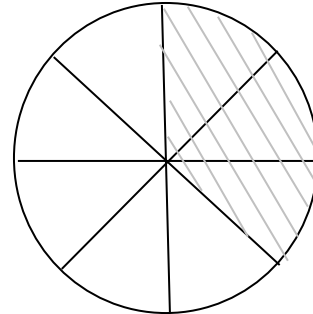# Fractions

With fractions it often helps to think in terms of cutting up a pizza or a block of chocolate.
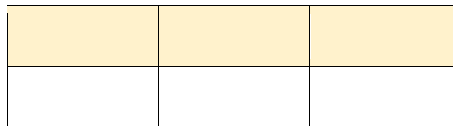
The bottom number (denominator) tells you how many (same sized) pieces you make from one whole pizza. The top number (numerator) tells you how many pieces you get to eat.

Example: $\frac{3}{8}$ of the pizza is shaded

Note that you cannot divide a pizza into 0 pieces, so the denominator should never be 0.

EQUIVALENT FRACTIONS

The shaded section of the chocolate block can be thought of as $\frac{1}{2}$ the block or as $\frac{3}{6}$ of the block. Clearly they are both the same amount of chocolate. So $\frac{1}{2} = \frac{3}{6}$ and we call them equivalent fractions. Similarly $\frac{4}{8}, \frac{6}{12}, \frac{5}{10}, \frac{10}{20}, \frac{100}{200}$ are all equivalent to $\frac{1}{2}$.

If you have a fraction where the numerator and the denominator have factors in common then you simplify the fraction by dividing the top and the bottom by the same number. This will reduce the fraction to simplest form (i.e. write it using the smallest numbers possible).

$\div 12$

$\frac{12}{36} = \frac{1}{3}$

$\div 12$

You can take a few steps to do this if you need.

$$\frac{12}{36} = \frac{4}{12} = \frac{1}{3}$$

Sometimes we need to write fractions using larger numbers so we reverse the process and multiply the top and the bottom of the fraction by the same number.

$$\frac{1}{3} \overset{\times 12}{\underset{\times 12}{=}} \frac{12}{36}$$

$$\frac{2}{3} \overset{\times 5}{\underset{\times 5}{=}} \frac{10}{15}$$

ADDING AND SUBTRACTING FRACTIONS

To add two fractions, they must have the same denominator, then we can add them as if you are counting objects. For example,

Just add the tops

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4} \quad \text{and} \quad \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

(Remember if the denominators are the same then we are talking about the same sized pieces so we can just add up how many pieces we have.)

Subtracting is the same except you subtract the numerators. For example,

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4} \quad \text{and} \quad \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

Example

If we want to add $\frac{1}{2} + \frac{1}{4}$ we are adding two different sized pieces of pizza. Before we can add them we need to write them both with the same denominator. This time we can use 4.

From our previous work with equivalent fractions we know that

$$\frac{1}{2} + \frac{1}{4} = \frac{1 \times 2}{2 \times 2} + \frac{1}{4}$$ Write ½ as a number of quarters

$$= \frac{2}{4} + \frac{1}{4}$$ Now add the numerators

$$= \frac{3}{4}$$

In this example we only had to change one denominator to match the other. Sometimes we have to change both of them.

Example

Find $\frac{1}{2} + \frac{2}{5}$

To find our new denominator we will multiply the two denominators together, 2×5 = 10.
Now convert both fractions to equivalent fractions with denominator 10

$$\frac{1}{2} + \frac{2}{5} = \frac{1 \times 5}{2 \times 5} + \frac{2 \times 2}{5 \times 2}$$
$$= \frac{5}{10} + \frac{4}{10}$$
$$= \frac{9}{10}$$

If one denominator will divide evenly into the other you only need to change the smaller denominator to match the larger one.

You can always find the new denominator by multiplying the two denominators together (as in the above example), but sometimes you can use a smaller number than this for your new denominator.

Now for the case when the two denominators have a common factor but neither divides evenly into the other. Here you use the lowest common multiple of the two denominators as your new denominator.

Example

15 = 3 × 5
12 = 3 × 4
New denominator = 3 × 5 × 4 = 60

15 × 4 = 60
So × 4

12 × 5 = 60
So × 5

$$\frac{2}{15} + \frac{7}{12} = \frac{2 \times 4}{15 \times 4} + \frac{7 \times 5}{12 \times 5}$$
$$= \frac{8}{60} + \frac{35}{60}$$
$$= \frac{43}{60}$$

MIXED NUMBERS AND IMPROPER FRACTIONS
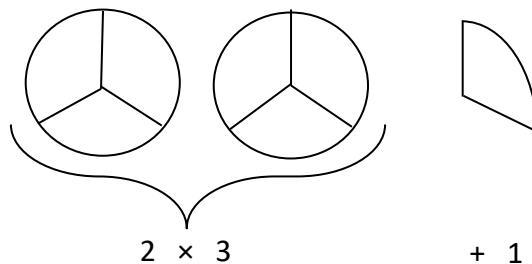
A mixed number is a combination of a whole number and a fraction, for example 2⅓. It makes it easy to picture how much you have.

An improper fraction is one where the numerator is bigger than the denominator, for example $\frac{7}{3}$. This form is easier to use when you have to do calculations with your fraction.

Sometimes we need to convert a mixed number into an improper fraction.

Example

$$2\frac{1}{3} = \frac{(2 \times 3) + 1}{3}$$
$$= \frac{7}{3}$$

2 × 3   + 1

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…….and converting in the other direction….

To convert an improper fraction into a mixed number you divide the numerator by the denominator and write the remainder as a fraction.

$$\frac{14}{3} = 4\frac{2}{3}$$

3 into 14 goes 4 times with 2 left over

MULTIPLYING AND DIVIDING FRACTIONS

To multiply two fractions, multiply the numerators together, and multiply the denominators together. For example,

$$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$$

$$= \frac{1}{6}$$

Remember to always write your answer in simplest terms

Particularly when you are working with big numbers it can help to simplify the fractions first

Example

$$\frac{3}{20} \times \frac{10}{2} = \frac{3}{2\,20} \times \frac{10}{2}^{1}$$

$$= \frac{3}{2} \times \frac{1}{2}$$

$$= \frac{3}{4}$$

Divide top and bottom by 10

Example

$$\frac{6}{50} \times \frac{5}{4} = \frac{6}{10\,50} \times \frac{5}{4}1$$

$$= \frac{6}{10} \times \frac{1}{4}$$

$$= \frac{6}{10}^{3} \times \frac{1}{4}2$$

$$= \frac{3}{10} \times \frac{1}{2}$$

$$= \frac{3}{20}$$

Divide top and bottom by 5

Divide top and bottom by 2

When multiplying a fraction and a whole number you treat the whole number like the top of a fraction. You can always write the whole number as a fraction over 1 to help you line it up in the right place. For example,

$$12 = \frac{12}{1} \qquad \text{or} \qquad 25 = \frac{25}{1}$$

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Example

$$\frac{2}{3} \times 60 = \frac{2}{3} \times \frac{60}{1}$$

$$= \frac{2}{3} \times \frac{\overset{20}{\cancel{60}}}{1}$$

$$= \frac{2}{1} \times \frac{20}{1}$$

$$= 40$$

DIVIDING BY A FRACTION

Dividing by a fraction is very simple. To divide two fractions, 'flip' the dividing fraction and then multiply.

Example

$$\frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \times \frac{5}{1}$$ Flip and multiply

$$= \frac{10}{3}$$

Example

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$ Flip and multiply

$$= \frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{1}$$ Divide top and bottom by 2

$$= \frac{3}{2} \times \frac{1}{1}$$

$$= \frac{3}{2}$$

EXERCISES

Equivalent fractions

1. Reduce these fractions to their simplest terms

(a) $\dfrac{2}{6}$

(b) $\dfrac{30}{50}$

(c) $\dfrac{3}{6}$

(d) $\dfrac{3}{15}$

(e) $\dfrac{15}{25}$

(f) $\dfrac{12}{48}$

(g) $\dfrac{16}{20}$

(h) $\dfrac{30}{600}$

(i) $\dfrac{150}{800}$

(j) $\dfrac{40}{120}$

(k) $\dfrac{75}{250}$

(l) $\dfrac{55}{100}$

2. Write equivalent fractions making the denominator 12

(a) $\dfrac{2}{6}$ (c) $\dfrac{3}{2}$ (e) $\dfrac{2}{3}$

(b) $\dfrac{3}{4}$ (d) $\dfrac{5}{6}$ (f) $\dfrac{1}{4}$

3. Write equivalent fractions with denominator 100

(a) $\dfrac{35}{50}$ (c) $\dfrac{4}{25}$ (e) $\dfrac{15}{20}$

(b) $\dfrac{3}{10}$ (d) $\dfrac{8}{10}$ (f) $\dfrac{13}{25}$

Adding and subtracting fractions

4. Calculate the following and write your answer in simplest terms
(this means using the smallest numbers possible)

(a) $\dfrac{3}{8}+\dfrac{1}{8}$ (c) $\dfrac{3}{5}+\dfrac{1}{5}$ (e) $\dfrac{1}{4}+\dfrac{3}{8}$

(b) $\dfrac{15}{20}-\dfrac{9}{20}$ (d) $\dfrac{7}{12}-\dfrac{2}{12}$

5. Calculate the following and write your answers in simplest terms

(a) $\dfrac{3}{4}+\dfrac{1}{3}$ (d) $\dfrac{3}{10}+\dfrac{1}{4}$ (g) $\dfrac{3}{4}-\dfrac{2}{3}$

(b) $\dfrac{3}{5}+\dfrac{1}{2}$ (e) $\dfrac{2}{3}-\dfrac{1}{2}$ (h) $\dfrac{9}{10}-\dfrac{2}{3}$

(c) $\dfrac{2}{3}+\dfrac{3}{10}$ (f) $\dfrac{1}{2}-\dfrac{2}{5}$

6. Calculate the following and write your answers in simplest terms

(a) $\dfrac{9}{10}+\dfrac{7}{18}$ (d) $\dfrac{3}{20}+\dfrac{7}{30}$ (g) $\dfrac{12}{21}-\dfrac{1}{6}$

(b) $\dfrac{5}{24}+\dfrac{2}{9}$ (e) $\dfrac{9}{20}-\dfrac{1}{8}$ (h) $\dfrac{13}{15}-\dfrac{3}{50}$

(c) $\dfrac{5}{12}+\dfrac{11}{21}$ (f) $\dfrac{13}{15}-\dfrac{5}{6}$

Improper fractions and mixed numbers

7. Convert to improper fractions

(a) $2\dfrac{3}{4}$ (d) $1\dfrac{1}{4}$ (g) $8\dfrac{3}{4}$

(b) $7\dfrac{1}{2}$ (e) $4\dfrac{2}{3}$ (h) $2\dfrac{3}{10}$

(c) $5\dfrac{2}{3}$ (f) $10\dfrac{2}{5}$

8. Convert to mixed numbers

(a) $\dfrac{11}{3}$

(b) $\dfrac{7}{2}$

(c) $\dfrac{23}{10}$

(d) $\dfrac{17}{4}$

(e) $\dfrac{25}{3}$

(f) $\dfrac{12}{5}$

(g) $\dfrac{13}{4}$

(h) $\dfrac{22}{3}$

Multiplying fractions

9. Multiply these fractions

(a) $\dfrac{3}{5} \times \dfrac{1}{2}$

(b) $\dfrac{15}{2} \times \dfrac{1}{20}$

(c) $\dfrac{5}{7} \times \dfrac{14}{3}$

(d) $\dfrac{15}{4} \times \dfrac{3}{5}$

(e) $\dfrac{3}{25} \times \dfrac{5}{3}$

(f) $\dfrac{6}{2} \times \dfrac{8}{12}$

(g) $\dfrac{1}{4} \times \dfrac{12}{3}$

(h) $\dfrac{30}{10} \times \dfrac{1}{5}$

(i) $\dfrac{500}{60} \times \dfrac{3}{7}$

(j) $\dfrac{5}{250} \times \dfrac{20}{4}$

10. Multiply fractions and whole numbers

(a) $\dfrac{3}{5} \times 50$

(b) $\dfrac{200}{800} \times 20$

(c) $\dfrac{80}{25} \times 50$

(d) $\dfrac{650}{50} \times 10$

(e) $\dfrac{8}{100} \times 360$

(f) $\dfrac{1}{2} \times 16$

(g) $\dfrac{500}{60} \times 12$

(h) $\dfrac{5}{250} \times 60$

(i) $\dfrac{8}{100} \times 70$

(j) $\dfrac{25}{100} \times 1600$

Dividing fractions

11.

(a) $\dfrac{15}{7} \div \dfrac{5}{14}$

(b) $\dfrac{12}{5} \div \dfrac{24}{15}$

(c) $\dfrac{3}{7} \div \dfrac{10}{21}$

(d) $\dfrac{16}{15} \div \dfrac{8}{35}$

ANSWERS

Equivalent fractions

1. (a) $\dfrac{1}{3}$ (b) $\dfrac{3}{5}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{5}$ (e) $\dfrac{3}{5}$ (f) $\dfrac{1}{4}$ (g) $\dfrac{4}{5}$ (h) $\dfrac{1}{20}$ (i) $\dfrac{3}{16}$ (j) $\dfrac{1}{3}$

(k) $\dfrac{3}{10}$ (l) $\dfrac{11}{20}$

2. Write equivalent fractions making the denominator 12

(a) $\dfrac{4}{12}$ (b) $\dfrac{9}{12}$ (c) $\dfrac{18}{12}$ (d) $\dfrac{10}{12}$ (e) $\dfrac{8}{12}$ (f) $\dfrac{3}{12}$

3. Write equivalent fractions with denominator 100

(a) $\dfrac{70}{100}$ (b) $\dfrac{30}{100}$ (c) $\dfrac{16}{100}$ (d) $\dfrac{80}{100}$ (e) $\dfrac{75}{100}$ (f) $\dfrac{52}{100}$

Adding and subtracting fractions

4. Calculate the following and write your answer in simplest terms

(a) $\dfrac{4}{8} = \dfrac{1}{2}$ (b) $\dfrac{6}{20} = \dfrac{3}{10}$ (c) $\dfrac{4}{5}$ (d) $\dfrac{5}{12}$ (e) $\dfrac{5}{8}$

5. Calculate the following and write your answers in simplest terms

(a) $\dfrac{13}{12}$ (b) $\dfrac{11}{10}$ (c) $\dfrac{29}{30}$ (d) $\dfrac{11}{20}$ (e) $\dfrac{1}{6}$ (f) $\dfrac{1}{10}$ (g) $\dfrac{1}{12}$ (h) $\dfrac{7}{30}$

6. Calculate the following and write your answers in simplest terms

(a) $\dfrac{116}{90} = \dfrac{58}{45}$ (b) $\dfrac{31}{72}$ (c) $\dfrac{79}{84}$ (d) $\dfrac{23}{60}$ (e) $\dfrac{13}{40}$ (f) $\dfrac{1}{30}$ (g) $\dfrac{17}{42}$ (h) $\dfrac{121}{150}$

Improper fractions and mixed numbers

7. Convert to improper fractions

(a) $\dfrac{11}{4}$ (b) $\dfrac{15}{2}$ (c) $\dfrac{17}{3}$ (d) $\dfrac{5}{4}$ (e) $\dfrac{14}{3}$ (f) $\dfrac{52}{5}$ (g) $\dfrac{35}{4}$ (h) $\dfrac{23}{10}$

8. Convert to mixed numbers

(a) $3\dfrac{2}{3}$ (b) $3\dfrac{1}{2}$ (c) $2\dfrac{3}{10}$ (d) $4\dfrac{1}{4}$ (e) $8\dfrac{1}{3}$ (f) $2\dfrac{2}{5}$ (g) $3\dfrac{1}{4}$ (h) $7\dfrac{1}{3}$

Multiplying fractions

9. Multiply these fractions

(a) $\dfrac{3}{10}$ (b) $\dfrac{3}{8}$ (c) $\dfrac{10}{3}$ (d) $\dfrac{9}{4}$ (e) $\dfrac{1}{5}$ (f) 2 (g) 1 (h) $\dfrac{3}{5}$ (i) $\dfrac{25}{7}$ (j) $\dfrac{1}{10}$

10. Multiply fractions and whole numbers

(a) 30   (b) 5   (c) 160   (d) 130   (e) $\dfrac{144}{5}$ (f) 8   (g) 100   (h) $\dfrac{6}{5}$ (i) $\dfrac{28}{5}$ (j) 400

Dividing fractions

11. (a) $\dfrac{12}{7}$ (b) $\dfrac{3}{2}$ (c) $\dfrac{9}{10}$ (d) $\dfrac{14}{3}$ (e) $\dfrac{1}{5}$