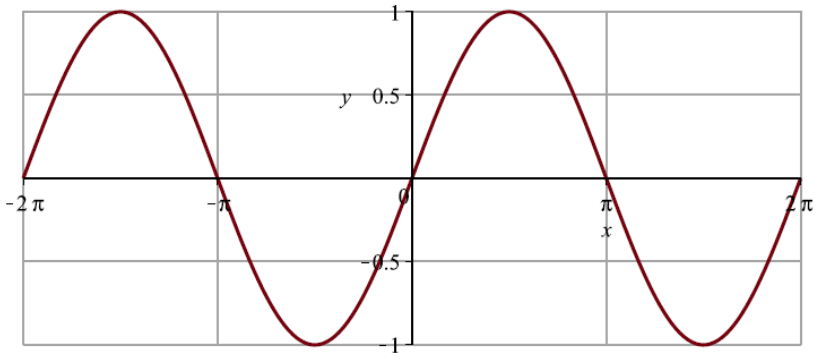
MATHS AND STATS

Graphing sine functions

The most basic sine function is $f(x) = \sin(x)$. Its graph is given below and continues on in the same way to the left and the right.



Vertical modifications

A general sine function has the form $f(x) = A\sin(Bx + C) + D$.

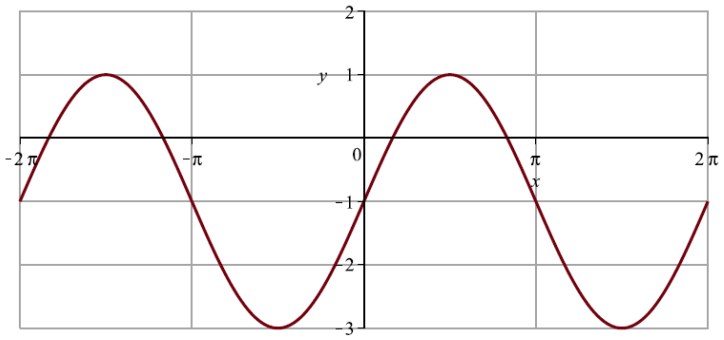
- Here $|A|$ is the amplitude,
- $Bx + C$ is called the argument of sine and
- D is the vertical shift.

The value of D gives the midline of the graph $y = D$ (the line the graph oscillates about). An increase in $|A|$ stretches the graph of sine vertically, whereas a decrease in $|A|$ compresses the graph vertically.

Example

Graph $f(x) = 2\sin x - 1$.

- The amplitude is 2 so we stretch a standard sine graph vertically by a factor of 2.
- The midline is $y = -1$ so the shift the graph down one place.



Horizontal modifications

The argument of sine is responsible for the horizontal modifications to the graph. Using our general sine function $f(x) = A\sin(Bx + C) + D$, the value of B affects the period of the graph and C affects the horizontal shift of the graph (known as phase shift).

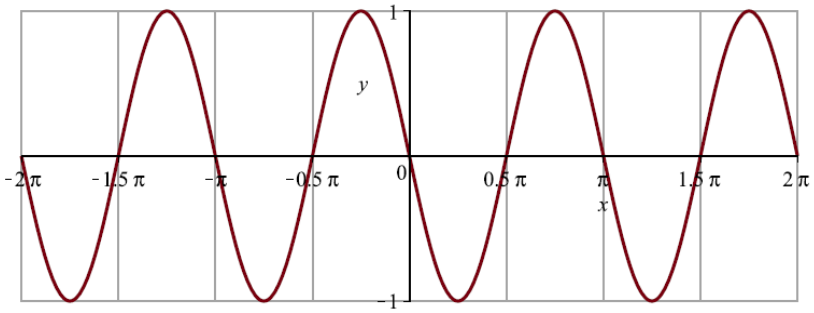
- The period is given by $\frac{2\pi}{B}$.
- The horizontal shift is given by $-\frac{C}{B}$

Notice that an increase in B results in a decrease in both the period and the shift. Also notice the negative sign means you might be shifting in the opposite direction to what you might expect.

Example

Graph $f(x) = \sin(2t + \pi)$.

The period is $\frac{2\pi}{2} = \pi$ so the graph repeats every π. The horizontal shift is $-\frac{\pi}{2}$ so every point on the graph will be moved $\frac{\pi}{2}$ to the left. The amplitude is 1 and the midline is $y = 0$.



Example

Graph $3\sin\left(\frac{\pi x}{2} - \frac{\pi}{2}\right) + 1$

- There is a midline of $y = 1$, and an amplitude of 3, so the graph will have a maximum of $1 + 3 = 4$ and a minimum of $1 - 3 = -2$.
- The period is $2\pi \div \frac{\pi}{2} = 2\pi \times \frac{2}{\pi} = 4$.
- The horizontal shift is $-\left(-\frac{\pi}{2}\right) \div \frac{\pi}{2} = 1$.