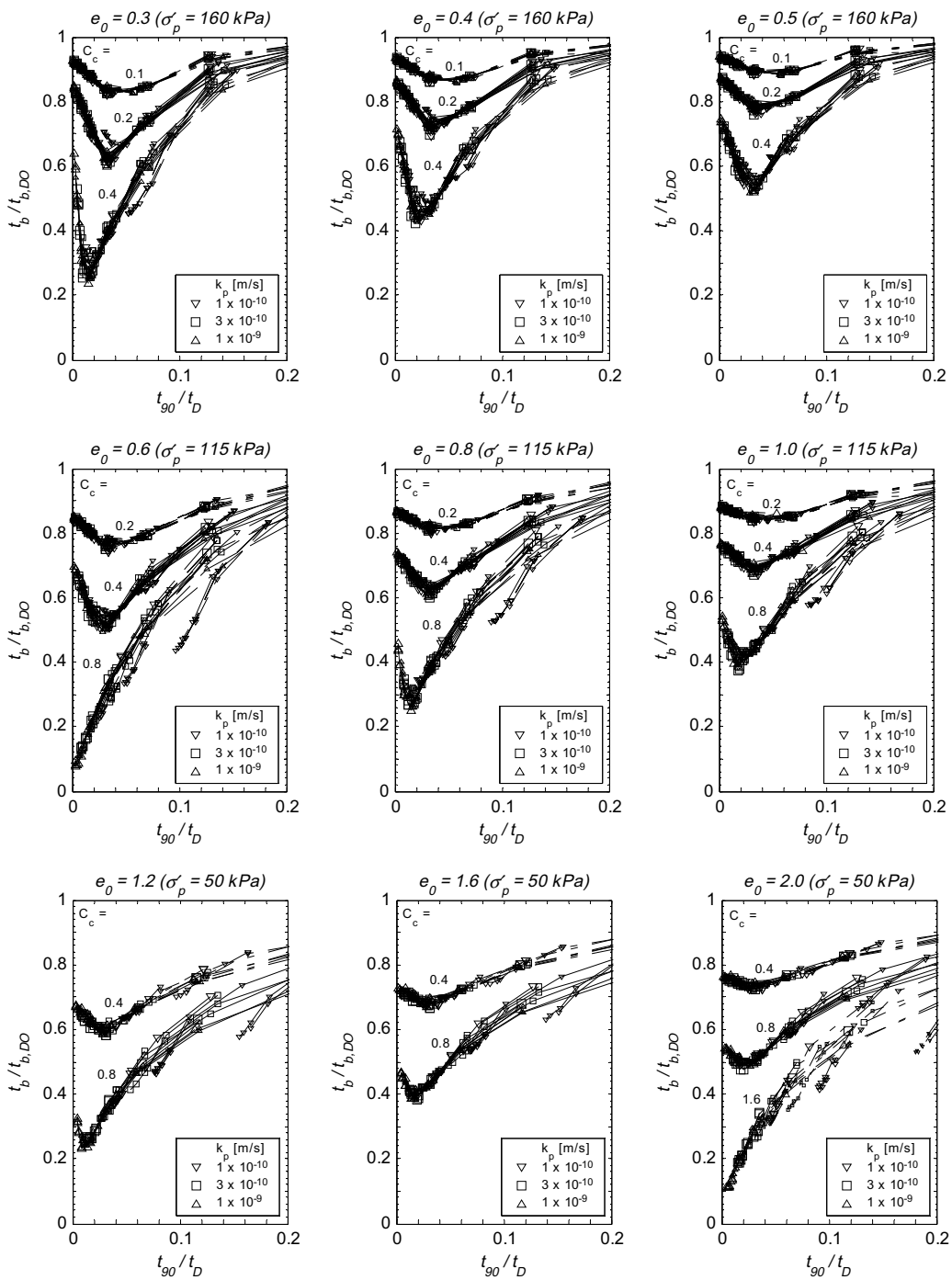

Fig. 4. Plots of breakthrough time ratio (BTR = $t_b/t_{b,DO}$) versus process timescale ratio (PTR = t_{90}/t_D) for all barrier thicknesses at different initial void ratios e_0 (data curves are presented in each plot corresponding to combinations of L, Δt_r, C_c, k_p, and D each having three different values – see Table 1; C_c – different curves; k_p – different marker types (triangle, square, inverted triangle); D – different marker size (large marker = low D, small marker = large D)).

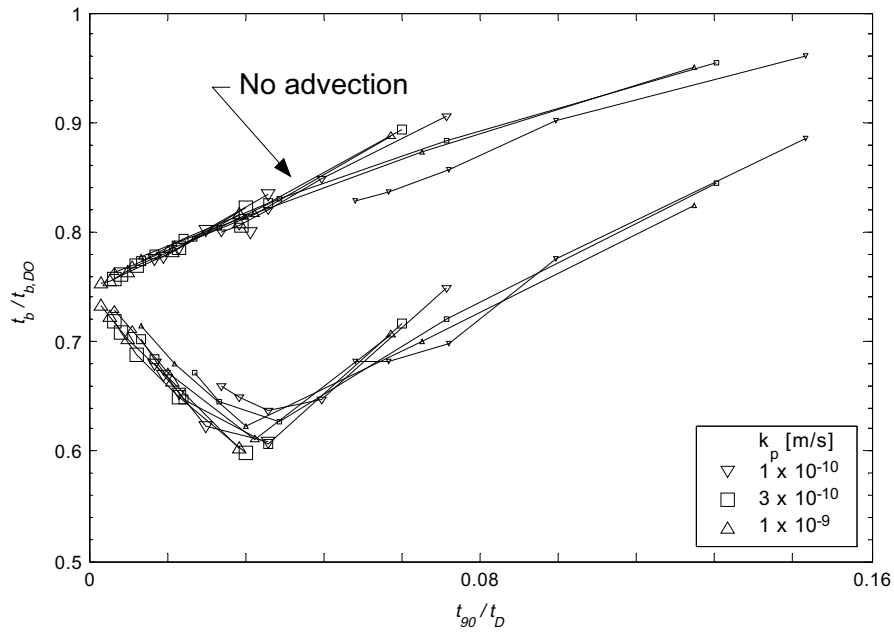

Fig. 5. Comparison of results from the coupled consolidation–transport model with those from the associated "no advection" form of the model ($e_0 = 0.3$; $C_c = 0.2$).

study of step loading of a composite landfill liner. Insight into this effect may be gained by comparing results from the consolidation–transport model with those from the associated "no advection" form presented in Fig. 5. It may be observed that the "no advection" model yields a monotonic BTR–PTR relation. Since advection is neglected, this is due to the effects of geometric and void ratio change only. The minimum turning point displayed in the consolidation–transport model BTR–PTR relation is thus attributable to the influence of advection. This finding may also explain the small but noticeable increase in the minimum BTR observed in Figs. 3–5 at low k_p and relatively high loading rates, since, as the loading rate increases, its influence on the rate of consolidation diminishes relative to the influence of k_p. Thus, for a given dispersion coefficient (assumed equivalent to the effective diffusion coefficient), the lower k_p values lead to a markedly increased PTR for the same loading rate, as well as lower fluid velocities and therefore less advection and an elevated minimum BTR turning point.

An additional feature which may be observed from Fig. 5 is that, for the lower D values, as the PTR approaches zero the data converges to a BTR of approximately 0.74. This is the same as the value obtained from calculating a BTR based upon the quotient of breakthrough times from the "instantaneous consolidation" model and the "diffusion only" model, respectively and affirms the "instantaneous consolidation" model is a limiting case of the general consolidation–transport model.

3.3.5. Advection

In Fig. 6, the spatial distribution of (a) the dimensionless concentration and Darcy velocity (q), and (b) the advective flux (f_a) and diffusive flux (f_d) are compared for a parameter combination which yielded a relatively low BTR ($L = 1.2$ m, $e_0 = 0.8$, $C_c = 0.4$, $k_p = 3 \times 10^{-10}$ m/s, $D = 4 \times 10^{-10}$ m^2/s, $\Delta t_r = 2$ years). It may be observed from the isochrones presented in Fig. 6a that the product of the Darcy flux and dimensionless concentration is maximised at approximately 580 days after the commencement of loading. This leads to the peak in advective flux at the same time, observed in Fig. 6b.[2]

[2] Note that fluxes are defined with respect to the (moving) material coordinate system.

These results indicate that significant advection relies on sufficient interaction between the concentration and fluid velocity. However, both of these are monotonic functions of x with their maximum values on opposite sides of the barrier. The extent of interaction thus depends upon the relative rates of the consolidation and diffusion processes, i.e., the PTR. At low values of the PTR the relatively fast consolidation leads to fluid velocities that are initially high. However, because the consolidation is short-lived, there is little overlap of concentration and velocity profiles and consequently, relatively little advection. In contrast, at high values of the PTR, although consolidation is relatively long-lasting, the fluid velocity is not high enough for advection to be significant. Advection is thus maximised (and the BTR consequently minimised) at intermediate PTR values.

3.3.6. Minimum BTR versus $\Delta e/e_0$

It is obvious both conceptually and from the results presented in Figs. 3 and 4 that in the limit of no consolidation a BTR of unity is obtained. It may be noted that in this case the proportion of pore water expelled is $\Delta e/e_0 = 0$. However, at the other extreme where all the pore water is expelled, $\Delta e/e_0 = 1$ and the minimum BTR approaches zero as the rate of consolidation increases (PTR approaches zero). Given these two limiting cases it is interesting to see how the minimum BTR values found from the parametric analysis correlate with $\Delta e/e_0$.

Fig. 7 indicates that for the parameter space covered in this investigation the minimum BTR is highly correlated with $\Delta e/e_0$. Although limited, this data suggests that $\Delta e/e_0$ may be used as a simple criterion for estimating the minimum BTR. In order to test the consistency of the observed correlation more rigorously, additional analysis incorporating a more extensive range of parameters is recommended.

4. Summary and conclusions

This paper employs a coupled consolidation–contaminant transport analysis to numerical simulate the transport of contaminants through engineered composite liner systems. In order to capture a large range of possible soil property and liner loading conditions encountered in practical situations, an extensive para-

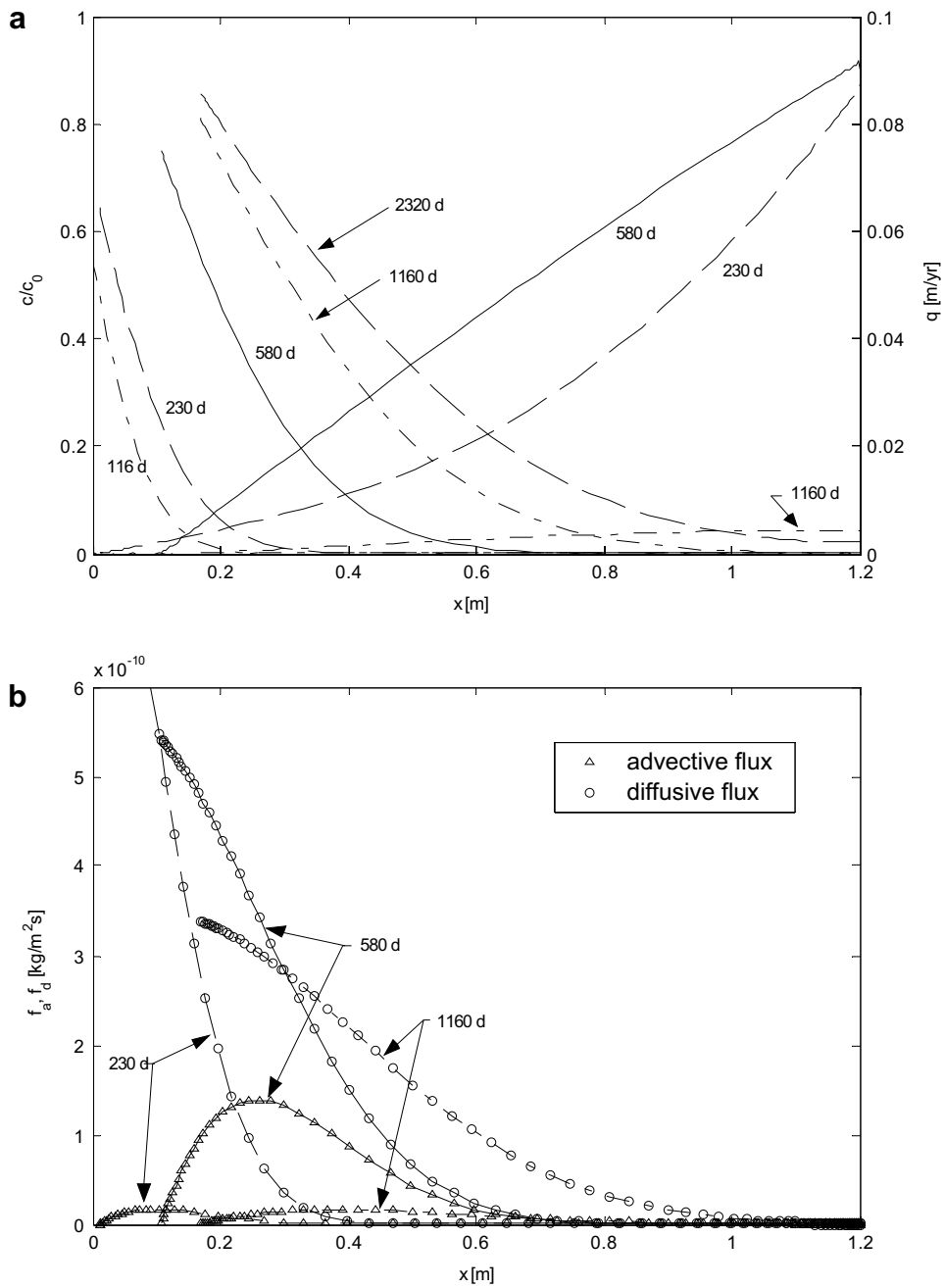

Fig. 6. Plots of (a) dimensionless concentration (c/c_0) and Darcy velocity (q), and (b) advective (f_a) and diffusive (f_d) fluxes as function of location in liner (x in m) at various time instants (in days) ($L = 1.2$ m, $e_0 = 0.8$, $C_c = 0.4$, $k_p = 3 \times 10^{-10}$ m/s, $D = 4 \times 10^{-10}$ m^2/s, $\Delta t_r = 2$ years).

metric study has been performed. Numerical results of this parametric sensitivity analysis are concisely presented using dimensionless quantities: a breakthrough time ratio (BTR) versus a process timescale ratio (PTR). The BTR is defined as the ratio of breakthrough time obtained from the coupled consolidation–transport model to that of a diffusion only (i.e., no advective transport due to consolidation) model. On the other hand, the PTR is the ratio of the time required to achieve 90% of the theoretical maximum settlement to the characteristic diffusion time (L^2/D). From the extensive parametric study the following conclusions can be drawn:

- BTR–PTR plots are significantly influenced by the (initial) void ratio and compression index, while being quite insensitive to variations in loading rate, (initial) hydraulic conductivity, dispersion coefficient and barrier thickness. A remarkably consistent BTR–PTR curve is thus obtained for each combination of (initial) void ratio and compression index despite variation of other parameters.
- Variation of BTR with PTR is generally non-monotonic characterized by a minimum (turning point) BTR, corresponding to a pessimum condition for contaminant transport through the barrier. The non-monotonicity is due to the contribution of advective flux to the total flux, which peaks at a PTR value slightly above the pessimum. When advection is neglected, effects of geometric and void ratio variation yield a monotonic reduction in BTR with decreasing PTR.
- Insight into understanding the pessimum (i.e., minimisation of the BTR) at an intermediate PTR is gained by noting that for significant advective transport to occur, a relatively strong interaction is required between the concentration profile and fluid

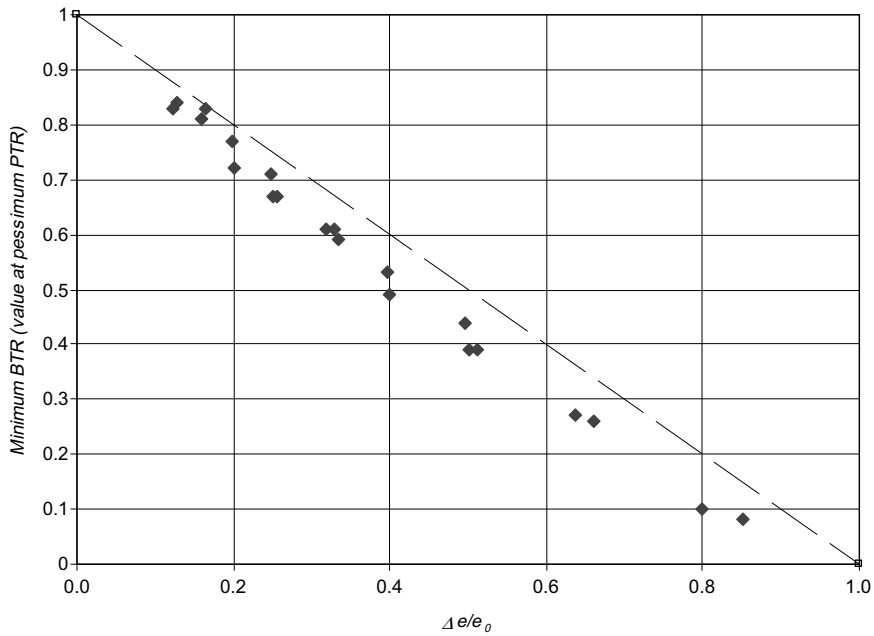

Fig. 7. Correlation of the minimum breakthrough time ratio (BTR) with variation in void ratio ($\Delta e/e_0$).

velocity profile in the liner system. It is noted that both are monotonic functions of x with their maximum values on opposite sides of the barrier. Relatively fast consolidation (low PTR) leads to fluid velocities that are high but short-lived. In contrast, slow consolidation (high PTR) produces a longer period of advective flow, but with low velocity. Advection is thus limited at low or high PTR and maximised at intermediate values. It is this effect, in combination with the effects of geometric and void ratio variation that leads to the minimisation of the BTR at the pessimum PTR.

- Reduction of breakthrough time strongly depends on the compressibility of the contaminant barrier. The BTR values increase with increasing compressibility. For moderate compressibility BTR values are generally higher than 0.6 (i.e., a 40% reduction in breakthrough time). For well-constructed composite compacted clay landfill liners, which are typically of low compressibility and void ratio, results show that contaminant transit time can still be reduced by up to 30% by taking into account coupling between consolidation and solute transport.
- Minimum BTR values are found to be strongly correlated with the proportion of pore water expelled from the barrier. This may provide a useful practical criterion for determining if consolidation is likely to have significant influence on transport in a composite barrier.

The numerical results presented here suggest that employment of coupled consolidation–contaminant transport models are required in cases of soft clays in order to guarantee conservative contaminant barrier design. In future research, we plan to extend the parametric study to variable coefficients of consolidation and investigate the influence of different solute transport boundary conditions on contaminant breakthrough time.

References

[1] Acar YB, Haider L. Transport of low – concentration contaminants in saturated earthen barriers. J Geotech Eng 1990;116(7):1031–52.
[2] Alshawabkeh AN, Rahbar N, Sheahan TC, Tang G. Volume change effects in solute transport in clay under consolidation. Geo Jordan, Irbid, Jordan; 2004. p. 105–15.
[3] Benson CH, Daniel DE. Minimum thickness ofcompacted soil liners: II. Analysis and case histories. J Geotech Eng 1994;120(1):153–72.
[4] Benson CH, Daniel DE, Boutwell GP. Field performance of compacted clay liners. J Geotech Eng 1999;125(5):390–403.
[5] Benson CH, Zhai H, Wang X. Estimating hydraulic conductivities of compacted clay liners. J Geotech Eng 1994;120(2):366–87.
[6] Burland JB. On the compressibility and shear strength of natural clays. Geotechnique 1990;40(3):329–78.
[7] Carter M, Bentley SP. Correlations of soil properties. London: Pentech Press Ltd.; 1991.
[8] COMSOL I. COMSOL multiphysics: chemical engineering model user's guide. Stockholm, Sweden (FEMLAB); 2005.
[9] Djoenaidi WJA. Compendium of soil properties and correlations. MEng Sc Thesis, University of Sydney, Sydney, Australia; 1985.
[10] Edil TB. A review of aqueous phase VOC transport in modern landfill liners. Waste Manage 2003;23(7):561–71.
[11] Farquhar GJ. Experiences with liners using natural materials. London: E and FN Spon; 1994.
[12] Foose TBG, Benson C, Edil T. Equivalency of composite geosynthetic clay liners as a barrier to volatile organic compounds. Geosynthetics 99, Industrial Fabrics Association International, St. Paul, MN; 1999. p. 321–34.
[13] Janbu N. Soil compressibility as determined by oedometer and triaxial tests. In: Third European conference on soil mechanics and foundation engineering, Wiesbaden, Germany; 1963.
[14] Kumar S, Yong W. Effect of bentonite on compacted clay landfill barriers. Soil Sediment Contamin 2002;11(1):71–9.
[15] TW, Pivonka P, Smith DW. Theoretical investigation of the effects of consolidation on contaminant transport through clay barriers. Int J Numer Anal Methods Geomech, in press.
[16] Lewis TW, Smith DW. A parametric study of contaminant transport in saturated clay soil. In: 12th Panamerican conference on soil mechanics and geotechnical engineering, Boston; 2003.
[17] Mitchell JK. Fundamentals of soil behaviour. New York: Wiley; 1993.
[18] Peters GP, Smith DW. Solute transport through a deforming porous medium. Int J Numer Anal Method Geomech 2002;26(7):683–717.
[19] Rabideau A, Khandelwal A. Boundary conditions for modeling transport in vertical barriers. ASCE J Environ Eng 1998;124(11):1135–9.
[20] Rowe RK. Geosynthetics and the minimization of contaminant migration through barrier systems beneath solid waste. In: Sixth international conference on geosynthetics, Atlanta; 1998.
[21] Rowe RK, Sangam HP, Lake CB. Evaluation of an HDPE gomembrane after 14 years as a leachate lagoon liner. Can Geotech J 2003;40(3):536–50.
[22] Smith DW. One-dimensional contaminant transport through a deforming porous medium: theory and a solution for a quasi-steady-state problem. Int J Numer Anal Methods Geomech 2000;24(8):693–722.
[23] Sridharan A, Nagaraj HB. Compressibility behaviour of remoulded, fine-grained soils and correlation with index properties. Can Geotech J 2000;37(3):712–22.
[24] Trast JM, Benson CH. Estimating field hydraulic conductivity of compacted clay. J Geotech Eng 1995;121(10):736–9.