Coupling hydraulic with mechanical models for unsaturated soils

Daichao Sheng and An-Nan Zhou

Abstract: This paper presents an alternative method to couple the hydraulic component with the mechanical component in a constitutive model for unsaturated soils. Some pioneering work on hydromechanical coupling is reviewed. Generalized constitutive relations on coupled hydromechanical behaviour are introduced. These generalized constitutive relations are then incorporated into existing mechanical and hydraulic models for unsaturated soils. A new coupling mechanism is proposed based on the fact that soil-water characteristic equations are usually obtained for constant stress, not constant volume. The proposed coupling mechanism also satisfies the intrinsic relationship between the degree of saturation and the volumetric strain for undrained compression. Numerical examples are presented to show the performance of the proposed model in predicting soil behaviour along drying and loading paths. Finally, the model is validated against experimental data for different soils.

Key words: hydromechanical coupling, unsaturated soils, soil-water retention, density effect, soil-water characteristic curve (SWCC).

Résumé : Cet article présente une méthode alternative pour coupler la composante hydraulique avec la composante mécanique dans un modèle constitutif pour les sols non saturés. Une revue de certains travaux innovateurs sur le couplage hydromécanique a été réalisée. Des relations constitutives généralisées sur le comportement hydromécanique sont introduites. Ces relations constitutives généralisées sont ensuite incluses dans des modèles hydrauliques et mécaniques existants pour des sols non saturés. Un nouveau mécanisme de couplage est proposé, basé sur le fait que les équations de rétention d'eau sont normalement obtenues dans des conditions de contrainte constante, et non pas de volume constant. Le mécanisme de couplage proposé satisfait aussi la relation intrinsèque entre le degré de saturation et la déformation volumétrique pour la compression non drainée. Des exemples numériques sont présentés pour démontrer la performance du modèle proposé à prédire le comportement du sol en séchage et en chargement. Enfin, le modèle est validé à l'aide de résultats expérimentaux pour différents sols.

Mots-clés : couplage hydromécanique, sols non saturés, rétention de l'eau du sol, effet de densité, courbe de rétention d'eau (CRE).

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Introduction

Elastoplastic modelling of stress–strain behaviour of unsaturated soils was pioneered by Alonso et al. (1990). Since then, a number of constitutive models have been proposed, and reviews of these models can be found in, e.g., Gens (1996), Gens et al. (2006), and Sheng et al. (2008*c*). In early models, the mechanical component that describes the stress–strain relationship was usually not coupled with the hydraulic component that describes the saturation–suction relationship. The issue of interaction between the mechanical and hydraulic behaviour was perhaps first raised by Wheeler (1996) and then by Dangla et al. (1997). The first complete model that deals with the coupled hydromechanical behaviour of unsaturated soils was perhaps proposed by Vaunat et al. (2000). A number of other models soon followed (e.g., Gallipoli et al. 2003*a*; Wheeler et al. 2003; Sheng et al. 2004; Pereira 2005; Santagiuliana and Schrefler 2006; Li 2007; Sun et al. 2007*a*; Nuth and Laloui 2008; Romero and Jommi 2008; Sheng et al. 2008*a*). In most of these coupled models, the influence of the hydraulic behaviour on the mechanical behaviour is accentuated.

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With respect to hydraulic behaviour of unsaturated soils, many models (van Genuchten 1980; Fredlund and Xing 1994) take advantage of the fact that the influence of suction on degree of saturation is more significant than the influence of deformation. The dependency of degree of saturation on suction is usually described by a soil-water characteristic curve (SWCC), also called soil-water retention curve (SWRC). Such a curve is obtained under constant stress. Only until recently, the effects of deformation or stress on SWCCs have been considered in coupled hydromechanical models for unsaturated soils (e.g., Gallipoli et al. 2003*b*; Wheeler et al. 2003; Simms and Yanful 2005; Sun et al. 2007*b*; Nuth and Laloui 2008; Tarantino 2009; Masin 2010). In addition to mechanical variables such as deformation or stress, SWCC is also influenced by many factors such as the pore size and pore-shape distribution, specific surface area, plastic index, particle size distribution, chemo-physical properties of the soil phases, and even temperature (Arya and Paris 1981; Romero et al. 2001; Simms and Yanful 2002, 2005; Aubertin et al. 2003; Haverkamp et al. 2005; Wang et al. 2008; Chateau and Viet 2009). However, it should also be realised that most existing SWCC equations are phenomenological in nature. In this regard, it is anticipated that some of the above-mentioned factors are incorporated into parameters used in SWCC equations. In other words, samples with, for example, different particle size distributions are effectively modelled as different soils using different parameter values. One specific parameter that affects the SWCC is the density or porosity of the soil. The density or porosity of one soil can change considerably, particularly in geotechnical problems. It would be difficult to justify that samples with the same particle size distribution but different initial densities should be treated as different soils for the purpose of modelling. In this regard, this paper focuses only on the effect of deformation on the soil-water retention properties.

As pointed out by Wheeler et al. (2003), the behaviour of an unsaturated soil depends on the degree of saturation even if the suction, net stress, and specific volume are kept the same for the soil. Separate treatment of mechanical and hydraulic components in modelling unsaturated soil behaviour has certain limitations in reproducing some experimental observation. It would be difficult to consider the saturation dependency in a mechanical model that is independent of the hydraulic behaviour. Similarly, in a hydraulic model that is independent of mechanical behaviour, the effects of soil density on the SWCC cannot be easily tackled. Experimental data generally indicate the following two points:

1. A SWCC obtained at a higher net mean stress tends to shift towards the higher suction (Ng and Pang 2000; Gallipoli et al. 2003*b*; Lee et al. 2005). This means that the incremental relationship between degree of saturation (S_r) and suction (s) depends on net mean stress ($\overline{p}$):

$$[1] \qquad dS_r = f(\overline{p}, s)\, ds, \quad d\overline{p} = 0$$

2. When the suction is kept constant, isotropic loading and unloading can also change the degree of saturation of an unsaturated soil (Sharma 1998; Wheeler et al. 2003). This implies that the change of degree of saturation is not only related to the change of suction, but also to the change of stress:

$$[2] \qquad dS_r = g(\overline{p}, s)\, d\overline{p}, \quad ds = 0$$

In eqs. [1] and [2], f and g are generally functions of net mean stress and suction.

One of the early models that consider the mutual influences of the hydraulic and mechanical components in unsaturated soil behaviour is that by Wheeler et al. (2003). Models that appeared before or soon after Wheeler et al. (2003) tend to emphasise the influences of the hydraulic component on the mechanical component (e.g., Vaunat et al. 2000; Sheng et al. 2004). The interaction between the mechanical and hydraulic components in the model by Wheeler et al. (2003) was realised through the use of the average soil skeleton stress and the modified suction:

$$[3] \qquad \begin{cases} \sigma_{ij}^* = \sigma_{ij} - [S_r u_w + (1 - S_r) u_a]\delta_{ij} \\ s^* = ns = n(u_a - u_w) \end{cases}$$

where σ_{ij}^* is the average soil skeleton stress, s^* is the modified suction, s is the matric suction (also called the matrix suction), n is the porosity, σ_{ij} is the total stress, S_r is the degree of saturation, u_w is the pore-water pressure, u_a is the pore-air pressure, and δ_{ij} is the Kronecker delta. The average soil skeleton stress (σ_{ij}^*) is an amalgam of stress, suction, and degree of saturation. The modified suction (s^*) is a combination of suction and porosity. Therefore, the influence of hydraulic behaviour on the stress–strain–suction relationship is considered via the definition of the average stress. The influence of porosity on the hydraulic relationship between degree of saturation and modified suction is considered via the definition of the modified suction. The model by Wheeler et al. (2003) is one of the few models that are qualitatively tenable in terms of coupling mechanical behaviour with hydraulic behaviour for unsaturated soils. However, the use of the modified suction and the soil skeleton stress, which is one of the advantages that make the model thermodynamically consistent, can become a disadvantage as well, particularly in terms of quantitative prediction and the application of the model. For example, it is difficult to experimentally calibrate the synchronized evolution of the loading-collapse surface and the suction-increase and suction-decrease surfaces adopted in the model.

In more recent models, the influences of mechanical properties on the hydraulic behaviour are usually represented via the dependency of the SWCC on soil volume (Vanapalli et al. 1999; Gallipoli et al. 2003*b*; Pham 2005), soil density (Simms and Yanful 2005; Mbonimpa et al. 2006; Sun et al. 2007*b*, 2008; Tarantino 2009; Masin 2010), volumetric strain (Nuth and Laloui 2008), or stress (Miller et al. 2008). Based on the experimental study on Speswhite kaolin, Gallipoli et al. (2003*b*) suggested including a function of specific volume (v) in the SWCC equation proposed by van Genuchten (1980). Simms and Yanful (2002, 2005) related volume change and the SWCC equation to the pore-size distribution through pore-network modelling. Mbonimpa et al. (2006) extended the modified Kovács model (Kovács 1981), which was developed initially for incompressible soils to predict SWCC of deformable soils by incorporating the volume shrinkage curve.

More recently, Sun et al. (2008) proposed a hydraulic model that takes into consideration the influence of void ratio (e):

$$[4] \qquad dS_r = -\lambda_{se}\, de - \lambda_{sr}\, ds/s$$

where λ_{se} is the slope of degree of saturation versus void ratio curve under constant suction, and λ_{sr} is the slope of the main drying or wetting curve. In theory, λ_{sr} can only be determined from drying or wetting tests under constant volume. However, such tests are not common. Nuth and Laloui (2008) provided a new explanation of the SWCC for a deforming soil. They assume there is an intrinsic SWCC for a nondeforming soil, and deformation of the soil can shift this intrinsic SWCC along the logarithmic suction axis. The shift is governed by an air-entry value that depends on the volumetric strain. Tarantino (2009) proposed a water retention model

(SWCC) for deformable soils based on an empirical power function for water ratio (e_w). This model is very similar to the model by Gallipoli et al. (2003*b*), but with one parameter less than Gallipoli's model. Masin (2010) presented a hydraulic model that can predict the dependency of degree of saturation on void ratio and suction using an effective stress concept. In this model, both air-entry value (s_{ae}) and the slope of main drying curve are shown to vary with void ratio.

A glaring omission of the models discussed above is the volume change along SWCCs, which are obtained under constant stress. A change in suction at high degrees of saturation can cause a significant volume change albeit a constant net mean stress. It is difficult to obtain a SWCC for a soil with constant volume. In addition, the vast available data on SWCCs were obtained under constant stresses, and these data would be of limited use in these models. On the other hand, if the SWCC is assumed to be a function of stress, it would then be difficult to differentiate loading and unloading paths.

To overcome the limitations of the above models, a new approach for coupling mechanical with hydraulic behaviour for unsaturated soils is proposed in this paper. Starting from basic phase relationships for a three-phase mixture, two general constitutive equations are proposed, respectively, for volume and saturation changes. These generalized constitutive equations are then incorporated into existing constitutive models for unsaturated soils. A new approach is proposed to couple the hydraulic and mechanical components. The predictions of the proposed model are then compared with several sets of experimental data.

Intrinsic phase relationships

In a three-phase mixture, there are certain intrinsic relationships between the volume and mass of each phase. Clarification of these simple relationships is prerequisite for more complex constitutive modelling.

In an element of unsaturated soil, let V_w and V_s be the volume of pore water and volume of solid particles, respectively. Let V_v be the volume of voids, and V be the total volume of the soil. The void ratio (e), porosity (n), and specific volume (v) are common mechanical state variables and are related to each other:

$$[5]\qquad e=\frac{V_v}{V_s},\quad n=\frac{V_v}{V},\quad v=1+e,\quad n=\frac{e}{1+e}=\frac{v-1}{v}$$

Provided that the solid particles are incompressible (V_s is constant), the increment of volumetric strain is defined as

$$[6]\qquad \mathrm{d}\varepsilon_v=-\frac{\mathrm{d}V}{V}=-\frac{\mathrm{d}V_v}{V}=-\frac{V_s}{V_s+V_v}\mathrm{d}e =-\frac{\mathrm{d}e}{1+e}=-\frac{\mathrm{d}n}{1-n}$$

The degree of saturation (S_r), gravimetric water content (w), and volumetric water content (θ) are usually adopted as hydraulic state variables. The definitions of S_r, w, and θ as well as their relationship are given below:

$$[7]\qquad S_r=\frac{V_w}{V_v},\quad w=\frac{M_w}{M_s},\quad \theta=\frac{V_w}{V},\quad \frac{S_r}{w}=\frac{G_s}{e},\quad \theta=nS_r$$

where M_w is the mass of pore water, M_s is the mass of solids, and G_s is the specific gravity and is usually treated as a constant for a soil.

Additionally, there are intrinsic relationships between the mechanical state variables (e and n) and the hydraulic state variables (w, θ, and S_r), which can be derived from eq. [7]:

$$[8]\qquad -S_r\,\mathrm{d}\varepsilon_v+n\mathrm{d}S_r=\frac{G_s}{1+e}\mathrm{d}w$$

$$[9]\qquad \mathrm{d}\theta=n\,\mathrm{d}S_r-S_r(1-n)\,\mathrm{d}\varepsilon_v$$

These equations are always true as along as the solids are assumed to be incompressible.

An additional constraint can be derived from undrained conditions. For an undrained condition, the gravimetric water content (w) in the soil does not change. Equation [8] can then be rewritten in the following form:

$$[10]\qquad \mathrm{d}S_r=\frac{S_r}{n}\mathrm{d}\varepsilon_v$$

Equation [10] can be regarded as constraints for undrained conditions.

General constitutive laws

Under isotropic stress states, an increment in the net mean stress ($\mathrm{d}\overline{p}$) and an increment in suction ($\mathrm{d}s$) can both cause an increment in volumetric strain ($\mathrm{d}\varepsilon_v$):

$$[11]\qquad \mathrm{d}\varepsilon_v=A\,\mathrm{d}\overline{p}+B\,\mathrm{d}s$$

where A and B are two general functions ($A=\partial\varepsilon_v/\partial\overline{p}$, and $B = \partial\varepsilon_v/\partial s$) and may be expressed as functions of mechanical variables (such as $\overline{p}$, e, etc.) and hydraulic variables (such as S_r, s, etc.). These parameters also depend on the suction–stress path and can take different values on a loading and unloading path, respectively.

As suggested by Sun et al. (2007*b*) and Masin (2010), the degree of saturation depends on suction, void ratio, and suction–stress path. Therefore, an increment in the degree of saturation ($\mathrm{d}S_r$) can be caused by an increment in suction and (or) an increment in volumetric strain:

$$[12]\qquad \mathrm{d}S_r=C\,\mathrm{d}s+D\,\mathrm{d}\varepsilon_v$$

where C and D are two general functions like A and B ($C = \partial S_r/\partial s$ and $D = \partial S_r/\partial\varepsilon_v$).

Substituting eq. [11] into eq. [12] gives

$$[13]\qquad \begin{aligned}\mathrm{d}S_r&=C\,\mathrm{d}s+D(A\,\mathrm{d}\overline{p}+B\,\mathrm{d}s)\\&=(C+DB)\,\mathrm{d}s+DA\,\mathrm{d}\overline{p}=E\,\mathrm{d}s+F\,\mathrm{d}\overline{p}\end{aligned}$$

where $E = C + DB$ and $F = DA$. Equation [13] states that both a change of suction and a change of net stress can lead to a change of degree of saturation. A noticeable difference between eqs. [12] and [13] is that the S_r–s relationship in eq. [12] is for constant volume and that in eq. [13] is for constant stress. In eq. [13], the effect of suction-induced volume change on the degree of saturation is included in function E.

In drying or wetting tests for determining SWCCs, the volume of the soil is not constant, but the net mean stress is constant. Therefore, a SWCC corresponds to eq. [13] with $\mathrm{d}\overline{p}=0$:

$$[14]\qquad E = \frac{\partial S_r^{\text{SWCC}}}{\partial s}$$

where S_r^{SWCC} defines the equation for the SWCC. Therefore, it is function E, not function C, that corresponds to the slope of a SWCC, which is not necessarily a constant.

Equations [11]–[13] are general constitutive equations for coupled hydromechanical behaviour of unsaturated soils. These equations are generally true and transcend the differences in stress variables and constitutive models used to model the soil behaviour. Indeed, most existing models for coupled hydromechanical behaviour can be fit into these equations. These equations are also valid for both drained and undrained conditions. For undrained conditions, an additional constraint defined by eq. [10] has to be imposed to deduce the variation of suction for a given variation of stress. It should also be noted that these equations are in incremental forms, and as such they only define the variation of the variable (ε_v or S_r) caused by a variation of stress or a variation of suction. These equations can be solved by integration, and their solutions will depend on the initial state (e.g., a reference stress level or a reference void ratio). However, eqs. [11]–[14] cannot directly be used to predict soil behaviour. We need to determine at least four general functions involved: A, B, E, and F. To do that, we need to seek use of specific constitutive laws.

Specific constitutive relations

In the former section, generalized constitutive laws are proposed to model the coupled hydromechanical behaviour of unsaturated soil. To apply these generalized constitutive laws, we need to define specific expressions for functions like A and B in eq. [11] and E and F in eq. [13].

Relationship between volume, stress, and suction

A number of constitutive models have been used to describe the volume change caused by stress and suction changes. In this paper, we use the following relationship:

$$[15]\qquad d\varepsilon_v = \frac{\lambda_{vp}}{\overline{p}+f(s)}d\overline{p} + \frac{\lambda_{vs}}{\overline{p}+f(s)}ds$$

where λ_{vp} and λ_{vs} are two soil parameters. As a first approximation, λ_{vp} can be assumed to be independent of suction, but more realistically, it depends on suction and perhaps on degree of saturation as well. Parameter λ_{vs} must equal λ_{vp} when the soil is fully saturated and decreases with increasing suction. Sheng et al. (2008*a*) suggested the following function:

$$[16]\qquad \lambda_{vs} = \begin{cases} \lambda_{vp} & s \le s_{sa} \\ \lambda_{vp}\dfrac{s_{sa}+1}{s+1} & s > s_{sa} \end{cases}$$

where s_{sa} is the saturation suction, which is the unique transition suction between saturated and unsaturated states (see Sheng et al. 2008*a*). We note that the number "1" in eq. [16] is not needed if s_{sa} is larger than zero:

$$[17]\qquad \lambda_{vs} = \begin{cases} \lambda_{vp} & s \le s_{sa} \\ \lambda_{vp}\dfrac{s_{sa}}{s} & s > s_{sa} \end{cases}$$

Equation [17] is preferred to eq. [16], as the transition suction is seldom absolutely zero. Both λ_{vp} and λ_{vs} vary with stress path and can take different values on a loading and unloading path, respectively.

Now, the two functions A and B in the volume change, eq. [11], can be defined as follows:

$$[18]\qquad \begin{cases} A = \dfrac{\lambda_{vp}}{\overline{p}+f(s)} \\ B = \dfrac{\lambda_{vs}}{\overline{p}+f(s)} \end{cases}$$

The function $f(s)$ in eqs. [15] and [18] was set to s in the Sheng, Fredlund, and Gens (SFG) model by Sheng et al. (2008*a*). This is its simplest form possible. Even with this simplest form, Zhou and Sheng (2009) showed that eqs. [15] and [16] can predict a large set of experimental data, both in terms of volume change and shear strength. Other forms of $f(s)$ are possible. For example, $f(s) = S_r^k s$, with k being a positive real number, would also guarantee a smooth transition between saturated and unsaturated states and does not result in a significant change of the yield stress – suction and shear strength – suction relationships. Further research in this direction is warranted.

SWCCs

Extensive research has been carried out on SWCCs, first in the field of soil physics and later within geotechnical engineering. Numerous empirical equations have been proposed in the literature (see, e.g., Gardner 1956; Hillel 1971; van Genuchten 1980; Fredlund and Xing 1994; Simms and Yanful 2002; Aubertin et al. 2003). These relations are usually written as continuous functions and do not explicitly consider the hysteretic behaviour during a drying–wetting cycle. More recently, incremental SWCC relationships with smooth hysteretic responses to arbitrary wetting or drying paths have also been proposed by Li (2005), Lins et al. (2007), and Pedroso et al. (2008).

Most of the existing SWCC equations are for constant stress and can thus be directly used to find function E in the generalized hydraulic model, i.e., eq. [13]. In the SFG model, a simplified SWCC function (see Fig. 1) is used, and it is used here as an example. As suggested by Sheng et al. (2008*a*), the SWCC can be expressed as

$$[19]\qquad dS_r = dS_r^{\text{SWCC}} = -\lambda_{ws}\,d(\ln s) = -\frac{\lambda_{ws}}{s}ds,\ \ d\overline{p} = 0$$

where parameter λ_{ws} can take a value of zero, κ_{ws}, or λ_{ws}, dependent on the suction value and suction path, as shown in Fig. 1. In Fig. 1, λ_{ws} is the slope of the main drying or wetting curve and κ_{ws} is the slope of the scanning curve.

Now, function E in eq. [13] can be specified as

$$[20]\qquad E = \frac{\partial S_r^{\text{SWCC}}}{\partial s} = -\frac{\lambda_{ws}}{s}$$

Alternatively, closed-form equations like van Genuchten (1980) and Fredlund and Xing (1994) can also be used to find E. For example, the van Genuchten equation is defined as

$$[21] \qquad S_{\mathrm{r}}^{\mathrm{SWCC}} = \frac{1}{\left[1+(s/a)^{\alpha}\right]^{\beta}}$$

where a, α, and β are model parameters. In the van Genuchten equation and in that of Fredlund and Xing, no clear threshold suctions like the air-entry value are used. These equations are usually used for the drying curve only. Hydraulic hysteresis has to be incorporated if important.

Relationship between degree of saturation and stress

The influence of net mean stress on the degree of saturation is usually reflected by two related phenomena: (*i*) the SWCC is shifted as the initial void ratio of the soil changes; (*ii*) the change of net stress under constant suction ($\mathrm{d}s = 0$) can result in a change of degree of saturation. These two effects are related, but not always equivalent. In some models for coupled hydromechanical behaviour of unsaturated soils, it is assumed that the air-entry suction is a function of specific volume or volumetric strain (Sun et al. 2007*b*; Nuth and Laloui 2008; Masin 2010). As such, the SWCC is shifted with the air-entry value. This approach of coupling mechanical properties with the SWCC is straightforward and simple to use. However, we are not adopting such an approach because of the following reasons:

1. As mentioned above, the volume change due to suction change is already included in most SWCC equations, since these equations are obtained from constant stress tests. We need only to consider the volume change caused by stress change when considering the effect of mechanical properties on the SWCC.
2. The intrinsic relationship between the degree of saturation and the volumetric strain must be satisfied. For example, constraints like eqs. [8] and [10] must hold. Setting the air-entry value to a function of specific volume or volumetric strain does not necessarily lead to satisfaction of these constraints.
3. Shifting the air-entry value may not sufficiently reflect the effect of the initial void ratio on the SWCC. For example, the slope of the SWCC may also change (Huang 1994; Vanapalli et al. 1999; Lee et al. 2005; Masin 2010).
4. Common SWCC equations such as that by van Genuchten (1980) and Fredlund and Xing (1994) do not explicitly use an air-entry value, even though some parameters may be related to it.

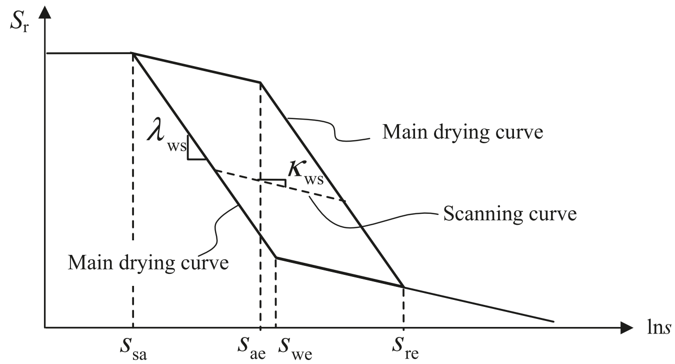
Based on the above reasons, we seek an alternative method to couple the mechanical equation with the hydraulic equation. To achieve the goal, we first study the change of degree of saturation during isotropic compression under constant suction. We also study the changes of suction and volume during undrained isotropic compression.

To define the effect of stress on the degree of saturation, it is equivalent to find function F in eq. [13]. Using [18], F can be written as

$$[22] \qquad F = DA = D\frac{\lambda_{\mathrm{vp}}}{\overline{p}+f(s)}$$

Substituting eq. [22] into eq. [13] and setting $\mathrm{d}s = 0$, we have

Fig. 1. SWCC in the SFG model. s_{sa}, saturation suction; s_{ae}, air-entry value; s_{we}, water-entry value; s_{re}, residual suction.



$$[23] \qquad \mathrm{d}S_{\mathrm{r}} = F\,\mathrm{d}\overline{p} = D\frac{\lambda_{\mathrm{vp}}}{\overline{p}+f(s)}\mathrm{d}\overline{p}, \qquad \mathrm{d}s = 0$$

Equation [23] indicates that the change of degree of saturation ($\mathrm{d}S_{\mathrm{r}}$) is related to the change of net stress ($\mathrm{d}\overline{p}$) when suction is kept constant ($\mathrm{d}s = 0$). The relationship between $\mathrm{d}S_{\mathrm{r}}$ and $\mathrm{d}\overline{p}$ shown in eq. [23] can be calibrated by isotropic compression tests under different suctions. Figure 2 is a replot from Wheeler et al. (2003) using data by Sharma (1998). It shows the volume and saturation changes of a bentonite–kaolin mixture during isotropic compression at constant suction ($s = 200$kPa).

A comparison between the two curves in Figs. 2*a* and 2*b* shows that the relationship between $\mathrm{d}S_{\mathrm{r}}$ and $\mathrm{d}\overline{p}$ is quite similar to that between $\mathrm{d}v$ and $\mathrm{d}\overline{p}$. This suggests that the constitutive function between $\mathrm{d}S_{\mathrm{r}}$ and $\mathrm{d}\overline{p}$ can be similar to that between $\mathrm{d}v$ and $\mathrm{d}\overline{p}$. In addition, there are some other constraints that may be useful for determining the unknown general function D in eq. [23]:

1. For a fully saturated soil, the degree of saturation is independent of net stress, i.e., $D = 0$ when $S_{\mathrm{r}} = 1$;
2. D is positive ($\mathrm{d}S_{\mathrm{r}} > 0$ when $\mathrm{d}\overline{p} > 0$, see Fig. 2);
3. Equation [10] must be satisfied for undrained isotropic compression:

$$[24] \qquad \mathrm{d}S_{\mathrm{r}} = \frac{S_{\mathrm{r}}}{n}\mathrm{d}\varepsilon_{\mathrm{v}}, \qquad \mathrm{d}_{w} = 0$$

Substituting eq. [24] into eq. [12] leads to

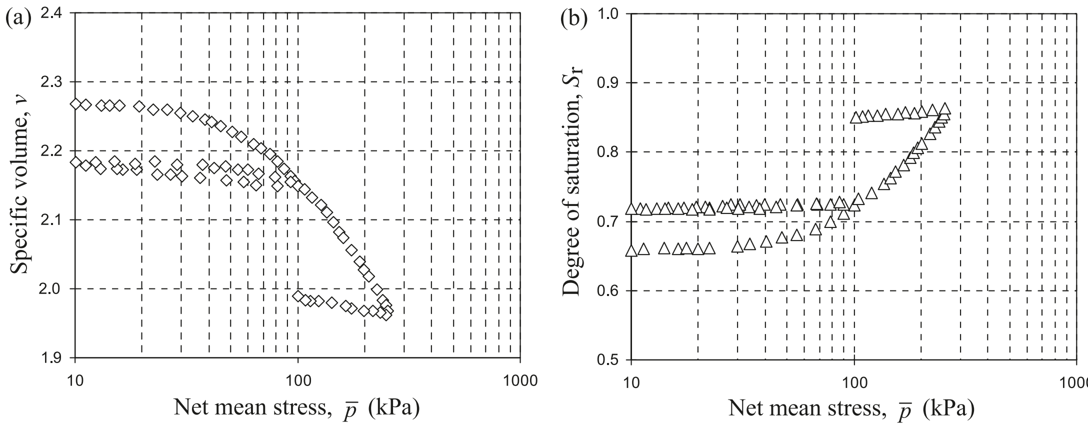
$$[25] \qquad \left(\frac{S_{\mathrm{r}}}{n} - D\right)\mathrm{d}\varepsilon_{\mathrm{v}} = C\,\mathrm{d}s, \qquad \mathrm{d}w = 0$$

The above equation actually defines the variation of suction in terms of volumetric strain under undrained compression. According to the experimental data by Sun et al. (2008), both the volume and suction decrease during undrained isotropic compression of an unsaturated soil, i.e., $\mathrm{d}\varepsilon_{\mathrm{v}} > 0$ and $\mathrm{d}s \leq 0$. Parameter C in eq. [12] is usually negative because an increase in suction usually results in a decrease in saturation. Therefore, we have

$$[26] \qquad D \leq \frac{S_{\mathrm{r}}}{n}$$

The equal sign ($D = S_{\mathrm{r}}/n$) is valid only if $\mathrm{d}s = 0$. To meet the above constraints, function D is set to the following function:

Fig. 2. Behaviour of bentonite–kaolin during isotropic loading at constant suction (after Wheeler et al. 2003): (*a*) specific volume versus net mean stress; (*b*) degree of saturation versus net mean stress.



$$[27] \qquad D = \frac{S_r}{n}(1 - S_r)^m$$

where m is a fitting parameter.

The last unknown function F can now be specified as

$$[28] \qquad F = \frac{S_r}{n}(1 - S_r)^m A = \frac{S_r}{n}(1 - S_r)^m \frac{\lambda_{vp}}{\bar{p} + f(s)}$$

Figure 3*a* shows the calculated results by eqs. [11] and [18] for isotropic compression under constant suction, and Fig. 3*b* shows the results by eqs. [13] and [28]. The compression indexes for loading and unloading are set as follows: $\lambda_{vp} = 0.21$, $\kappa_{vp} = 0.06$. The preconsolidation pressure is set to 40 kPa. Parameter m in eq. [27] varies between 0 and ~1.0, with $m = 0.05$ giving the best fit of the data of Sharma (1998). Setting $m = 0$ also gives very good prediction of Sharma's data, but may lead to inconsistent prediction of undrained behaviour. Figure 3 demonstrates that eq. [27] captures very well the observed behaviour of the bentonite–kaolin mixture during isotropic compression at constant suction.

Shifting of SWCC due to change of initial void ratio

As mentioned above, the shifting the SWCC due to the change of initial void ratio is controlled by the air-entry value in some models. In these models, it is assumed that the air-entry value is related to the void ratio or the volumetric strain (Sun et al. 2007*b*; Nuth and Laloui 2008; Masin 2010). In this section, we demonstrate the connection between the approach adopted in this paper and the approach used in the above models. The coupling between the SWCC and mechanical properties of the soil is realised through function F in the model proposed in this paper. Indeed, the shifting of the SWCC due to the change of initial void ratio can be derived from function F. Assuming that the degree of saturation in the SWCC equation is related to the initial void ratio of the soil, we then have

$$[29] \qquad dS_r = \frac{\partial S_r^{SWCC}}{\partial s} ds + \frac{\partial S_r}{\partial e_0} de_0 = E\, ds - \frac{\partial S_r}{\partial e_0}(1+e_0)\, d\varepsilon_{vp}$$

The variation of the void ratio (de_o) in the equation above refers to that due to stress change only and hence refers to the initial void at the start of a conventional SWCC test where stress does not change. This initial void ratio can change, due to, for example, a change in the consolidation pressure. The volumetric strain ε_{vp} in eq. [29] is purely due to stress change. Therefore, we have

$$[30] \qquad dS_r = E\, ds - \frac{\partial S_r}{\partial e_0}(1+e_0) A\, d\bar{p}$$

Comparing eq. [30] with eq. [13] leads to

$$[31] \qquad F = -\frac{\partial S_r}{\partial e_0}(1+e_0) A$$

Substituting eq. [28] into eq. [31] gives

$$[32] \qquad \frac{\partial S_r}{\partial e_0} = -\frac{S_r(1 - S_r)^m}{e_0}$$

The equation above shows that the SWCC can indeed be related to the initial void ratio of the soil in the model proposed in this paper. It should be noted that eq. [32] can only be used to shift SWCCs according to the initial void ratio. In more general cases where both stress and suction change, eq. [13] should be used.

Equation [32] can be integrated numerically for an arbitrary value of m. In the special cases of $m = 1$, $m = 0.5$, and $m = 0$, analytical solutions are available:

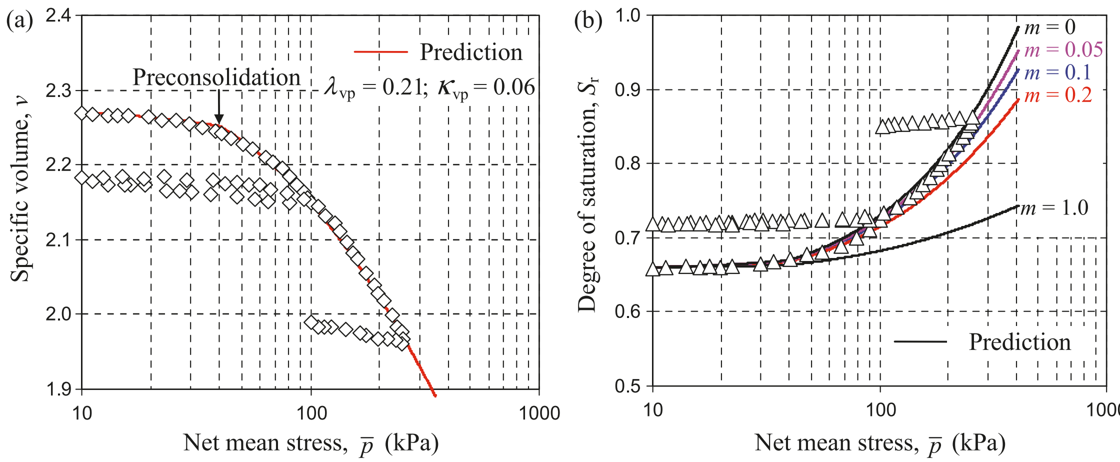
$$[33] \qquad \ln\frac{1 - S_r}{S_r} = \ln\frac{1 - S_{r0}}{S_{r0}} + \ln\frac{e_{r0}}{e_0}, \qquad m = 1$$

$$[34] \qquad \frac{1 - \sqrt{1 - S_r}}{1 + \sqrt{1 - S_r}} = \frac{e_{r0}}{e_0}\frac{1 - \sqrt{1 - S_{r0}}}{1 + \sqrt{1 - S_{r0}}}, \qquad m = 0.5$$

$$[35] \qquad S_r = S_{r0}\frac{e_{r0}}{e_0}, \qquad m = 0$$

where e_{r0} is a reference void ratio and S_{r0} is the SWCC equation for the soil at the reference void ratio. We note that eq. [35] cannot be used when $S_r = 1$ because the constraint for saturated states ($D = 0$) is not satisfied when $m = 0$. Setting $m = 0$ also implies that the suction does not change under undrained compression (see discussion above). Setting $m = 1$ may underestimate the effect of void ratio on the

Fig. 3. Calculated results during isotropic loading under constant suction: (*a*) specific volume versus net mean stress; (*b*) degree of saturation versus net mean stress.



SWCC, according to Fig. 3. Nevertheless, eqs. [33]–[35] can be used to illustrate the effect of initial void ratio on the degree of the SWCC.

In Fig. 4, eqs. [33] and [35] are used to plot the SWCCs corresponding to different initial void ratios. The reference SWCC at e_0 is determined according to the van Genuchten equation. Figure 4 shows that both the air-entry value and slope of the SWCC may change with the initial void ratio, which is qualitatively consistent with experimental data (Huang 1994; Vanapalli et al. 1999; Lee et al. 2005).

Equation [32] can also be applied to piecewise SWCCs, such as that in Fig. 4. In this case, the degree of saturation must be written in terms of suction and a control parameter such as the air-entry value. The control parameter is then assumed to be related to the void ratio, and a relationship can be derived following the same procedure above.

Summary of the model

The proposed model consists of two general constitutive equations, i.e., eq. [11] for the mechanical stress–strain–suction relationship and eqs. [12] or [13] for the hydraulic saturation–suction–strain relationship. These general constitutive equations can be incorporated into existing mechanical and hydraulic models for unsaturated soils. As an example, the SFG model is used to specify general functions A and B. Function E is defined according to a SWCC equation. A new function (F) is defined based on the intrinsic relationship between degree of saturation and void ratio. When defining these general functions, we have attempted to limit the number of material parameters. There are two material parameters in the mechanical component, namely λ_{vp} and κ_{vp}, and these are the same as for saturated soils. In the hydraulic part, in addition to the basic SWCC equation, one new parameter is introduced: m, which is a fitting parameter used in the saturation–volume relationship. This is the only new parameter introduced into the coupled hydromechanical model.

The two key equations of the model are repeated below:

$$[36]\qquad \mathrm{d}\varepsilon_v = A\,\mathrm{d}\bar{p} + B\,\mathrm{d}s$$

$$[37]\qquad \mathrm{d}S_r = E\,\mathrm{d}s + \frac{S_r}{n}(1-S_r)^m A\,\mathrm{d}\bar{p}$$

Supplementary constitutive equations specific to the SFG model (Sheng et al. 2008*a*) are

$$[38]\qquad A = \frac{\lambda_{vp}}{\bar{p}+f(s)},\quad B = \frac{\lambda_{vs}}{\bar{p}+f(s)},\quad f(s) = s,$$

$$\lambda_{vs} = \begin{cases} \lambda_{vp} & s \le s_{sa} \\ \lambda_{vp}\dfrac{s_{sa}}{s} & s > s_{sa} \end{cases}$$

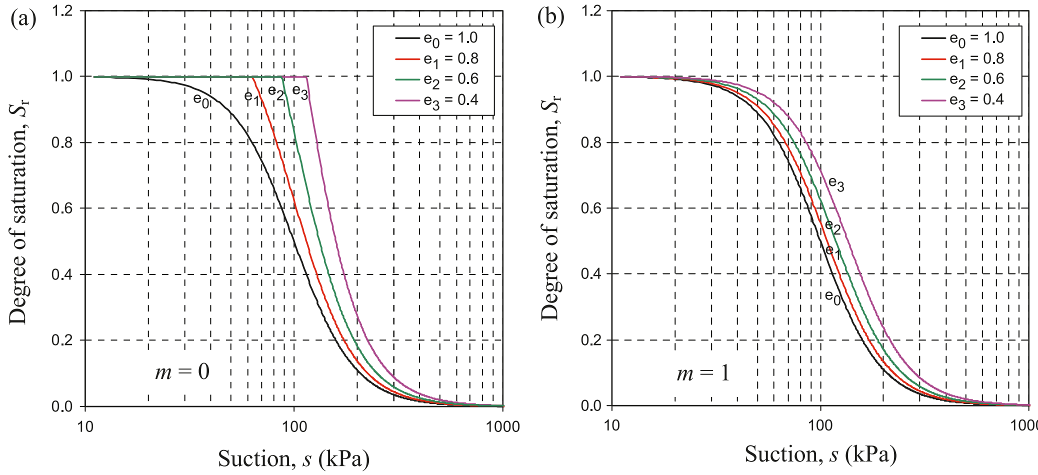
Note that the stiffness parameters λ_{vp} and λ_{vs} are usually bivalued, dependent on if the stress path is elastic (κ_{vp} and κ_{vs}) or elastoplastic (λ_{vp} and λ_{vs}), and κ_{vs} is defined by eq. [38], with λ_{vp} replaced by κ_{vp}.

Function E in eq. [37] refers to the gradient of the SWCC ($E = \partial S_r^{\mathrm{SWCC}}/\partial s$) and may also be multivalued in a piecewise equation, dependent on the suction path. Because eq. [37] is in an incremental form, integration of the equation requires one specific SWCC that corresponds to a reference initial void ratio. In other words, the conventional SWCC equation is only used for the reference initial void ratio, and the new SWCC for a new initial void ratio is obtained by integration of eq. [37]. The previous section "Shifting of SWCC due to change of initial void ratio" gives the details on how to use an existing SWCC equation for different initial void ratios.

It should be noted that the hydraulic model presented above is different from the conventional state-surface approach in the S_r–s–$\bar{p}$ space (e.g., by Matyas and Radhakrishna 1968; Lloret and Alonso 1985). Equation [37] does not lead to a state surface in the S_r–s–$\bar{p}$ space because of its incremental nature. It is also different from the conventional state-surface approach in the S_r–s–e space, because the S_r–s relationship for constant void ratio is differentiated with the conventional SWCC equations. We note that substituting eq. [36] into eq. [37] leads to

$$[39]\qquad \mathrm{d}S_r = \left[E - B\frac{S_r}{n}(1-S_r)^m\right]\mathrm{d}s - \frac{S_r}{e}(1-S_r)^m\,\mathrm{d}e$$

Fig. 4. Shifting of SWCC due to change of initial void ratio (SWCC for $e_0 = 1.0$ was obtained using the van Genuchten equation, with a = 100 kPa, $\alpha = 3$, $\beta = 1$): (*a*) $m = 0$; (*b*) $m = 1$.



The above equation clearly indicates that the S_r–s relationship for constant void ratio ($de = 0$) is not the same as the SWCC equation ($dS_r = E\,ds$). Equations [37] and [39] are equivalent and interchangeable.

It should also be noted that the proposed constitutive eqs. [36], [37], and [39] are all in incremental forms. Integration of these equations requires a known reference state. For example, the effect of the actual stress level on SWCC is reflected through the difference between this actual stress and the reference stress level.

In this paper, we limit our attention to isotropic stress state. Extension to three-dimensional stress states will depend on the constitutive model for saturated states. We also note that if the plastic volumetric strain is used as the hardening parameter, extension of the model to shear stress does not involve any new material parameters. The variation of yield stress and shear strength with suction can be derived from equations like eq. [36] (Sheng et al. 2008*a*). Some numerical examples are shown here to demonstrate the performance of the model.

Drying under different net stresses

In the first example, we study the model predictions for soils in drying tests under different net stresses. Slurry soil specimens are first consolidated to 1 kPa (point A_1 in Fig. 5*a*), 20 kPa (point A_2), 100 kPa (point A_3), 400 kPa (point A_4), respectively, and then dried to a suction of 1000 kPa (points B_1–B_4). The void ratio under 1 kPa (initial state) is assumed to be 2.0. The compression index λ_{vp} is assumed as 0.1. The SWCC for $e_0 = 2.0$ is obtained using the van Genuchten equation, with $a = 100$ kPa, $\alpha = 2$, $\beta = 1$. The fitting parameter, m, in eq. [37] is assumed to be 0.5.

The stress paths are shown in Fig. 5*a*. The predicted volume change during the consolidation is shown in Fig. 5*b*. The volume change during the drying phase is shown in Fig. 5*c*, which demonstrates that the shrinkability due to suction change depends on the stress, which is consistent with experimental observation by Richards et al. (1984), Huang (1994), and Delage and Graham (1996). Figure 5*c* also shows that a suction change at high degrees of saturation can cause a significant change in void ratio if the net mean stress is kept at a small value. Figure 5*d* shows the predicted SWCCs under different net stresses. The predicted shift of the SWCC due to the change of the initial void ratio is consistent with experimental observation by Huang (1994).

Isotropic compression under different suctions

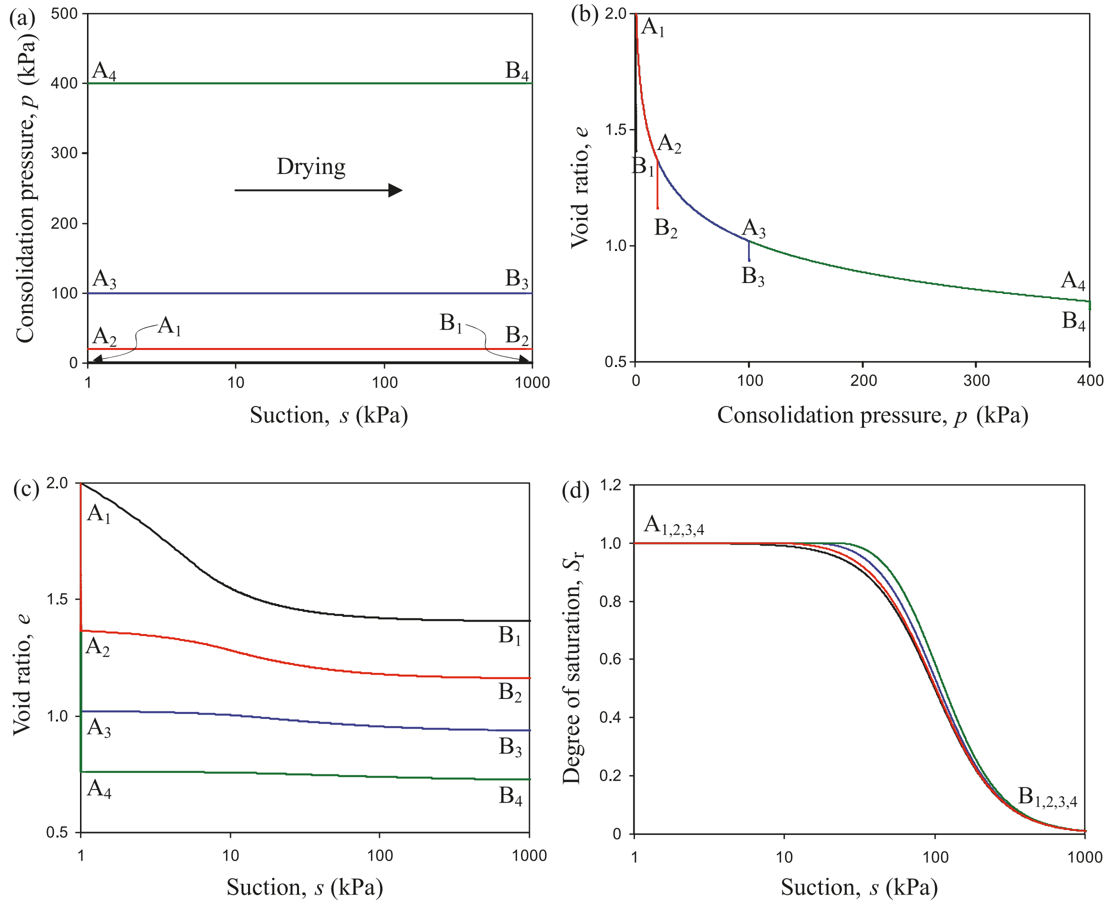
In the second example, we study the model prediction for another common laboratory test: isotropic compression under constant suction. Slurry specimens (consolidated to 1 kPa) are dried to different suctions such as 1 kPa (point A_1 in Fig. 6*a*), 20 kPa (point A_2), 100 kPa (point A_3), and 200 kPa (point A_4). The dried specimens are then compressed to points B_1, B_2, B_3, or B_4 in Fig. 6*a*, respectively. The stress paths are shown in Fig. 6*a*. The material and fitting parameters are kept the same as in the first example. Figure 6*b* shows the predicted compression curves (void ratio versus net mean stress). These curves are all normal compression lines, as the recompression index (κ_{vp}) is not used in the calculation. The saturation suction, which is required for volume calculation, is assumed to be 10 kPa. Figure 6*c* shows the SWCC (degree of saturation versus suction) and the change of S_r due to the isotropic compression. Figure 6*d* shows the predicted relationship between degree of saturation and net mean stress. The predicted variations of volume and degree of saturation in Fig. 6 are generally reasonable.

Isotropic compression under undrained conditions

One of the motivations of the study presented in this paper is due to a recent a discussion (Sheng et al. 2008*b*; Zhang and Lytton 2008), where it was noted that some existing models fall short in predicting undrained behaviour of unsaturated soils. The specific issue noted was related to the suction and saturation changes during undrained isotropic compression.

The issue raised in the discussion (Zhang and Lytton 2008) is illustrated in Fig. 7. Assume the initial state of a soil is inside the elastic zone, but on the main wetting curve, i.e., point A in Fig. 7. Compressing the soil under undrained conditions will lead to some suction decrease, say to point B. Assume B is still inside the elastic zone. Unloading from B

Fig. 5. Drying tests under different net stresses: (*a*) stress paths; (*b*) void ratio versus consolidation pressure; (*c*) void ratio versus suction; (*d*) SWCCs under different net stresses.



to A will then recover the initial volume of the soil at A, and hence the initial degree of saturation should be recovered as well. However, the S_r change along path AB follows the main wetting curve and is hence "elastoplastic," whereas the S_r change along path BA follows the scanning curve and is hence "elastic." It would seem unlikely that a model where the irreversible volume change is not synchronized with the irreversible saturation change could lead to a consistent prediction of saturation change over the closed path ABA (Zhang and Lytton 2009*a*, 2009*b*). In the following analysis, we show that this inconsistency is actually due to the assumption that SWCCs are defined for constant volume. If SWCCs are defined for constant stress (as in the model presented here), the inconsistence in Fig. 7 can be avoided.

In the model proposed in this paper, the intrinsic S_r–ε_v relationship for an undrained condition, i.e., eq. [10], is satisfied. Imposing eq. [10] on eqs. [36] and [37] leads to

$$\text{[40]} \qquad \left(A - \frac{n}{S_r}F\right)\mathrm{d}\bar{p} = \left(E\frac{n}{S_r} - B\right)\mathrm{d}s$$

The above equation actually imposes a constraint on the change of suction for isotropic compression under undrained conditions. We note that both terms in the parentheses on the right-hand side are negative, as $E \le 0$ and $B > 0$. Therefore, $\mathrm{d}s \le 0$ when $\mathrm{d}\bar{p} > 0$, i.e., the suction decreases during undrained isotropic compression. The change of degree of saturation can be found from eq. [37]

Both terms on the right-hand side of eq. [37] are positive. Therefore, the increase in the degree of saturation is larger than the corresponding increase caused by suction decrease along the SWCC.

Because eq. [10] is satisfied, the model will predict no saturation change as long as the volumetric strain is zero, irrespective of plastic yielding. A quantitative analysis of undrained compression requires the definition of yield surfaces like the loading-collapse yield surface and the suction-decrease and suction-increase surfaces, which is outside the scope of this paper. However, it is easy to understand that the loading path ACB is not on the initial main wetting curve and the unloading path BDA is not on the scanning curve because the mean stress is not constant. The initial main wetting curve at point A also changes to that at point B, as the mean stress changes. Equation [10] is satisfied as long as the suction changes according to eq. [40], leading to a closed S_r–s loop for a closed ε_v–$\bar{p}$. The synchronicity between the evolution of the loading-collapse surface and the evolution of the suction-increase and suction-decrease surfaces is thus not necessary, at least not for the undrained compression case studied here. Indeed, the suction path can

Fig. 6. The modelling of isotropic loading under constant suction: (*a*) stress path; (*b*) void ratio versus net mean stress; (*c*) degree of saturation versus suction; (*d*) degree of saturation versus net mean stress.

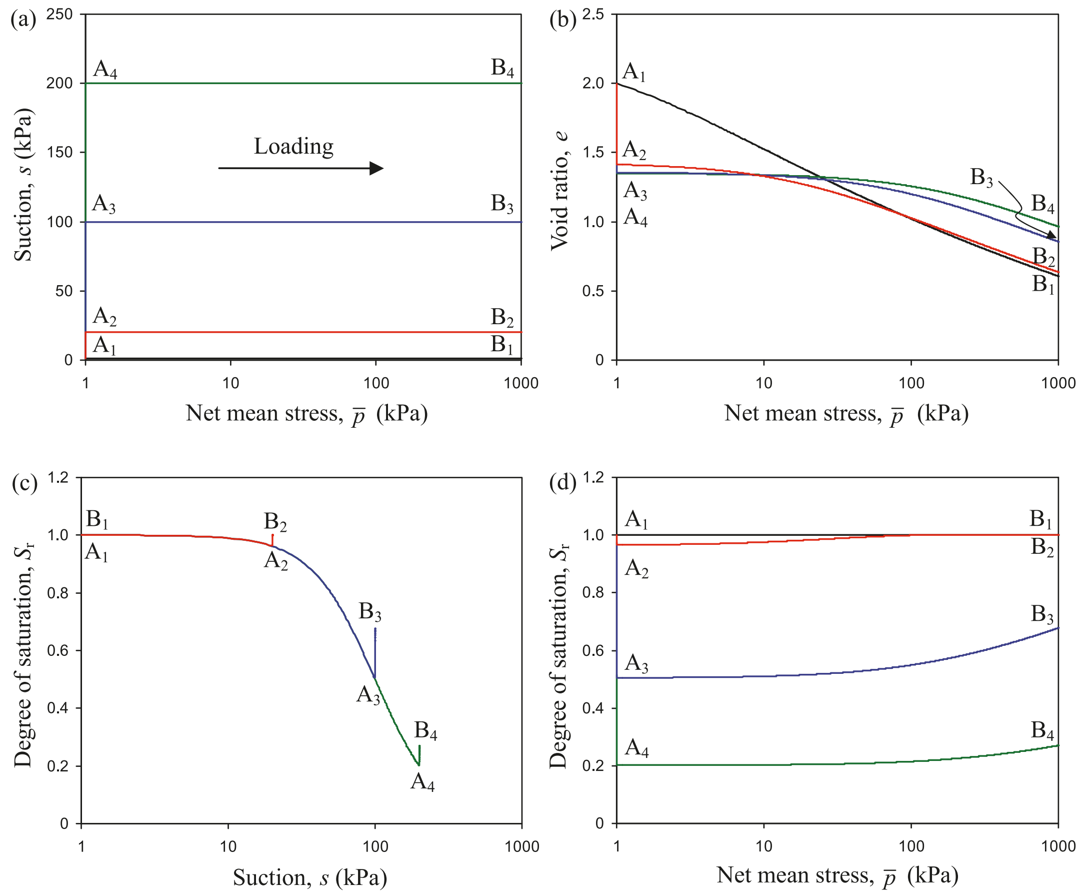
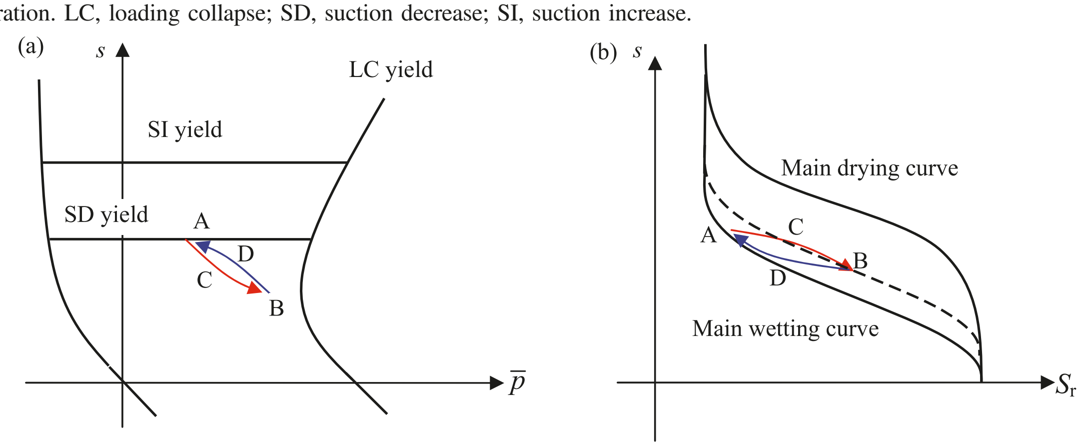


Fig. 7. Qualitative analysis of isotropic compression under undrained conditions: (*a*) stress path in net stress – suction space; (*b*) change of degree of saturation. LC, loading collapse; SD, suction decrease; SI, suction increase.



be elastoplastic albeit the elastic stress path. Experimental data do not support the synchronicity either (see recent discussion on “double yielding” in Raveendiraraj 2009).

Validation of the model

Isotropic compression of bentonite–kaolin

Sharma (1998) presented a series of isotropic compression test results for bentonite–kaolin specimens that were made of 20% bentonite and 80% kaolin. The tests were conventional suction-controlled oedometer tests where the axis-translation technique (Hilf 1956) is used to control suction. Water inflow and outflow are allowed to sustain the constant-suction levels. The matric suction is applied by the difference between the air pressure and water pressure. One of the series (series 2, with a suction of 200 kPa) was used to calibrate the proposed model, and the simulation was shown in Fig. 3 above. The results of other two series (series 1 and 3, with a suction

Fig. 8. Measured and predicted hydromechanical behaviour of bentonite–kaolin, s = 100 kPa (series 1) (data after Sharma 1998): (*a*) specific volume versus net mean stress; (*b*) degree of saturation versus net mean stress.

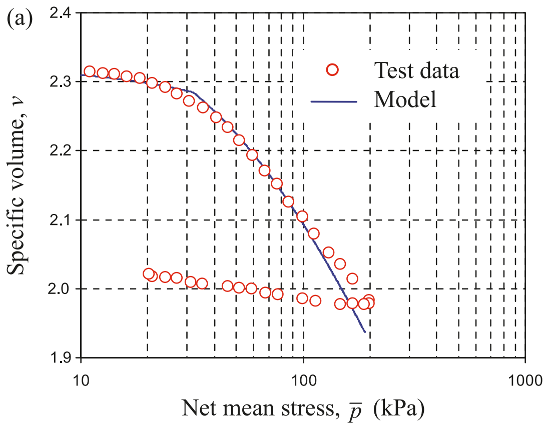
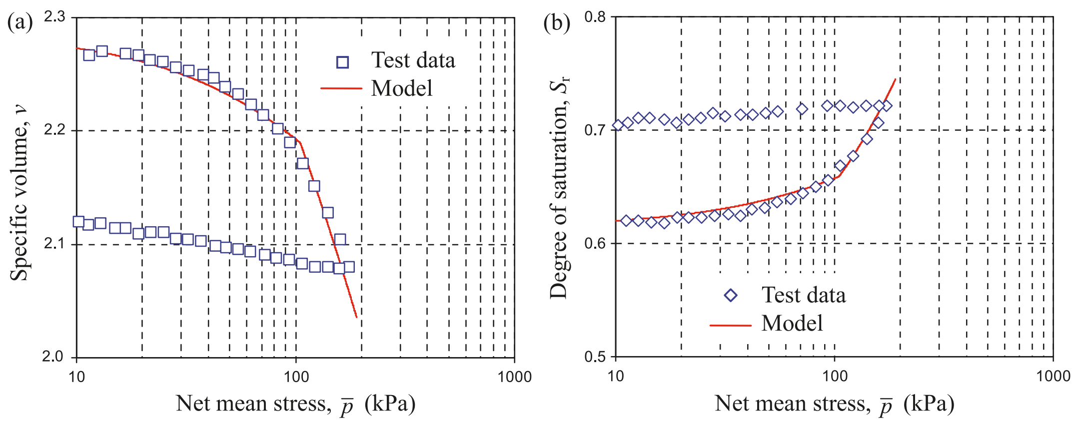


Fig. 9. Measured and predicted hydromechanical behaviour of bentonite–kaolin, s = 300 kPa (series 3) (data after Sharma 1998): (*a*) specific volume versus net mean stress; (*b*) degree of saturation versus net mean stress.



of 100 and 300 kPa, respectively) are used here to validate the model. The values of the parameters are kept the same as those in Fig. 3, i.e., λ_{vp} = 0.21, κ_{vp} = 0.06, and m = 0.01. In the case of series 1, the preconsolidation pressure is also set to 40 kPa. The predicted changes of volume and degree of saturation are compared with Sharma's data in Figs. 8*a* and 8*b*, respectively. It is shown that the prediction is reasonable. We also note that the prediction can be further improved if λ_{vp} is allowed to vary with suction.

In the case of series 3, where the suction was kept at 300 kPa, the preconsolidation pressure is set to 105 kPa. The predictions by the proposed model are compared with Sharma's data in Figs. 9*a* and 9*b*, respectively, for the change of volume and the change of degree of saturation. Again, the predictions are in a reasonable agreement with the test data.

Isotropic compression and triaxial tests of pearl clay

Sun et al. (2004, 2007*a*, 2007*b*, 2008) presented a series of isotropic compression tests on statically compacted pearl clay. Pearl clay is an industrial waste material with a moderate plasticity and very little expansive clay minerals. The main minerals are quartz, pyrophyllite, and kaolinite, in the order of dominance.

Figure 10 shows the results of isotropic compression tests by Sun et al. (2007*a*). In these tests, pearl clay specimens were compacted to different void ratios (e_0 = 1.17 to ~1.73). The suction was kept constant during isotropic compression (s = 147 kPa). The predicted compression curves are compared with the data in Fig. 10*a*. All predictions are performed using eq. [36], with one set of material parameters: λ_{vp} = 0.2, κ_{vp} = 0.06. The preconsolidation pressures for different initial void ratios are assumed as 25, 220, 420, and 1200 kPa, respectively, for e_0 = 1.73, 1.39, 1.28, and 1.17. It is shown that the volume change of the pearl clay can be well predicted by the proposed model. Equation [37] is used to predict the change of degree of saturation during isotropic compression. The fitting parameter, m, is set 0.25. As shown in Fig. 10*b*, the degree of saturation predicted by the proposed model is in a reasonable agreement with the experimental results.

Sun et al. (2007*b*) presented the experimental relationship between degree of saturation and void ratio based on isotropic compression tests and triaxial tests on pearl clay with different initial void ratios. During these tests, the suction level is kept as a constant (s = 147 kPa). For the two isotropic compression tests, the initial void ratios are 1.40 and 1.24, respectively. For the two triaxial tests, the initial void ratios are 1.65 and 1.34, respectively. The tests results are replotted in Fig. 11*a*, while the stress paths for all tests are illustrated in Fig. 11*b*. For the compacted pearl clay, the fitting parameter, m, is set to 0.25, same as above. The model simu-

Fig. 10. Isotropic compression tests with different initial void ratios (data after Sun et al. 2007*a*, 2007*c*): (*a*) predicted and measured void ratio versus net mean stress; (*b*) predicted and measured degree of saturation versus net mean stress.

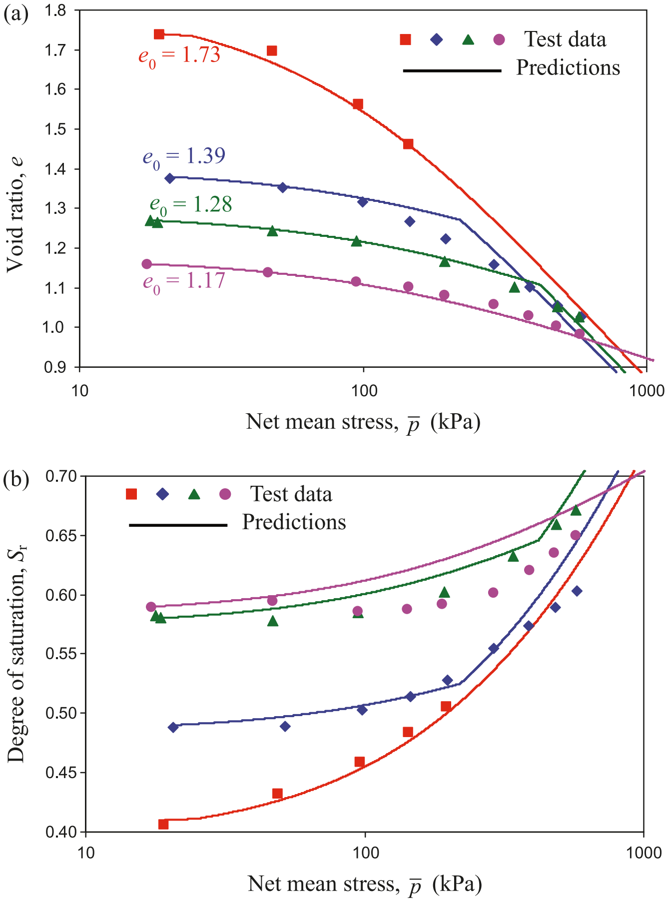
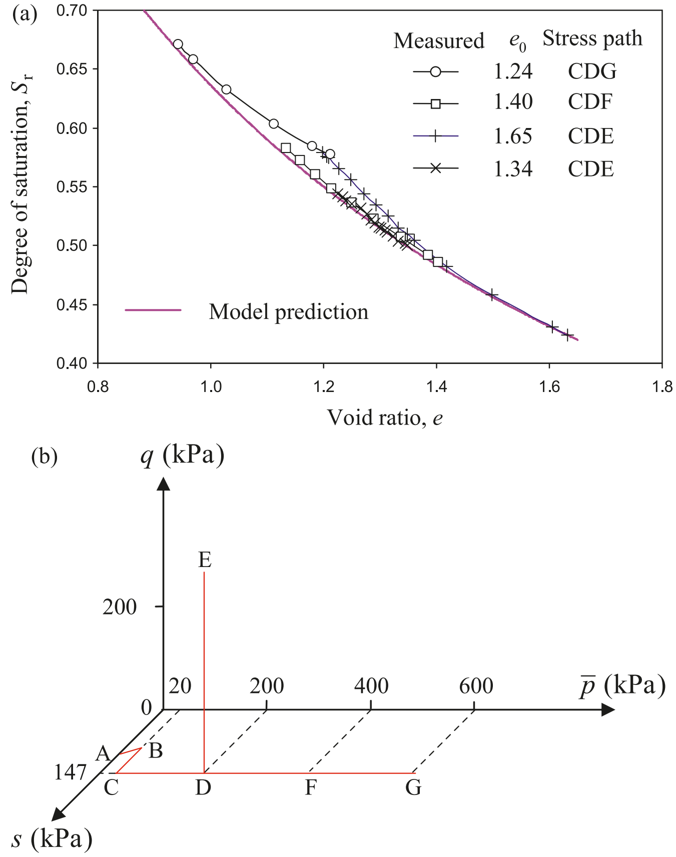


Fig. 11. (*a*) Measured and predicted relationship between degree of saturation and void ratio in both isotropic compression and triaxial tests. (*b*) Stress paths for all tests. (Data for (*a*) and (*b*) after Sun et al. 2007*b*.) *q*, deviator stress.



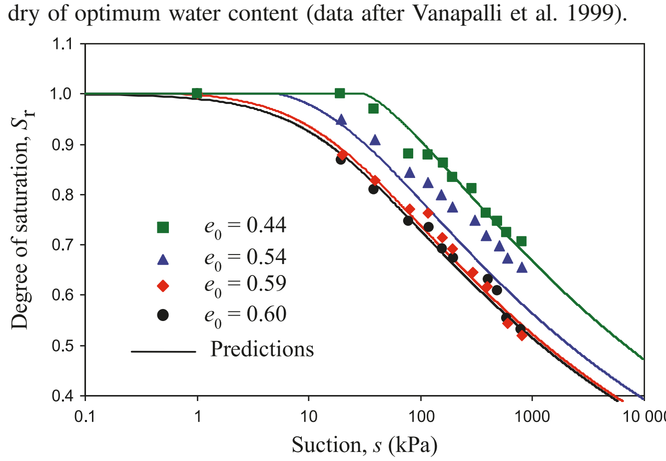
lation (eq. [27]) is also plotted in Fig. 11*a*. The comparison between predicted and measured results confirms the validation of the proposed model.

Drying test with different initial void ratios

Vanapalli et al. (1999) presented a series of drying tests to investigate the influence of stress history on the SWCC of a compacted till. The tested soil, a sandy till, consists of 28% sand, 42% silt, and 30% clay. In one group of the tests, specimens were compacted dry of optimum water content, to different initial void ratios. The SWCCs during the drying tests are shown in Fig. 12. As reported by Vanapalli et al. (1999), the calculations of the degree of saturation were made with reference to the initial void ratio because the change in void ratio with respect to the suction change was not found to be significant for this sandy clay till. It is shown that decreasing the initial void ratio shifts the SWCC to high suctions. The predicted SWCCs are compared with the data in Fig. 12. The prediction of the reference SWCC (e_0 = 0.6) is based on the van Genuchten equation, with a = 16 kPa, α = 1.0, β = 0.16. The fitting parameter, m, is set to 0.2. The predictions by the proposed model are in a good agreement with the test data.

In another group of tests, specimens were compacted to different initial void ratios at optimum water content. Figure 13 shows the drying test results of these specimens,

Fig. 12. Measured and predicted SWCCs for specimens compacted dry of optimum water content (data after Vanapalli et al. 1999).



with different symbols for different initial void ratios. The predicted SWCCs are shown as solid curves. The parameters used in the prediction are as follows: a = 65 kPa, α = 1.0, β = 0.15, and m = 0.03. The predicted SWCCs compare very well with the measured data, indicating that the proposed model can well capture the influence of the initial void ratio on the SWCC.

Fig. 13. Measured and predicted SWCCs for specimens compacted at optimum water content (data after Vanapalli et al. 1999).

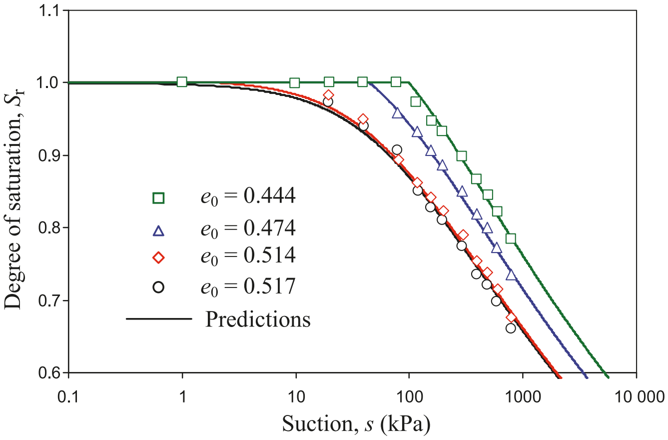
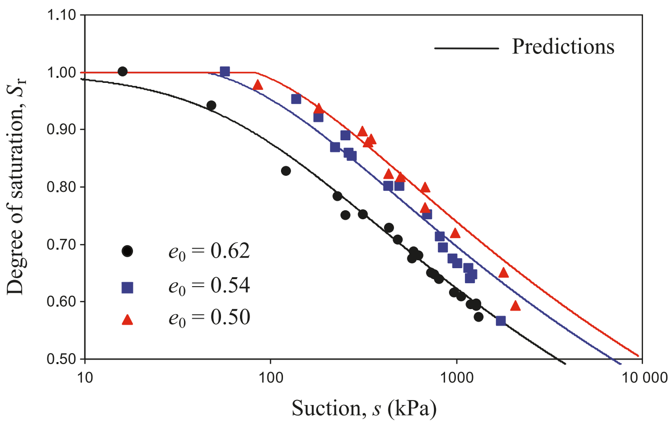


Fig. 14. Measured and predicted SWCCs of reconstituted Barcelona silt with different initial void ratios (data after Tarantino 2009).



Most recently, a series of laboratory SWCC tests under zero vertical stress on reconstituted Barcelona silt with different initial densities was reported by Tarantino (2009). Barcelona silt is a clayey silt. Dry powder of Barcelona silt was mixed with demineralised water to prepare a slurry, which was then consolidated to vertical stresses of 100, 300, or 500 kPa to obtain samples with different initial void ratios. The initial void ratios (e_0) at the start of drying process were 0.62, 0.54, and 0.50 for the samples consolidated to 100, 300, and 500 kPa, respectively. The SWCCs of the reconstituted Barcelona silt during the drying tests as well as the predictions are shown in Fig. 14. The parameters used in the prediction are as follows: a = 75 kPa, α = 1.2, β = 0.15, and m = 0.2. The predicted curves by eq. [32] compare very well with the measured data for different initial densities.

Conclusions

In constitutive models for coupled hydromechanical behaviour of unsaturated soils, the mechanical component that defines the stress–strain relation and the hydraulic component that defines the saturation–suction relation interact with each other. The influence of the hydraulic component on the mechanical component has been emphasised in early models. A handful of recent models also consider the influence of the mechanical component on the hydraulic component. In these recent models, it is common to link the air-entry value to the specific volume or volumetric strain. This paper proposes an alternative method to couple the mechanical component with the hydraulic component. The mechanical component is represented by a general equation where the volumetric strain is attributed to stress and suction variations. The hydraulic component is represented by a general equation where the variation of the degree of saturation is attributed to suction and soil volume variations. Realising that the SWCC is obtained under constant stress instead of constant volume, it is proposed that the SWCC equation is influenced by the volumetric strain caused by stress changes only. The coupling between the mechanical equation and the hydraulic equation is then realised based on the SFG model, the intrinsic relationship between degree of saturation and void ratio, and experimental observation. In the coupled model, both the air-entry value and the slope of the SWCC can be affected by the initial void ratio of the soil.

The parameters used in the proposed model include two compression indices, an equation for the SWCC, and a fitting parameter. As the compression indices and the SWCC equation are ordinary in mechanical and hydraulic models for unsaturated soils, the only new parameter introduced into the model is the fitting parameter. Numerical examples show that the proposed model gives reasonable predictions of the changes of soil volume and degree of saturation due to changes of suction and stress. The model is also validated against published experimental data.

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List of symbols

A general function defining the relation between volumetric strain and net stress under constant suction
a parameters for the van Genuchten function (van Genuchten 1980)
B general function defining the relation between volumetric strain and suction under constant net stress
C general function defining the relation between degree of saturation and suction under constant volume
D general function defining the relation between degree of saturation and volumetric strain under constant suction
E general function defining the relation between degree of saturation and suction under constant net stress
e void ratio
e_0 initial void ratio
e_{r0} initial void ratio for reference SWCC
e_w water ratio
F general function defining the relation between degree of saturation and net stress under constant suction
G_s specific gravity
k a real positive number in function $f(s) = S_r^k s$
M_s mass of solids
M_w mass of pore water
m new fitting parameter for hydromechanical coupling
n soil porosity
$\bar{p}$ net mean stress
S_r degree of saturation
S_r^{SWCC} experimental or simulated SWCC equation obtained from conventional water retention tests
S_{r0} reference SWCC equation
s matric suction
s^* modified suction (Wheeler et al. 2003)
s_{ae} air-entry value
s_{re} residual suction
s_{sa} saturated suction
s_{we} water-entry value
u_a pore-air pressure
u_w pore-water pressure
V total volume of a soil element
V_s volume of soil particles in a soil element
V_v volume of void in a soil element
V_w volume of pore water in a soil element
v specific volume
w gravimetric water content
α parameters for the van Genuchten function (van Genuchten 1980)
β parameters for the van Genuchten function (van Genuchten 1980)
δ_{ij} Kronecker delta
ε_v volumetric strain
ε_{vp} volumetric strain purely due to net stress change
θ volumetric water content
κ_{vp} elastic compression index due to net stress in SFG model (Sheng et al. 2008*a*)
κ_{vs} elastic compression index due to suction in SFG model (Sheng et al. 2008*a*)
κ_{ws} fitting parameter for the SWCC equation in SFG model (Sheng et al. 2008*a*)
λ_{se} slope of degree of saturation versus void ratio curve under constant suction (Sun et al. 2008)
λ_{sr} slope of the main drying or wetting curve (Sun et al. 2008)
λ_{vp} elastoplastic compression index due to net stress in SFG model (Sheng et al. 2008*a*)
λ_{vs} elastoplastic compression index due to suction in SFG model (Sheng et al. 2008*a*)
λ_{ws} fitting parameter for the SWCC equation in SFG model (Sheng et al. 2008*a*)
σ_{ij} total stress
σ_{ij}^* average soil skeleton stress