MATHS AND STATS

Differentiation Exercises II

1. Differentiate with respect to x:

$a.\ 5x^2$

$b.\ x^3 + 1$

$c.\ x - 5x^2 + 10x^3$

$d.\ (2x^3 + 4x)(x^2 - 1)$

$e.\ x^c + \frac{\sqrt{x}}{a} - \frac{3}{x}$

$f.\ \frac{2x}{x^2 - 4}$

$g.\ \frac{x^2 + 5x - 7}{x^2}$

$h.\ (4 + 5x)^6$

$i.\ \sqrt[4]{x^5}$

$j.\ (x^2 - 3)^8$

$k.\ 4x^4(3 - 5x)^3$

$l.\ \frac{7}{x^8} - \frac{1}{x^3} + ax^{-2}$

$m.\ \frac{1}{(2x-3)^3}$

$n.\ \frac{\sqrt{x}}{4 - x^2}$

2. If $f(x) = x^2 - 3x - 2$, evaluate:

$a.$ $f'(x)$

$b.$ The gradient of the function at $x = 2$

$c.$ Find when the gradient has a value of zero

$d.$ Find $f''(x)$

3. $a.$ For $y = x^3 + x^2$, find $\frac{dy}{dx}$ and explain what it is.

$b.$ Hence, or otherwise, find the point at which the gradient is 1.

Answers

1. Differentiate with respect to x:

$a.\ \frac{d}{dx}5x^2 = 10x$

$b.\ \frac{d}{dx}(x^3 + 1) = 3x^2$

$c.\ \frac{d}{dx}(x - 5x^2 + 10x^3) = 1 - 10x + 30x^2$

$d.\ \frac{d}{dx}(2x^3 + 4x)(x^2 - 1) = (2x^3 + 4x)2x + (x^2 - 1)(6x^2 + 4)$ by Product rule (PR)

$= 10x^4 + 6x^2 - 4$

$e.\ \frac{d}{dx}\left(x^c + \frac{\sqrt{x}}{a} - \frac{3}{x}\right) = \frac{d}{dx}\left(x^c + \frac{1}{a}x^{1/2} - 3x^{-1}\right)$

$= cx^{c-1} + \frac{1}{2a\sqrt{x}} + \frac{3}{x^2}$

$f.\ \frac{d}{dx}\left(\frac{2x}{x^2-4}\right) = \frac{(x^2-4)2 - 2x.2x}{(x^2-4)^2}$ by Quotient rule (QR)

$= \frac{-2x^2-8}{(x^2-4)^2}$

$g.\ \frac{d}{dx}\left(\frac{x^2+5x-7}{x^2}\right) = \frac{d}{dx}(1 + 5x^{-1} - 7x^{-2})$

$= -5x^{-2} + 14x^{-3}$

$h.\ \frac{d}{dx}(4 + 5x)^6 = 30(4 + 5x)^5$ by Chain rule (CR)

$i.\ \frac{d}{dx}\left(\sqrt[4]{x^5}\right) = \frac{d}{dx}x^{5/4}$
$= \frac{5}{4}x^{1/4}$
$= \frac{5}{4}\sqrt[4]{x}$

$j.\ \frac{d}{dx}(x^2-3)^8 = 16x(x^2-3)^7$ by CR

$k.\ \frac{d}{dx}4x^4(3-5x)^3 = -60x^4(3-5x)^2 + 16x^3(3-5x)^3$ by PR and CR

$l.\ \frac{d}{dx}\left(\frac{7}{x^8} - \frac{1}{x^3} + ax^{-2}\right) = \frac{d}{dx}(7x^{-8} - x^{-3} + ax^{-2})$
$= -56x^{-9} + 3x^{-4} - 2ax^{-3}$
$= \frac{-56}{x^9} + \frac{3}{x^4} - \frac{2a}{x^3}$

$m.\ \frac{d}{dx}\left(\frac{1}{(2x-3)^3}\right) = \frac{d}{dx}(2x-3)^{-3}$ by CR
$= -6(2x-3)^{-4}$
$= \frac{-6}{(2x-3)^4}$

$n.\ \frac{d}{dx}\left(\frac{\sqrt{x}}{4-x^2}\right) = \frac{(4-x^2)\frac{1}{2\sqrt{x}} - \sqrt{x}(-2x)}{(4-x^2)^2}$ by QR
$= \frac{\frac{2}{\sqrt{x}} + \frac{3}{2}x\sqrt{x}}{(4-x^2)^2}$

2. If $f(x) = x^2 - 3x - 2$, evaluate:
   $a.\ f'(x) = 2x - 3$
   $b.\ f'(2) = 2 \times 2 - 3 = 1$
   $c.\ f'(x) = 0 = 2x - 3$
   $x = 1\frac{1}{2}$
   $d.\ f''(x) = 2$

3. $a.$ For $y = x^3 + x^2$, find $\frac{dy}{dx}$ and explain what it is.
   $\frac{dy}{dx} = 3x^2 + 2x$, the gradient of the function at any point x

   $b.$ Hence, or otherwise, find the point at which the gradient is 1.

   $1 = 3x^2 + 2x$
   $0 = 3x^2 + 2x - 1$
   $0 = (3x-1)(x+1)$
   $x = \frac{1}{3}, \quad x = -1$
   Points:
   $(-1,0), \quad (\frac{1}{3}, \frac{4}{27})$