MATHS AND STATS

Solving basic equations

Equations come in many different forms. We will start with some simple equations.
Aim: To get the letter (e.g. x, y or z) on its own on the left hand side of the $=$ sign

Golden Rule: Do the same thing to both sides of the $=$ sign

One step equations

We solve these by picking something which will "undo" the operation being applied to the x, and applying it to both sides. Plus and minus are opposites, so are multiply and divide. These are called inverse operations.

You can check your solution to an equation by substituting your result back into the original equation.

Examples

$$x + 3 = 7$$
$$x + 3 - 3 = 7 - 3$$
$$x = 4$$

Subtract 3 from both sides

$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Divide both sides by 3

$$\frac{y}{4} = 5$$
$$\frac{y}{4} \times 4 = 5 \times 4$$
$$y = 20$$

Multiply both sides by 4

Exercises

1. $x + 5 = 13$
2. $5x = 40$
3. $x/7 = 3$
4. $y - 15 = 40$
5. $4z = 20$
6. $x/5 = 2$
7. $3y = 27$
8. $x - 6 = 11$

Answers

1. $x = 8$
2. $x = 8$
3. $x = 21$
4. $y = 55$
5. $z = 5$
6. $x = 10$
7. $y = 9$
8. $x = 17$

Two step equations

If more than one operation is being applied to the variable, we will need more than one inverse operation to undo them. We need to "unwrap" the x from the outside – the last thing we do to x needs to bethe first thing we undo.

Examples

1)
$$3x + 1 = 7$$
x is first multipled by 3, then the 1 is added last
$$3x + 1 - 1 = 7 - 1$$
Subtract 1 from both sides
$$3x = 6$$
$$\frac{3x}{3} = \frac{6}{3}$$
Divide by 3 on both sides
$$x = 2$$

2)
$$\frac{x}{6} - 2 = 3$$
Add 2 to both sides
$$\frac{x}{6} = 5$$
Multiply by 6 on both sides
$$x = 30$$

3)
$$3(x - 4) = -21$$
Divide both sides by 3
$$x + 4 = -7$$
Subtract 4 from both sides
$$x = -11$$

Note – the last example could also have been solved by expanding the brackets first. Both methods will give the same answer.

Multi-step equations

In more complex equations, we may have to think ahead to figure out how to get "$x =$" on its own. If there is more than one x in the equation, we will have to find a way to combine them, such as by collecting like terms.

The following examples show many different structures equations might have, and some differnet approaches to solving them.

Examples

1)
$$5x - 4 = 3x + 8$$
$-3x$ both sides
$$2x - 4 = 8$$
$+4$ both sides
$$2x = 12$$
Divide by 2 both sides
$$x = 6$$

2)
$$4(x - 2) = 3x + 5$$
Expand brackets
$$4x - 8 = 3x + 5$$
$-3x$ both sides
$$x - 8 = 5$$
$+8$ both sides
$$x = 13$$

3)
$$3(2x - 4) + 5(x - 5) = -4$$
Expand brackets
$$6x - 12 + 5x - 25 = -4$$
Collect like terms
$$11x - 37 = -4$$
$+37$ both sides
$$11x = 33$$
Divide by 11 both sides
$$x = 3$$

4)

$$\frac{12}{x} + 5 = 9$$ -5 both sides

$$\frac{12}{x} = 4$$ Multiply by x both sides

$$x \times \frac{12}{x} = 4 \times x$$

$$12 = 4x$$ Divide by 4 both sides

$$x = 3$$

5)

$$\frac{x}{4} + \frac{2x}{3} = 11$$ Make common denominator

$$\frac{3 \times x}{3 \times 4} + \frac{2x \times 4}{3 \times 4} = 11$$

$$\frac{3x}{12} + \frac{8x}{12} = 11$$ Multiply by 12 both sides

$$3x + 8x = 132$$ Collect like terms

$$11x = 132$$ Divide by 11 both sides

$$x = 12$$

6)

$$\frac{2x+2}{3} = \frac{4x-8}{2}$$ Make common denominator

$$\frac{2 \times (2x+2)}{2 \times 3} = \frac{(4x-8) \times 3}{2 \times 3}$$ Expand brackets

$$\frac{4x+4}{6} = \frac{12x-24}{6}$$ Multiply by 6 both sides

$$4x + 4 = 12x - 24$$ $+24$ both sides

$$4x + 28 = 12x$$ $-4x$ both sides

$$28 = 8x$$ Divide by 8 both sides

$$\frac{28}{8} = x$$ Simplify fraction and swap sides

$$x = 3\frac{1}{2}$$

Exercises

1. $6x - 9 = 27$
2. $3x + 8 = 7x - 10$
3. $4(x - 5) = 3x + 8$
4. $2(x - 3) = 4(2x + 6)$
5. $\frac{5x-8}{3} = 4$
6. $\frac{x}{5} = 2x + 9$
7. $\frac{x-3}{4} = \frac{x}{2} - 2$

Answers

1. $x = 6$
2. $x = \frac{18}{4} = 4\frac{1}{2}$
3. $x = 28$
4. $x = -5$
5. $x = 4$
6. $x = -5$
7. $x = 5$