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Prediction of Pressure and Density Dependent Failure in Sand-like Granular Materials under General Stress Conditions

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Abstract. This paper presents a theoretical study of failure modes in sand-like granular materials under general triaxial stress conditions based on a hypoplastic model and a bifurcation analysis. The well developed constitutive model contains the void ratio as a state variable which allows the pressure and density dependent behaviour of the granular materials to be described with a single set of state-independent constitutive parameters. Based on this model a shear bifurcation condition is derived. Loading along various stress paths that can be achieved with a true triaxial test apparatus are simulated numerically. Either localized failure or uniform diffuse failure in granular samples may occur, which is determined based on whether the shear bifurcation condition is met. Effects of confining pressure and initial density are discussed.

Introduction

Laboratory testing is crucial for calibration of constitutive models. A problem often encountered in tests of granular materials is the occurrence of shear localization: shear strain concentrated in narrow zones known as shear bands, leading to a heterogeneous deformation in the granular sample. As a consequence, the testing results no longer represent a material response and therefore cannot be used to calibrate a constitutive model, particularly when softening is modelled.

Shear localization has been observed in biaxial compression tests and triaxial extension tests ([1], [2]) and sometimes in triaxial compression tests [3]. In a more careful experimental study with a true triaxial testing apparatus, Wang and Lade [4] showed that either localized failure or uniform diffuse failure may occur in testing samples, depending on the intermediate principal stress. The uniform diffuse failure may be obtained in tests along stress paths with a low Lode angle, including triaxial compression test. This result is important as it indicates that constitutive models describing softening behaviour may be calibrated with laboratory data from triaxial compression tests.

This paper presents a theoretical analysis of the possible mode of failure in sand-like granular materials under general stress conditions. The material is described with a hypoplastic model proposed by Gudehus [5] and Bauer [6]. The constitutive model involves the current stress and void as state variables and can describe the pressure- and density-dependent behaviour with a single set of state-independent parameters. The condition of emergence of shear localization in a uniformly deforming granular material is obtained from bifurcation analysis following the concept developed by Rudnicki and Rice [7]. Numerical tests are performed to simulate the evolution of stress and density in granular samples during loading along various stress paths that can be achieved with a true triaxial testing apparatus. The mode of failure is determined based on whether the bifurcation condition is met during loading. The effects of confining pressure and initial density are discussed by assigning different initial conditions in the tests.

Hypoplastic description of granular materials

In the hypoplasticity, the irreversible material behaviour is described in terms of stress rate $\dot{\sigma}$ as a nonlinear function of total strain rate $\dot{\varepsilon}$ without decomposing it into an elastic part and a plastic

part. Granular materials, such as sand, exhibit a strong pressure- and density- dependent behaviour. The shear strength is proportional to the confining pressure. When sheared, a loose granular material shows a strain hardening response with a volumetric contraction, whereas a dense granular material displays a strain softening response with a volumetric dilatation. Such complicated behaviour can be well described with the following constitutive equations ([5] [6]):

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= f_s[\hat{a}^2\dot{\boldsymbol{\varepsilon}} + \hat{\boldsymbol{\sigma}}(\hat{\boldsymbol{\sigma}}:\dot{\boldsymbol{\varepsilon}}) + f_b\hat{a}(\hat{\boldsymbol{\sigma}}+\hat{\boldsymbol{\sigma}}^d)\sqrt{\dot{\boldsymbol{\varepsilon}}:\dot{\boldsymbol{\varepsilon}}}\,], \\ \dot{e} &= (1+e)\,\mathrm{tr}\,\dot{\boldsymbol{\varepsilon}}. \end{aligned} \tag{1}$$

Here the evolution of stress $\boldsymbol{\sigma}$ and void ratio e are defined. $\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/\mathrm{tr}\boldsymbol{\sigma}$ is the normalised stress tensor and $\hat{\boldsymbol{\sigma}}^d = \hat{\boldsymbol{\sigma}} - \mathbf{1}/3$ is its deviator with $\mathbf{1}$ being a unit tensor of rank 2, $\mathrm{tr}\dot{\boldsymbol{\varepsilon}}$ represents the trace of $\dot{\boldsymbol{\varepsilon}}$. Parameter $\hat{a}$ is a related to the critical shear strength of the material. Different limit stress conditions can be incorporated into this model with a proper interpolation for $\hat{a}$ with respect to the Lode angle θ. In the present study, the following interpolation for $\hat{a}$ is employed, which incorporates the Matsuoka-Nakai limit condition:

$$\hat{a} = \sqrt{\frac{8}{3}}\frac{\sin\varphi_c}{3+\sin\varphi_c}\left(\sqrt{\frac{3}{8}\|\hat{\boldsymbol{\sigma}}^d\|^2 + (1-\frac{3}{2}\|\hat{\boldsymbol{\sigma}}^d\|^2)\big/(1-\sqrt{\frac{3}{2}}\|\hat{\boldsymbol{\sigma}}^d\|\cos(3\theta))} + \sqrt{\frac{3}{8}}\|\hat{\boldsymbol{\sigma}}^d\|\right), \tag{2}$$

where φ_c denotes the critical friction angle, $\|\hat{\boldsymbol{\sigma}}^d\|$ is defined as $\|\hat{\boldsymbol{\sigma}}^d\| = \sqrt{\hat{\boldsymbol{\sigma}}^d:\hat{\boldsymbol{\sigma}}^d}$.

The scalar factors f_s and f_d in Eq. (1) define the pressure and density dependence behaviour of the model. They are given as functions of the mean pressure $p = -\mathrm{tr}\boldsymbol{\sigma}/3$ and the void ratio e in the forms of $f_s = (1/\hat{\boldsymbol{\sigma}}:\hat{\boldsymbol{\sigma}})(e_i/e)^{\beta}(3p/h_s)^{1-n}f_b^*$, and $f_d = I_d^{\alpha}$. Here f_b^* is a coefficient that can be determined from the consistency condition for isotropic compression [5], and I_d is a pressure-dependent density index defined as:

$$I_d = \frac{e-e_d}{e_c-e_d}, \tag{3}$$

where e_i, e_d and e_c represent the void ratios at the loosest state, the densest state and the critical state, respectively. Note that the density of a granular material under a given pressure is not unique. The void ratio e of a granular sample can vary between a maximum value e_i and a minimum value e_d, depending on how the sample is prepared. In large shearing, the void ratio of a material sample tends to a critical void ratio e_c. These void ratios, however, also vary with the mean pressure p applied on the material sample, which can be described by the following relations:

$$e_c/e_{co} = e_d/e_{do} = e_i/e_{io} = \exp[-(3p/h_s)^n]\ . \tag{4}$$

Here h_s and n are two parameters determines the degradation of void ratios with respect to the mean pressure. e_{io}, e_{do} and e_{do} represent, respectively, the loosest, the densest and the critical void ratios at a nearly stress-free state. Note that the pressure-dependent density index I_d defined in Eq. (3) decreases as the density increases, with $I_d = 0$ for $e = e_d$, $I_d = 1$ for $e = e_c$ and $I_d > 1$ for looser states. In the following numerical simulation, the initial void ratio is determined from the initial density I_{d0} at a given confining pressure p_0.

The model contains eight material parameters, namely, e_i, e_d, e_c, h_s, n, φ_c, α and β. They can be determined from granulometric properties, isotropic compression test and conventional triaxial compression tests, as discussed in detail by Bauer [6] and Herle and Gudehus [8].

Shearing of medium dense to dense sand will lead to a peak state, followed by strain-softening before a critical state is reached. At the peak state, stress changes vanish under a continuing dilatation so that $\overset{\circ}{\boldsymbol{\sigma}} = \mathbf{0}$, $\dot{e} > 0$ and $f_d < 1$. The stress and void ratio at the peak state fulfill the following condition [9]:

$$\psi_p(\hat{\boldsymbol{\sigma}}, e) = \frac{f_d^2}{\hat{a}^2}[\eta^2 \,\|\hat{\boldsymbol{\sigma}}\|^2 + (2\eta+1)\,\|\hat{\boldsymbol{\sigma}}^d\|^2] - 1 = 0, \tag{5}$$

where $\eta = (\hat{a}^2 - \|\hat{\boldsymbol{\sigma}}^d\|^2)/(\hat{a}^2 + \|\hat{\boldsymbol{\sigma}}\|^2)$.

Condition of shear bifurcation

According to Rudnicki and Rice [7], the phenomenon of shear localization can be directly associated with the weak discontinuity bifurcation properties of constitutive models. The onset of shear localization in a uniformly deforming material corresponds to a bifurcation point, at which the constitutive relations allow a non-uniform deformation pattern in the form of planar weak discontinuity to enter the solution under continuing equilibrium. Across the weak discontinuity plane the velocity gradient experiences a jump while the velocity and stress fields are continuous. A kinematical condition can be derived as:

$$[[\,\nabla \dot{\boldsymbol{u}}\,]] = \boldsymbol{g} \otimes \boldsymbol{n}\ , \tag{6}$$

where $\boldsymbol{n}$ represents the unit normal vector of the discontinuity plane and $\boldsymbol{g}$ is a characteristic bifurcation vector which must be non-trivial when weak discontinuity occurs. Condition for bifurcation can be obtained by considering continuing equilibrium along the weak discontinuity plane with application of the constitutive relations. An incrementally linear constitutive model usually leads to the nullification of the acoustic tensor [10]. For the present hypoplastic model, which belongs to the category known as incrementally nonlinear constitutive models, the following bifurcation condition has been obtained [9]

$$\psi_b(\hat{\boldsymbol{\sigma}}, e, \boldsymbol{n}) = \tfrac{1}{2} f_d^2[\hat{\boldsymbol{g}} \cdot \hat{\boldsymbol{g}} + (\hat{\boldsymbol{g}} \cdot \boldsymbol{n})^2] - 1 \geq 0 \tag{7}$$

Here $\hat{\boldsymbol{g}} = \boldsymbol{\Pi}^{-1}\boldsymbol{r}$ is a function of the normalized stress $\hat{\boldsymbol{\sigma}}$ and the orientation vector $\boldsymbol{n}$ only, with $\boldsymbol{r}$ and $\boldsymbol{\Pi}$ being defined as $\boldsymbol{r} = -\hat{a}(\hat{\boldsymbol{\sigma}} + \hat{\boldsymbol{\sigma}}^d)\boldsymbol{n}$ and $\boldsymbol{\Pi} \approx (\hat{a}/2)[\mathbf{1} + \boldsymbol{n} \otimes \boldsymbol{n} + (\hat{\boldsymbol{\sigma}}\boldsymbol{n}) \otimes (\hat{\boldsymbol{\sigma}}\boldsymbol{n})]$. Note that function ψ_b depends not only on stress and density state, but also on the orientation vector $\boldsymbol{n}$.

Prediction of failure in biaxial compression and in true triaxial tests

In this section, the mode of failure in granular samples is studied via numerical simulation of strain controlled loading in a granular element initially under isotropic compression. Different initial density and confining pressure are assumed for the tests. Various stress paths that can be achieved with a true triaxial test apparatus are tested. The bifurcation condition is examined along the stress paths with respect to the varying stress and density state and by searching the orientations of the potential shear plane corresponding to the maximum value of function ψ_b. The failure mode is determined based on whether there is a bifurcation point and if the bifurcation point is encountered before the peak point. A peak point is uniquely determined according to Eq. (5). For these element

tests, a set of material parameters calibrated for Karlsruhe sand (Herle and Gudehus, 1999) are used. Initial pressure and density are determined by given values of p_0 and I_{d0}.

Some results obtained from simulation of biaxial compression tests are presented in Figure 1. Figure 1(b) and 1(c) show the evolution of stress ratio and volumetric strain for tests with an initial mean pressure of $p_0 = 1000\text{kPa}$ and different initial density. The bifurcation points and peak points are marked with a cross symbol and an upward triangle symbol. It can be seen that the peak stresses and bifurcation stresses are strongly influenced by the initial density. The axial strain values corresponding to the bifurcation point and peak point increase as the initial density decreases. In all these cases, shear bifurcation condition is met before the reach of the peak. This result indicates that shear localization may always occur in biaxial compression tests and the localized failure is the typical failure mode in such tests.

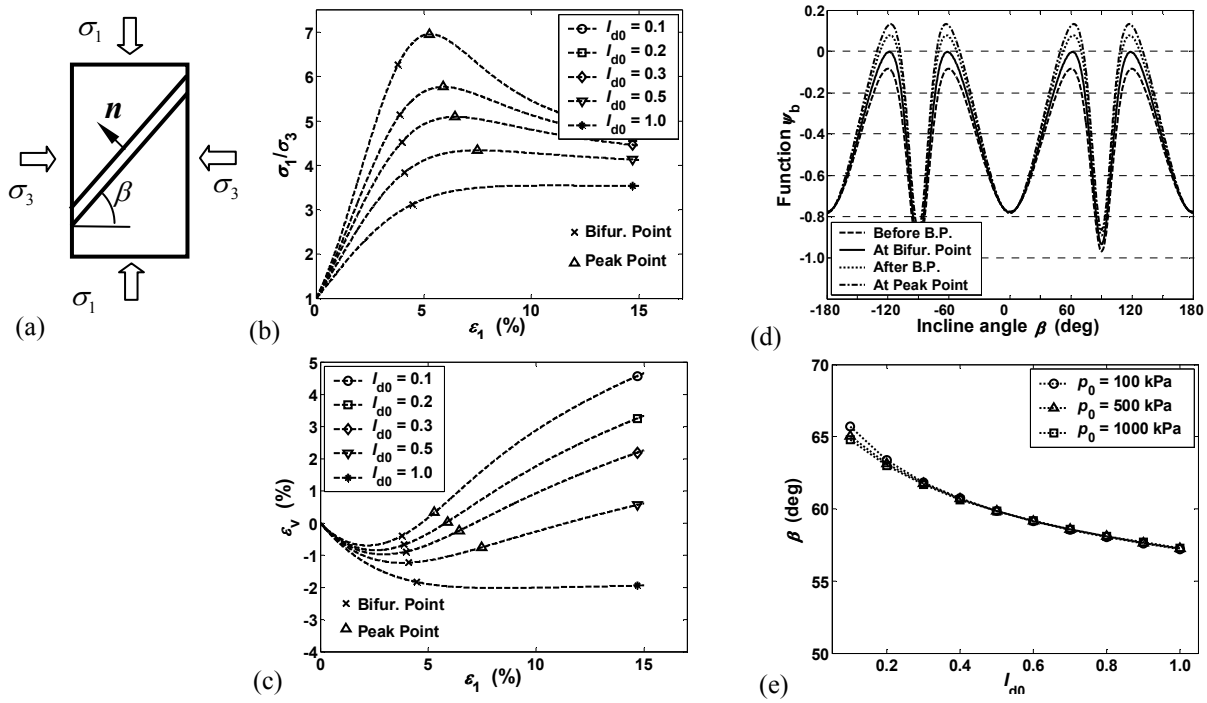

Figure 1. Shear bifurcation in plane strain biaxial compression tests: (a) Sketch of stress state and shear band inclination; (b) Evolution of stress ratio and (b) volumetric strain obtained for $p_0 = 1.0\text{MPa}$; (c) Variation of function ψ_b with respect to the incline angle at different points on the stress path from a typical test; (d) Predicted shear band inclination against initial density index.

In biaxial compression tests, the orientation of shear band or shear plane can be characterized by the inclination angle β as sketched in Figure 1(a). The values of function ψ_b vary with respect to β as shown in Figure 1(d) for a typical test. The inclination of shear plane can thus be determined at the bifurcation point with $\psi_b|_{\max} = 0$ (multi-valued due to symmetry). The predicted inclination of shear band varies between 56° and 66°, depending on the initial density. But it is not very sensitive to the initial mean pressure (Figure 1(e)).

Failure in granular materials under general stress condition is studied by simulating true triaxial tests. Results obtained from the true triaxial tests with constant mean pressure along stress paths defined by a fixed Lode angle are presented in Figure 2. Influence of the intermediate principal stress is represented by the variation of the Lode angle θ. Peak points obtained from different stress paths form a boundary of a closed area in the deviatoric stress plane. This area is larger for the material tested at a denser initial state as shown in Figure 2(a) and 2(b). Also shown in these two plots are bifurcation points (represented by dashed curves).

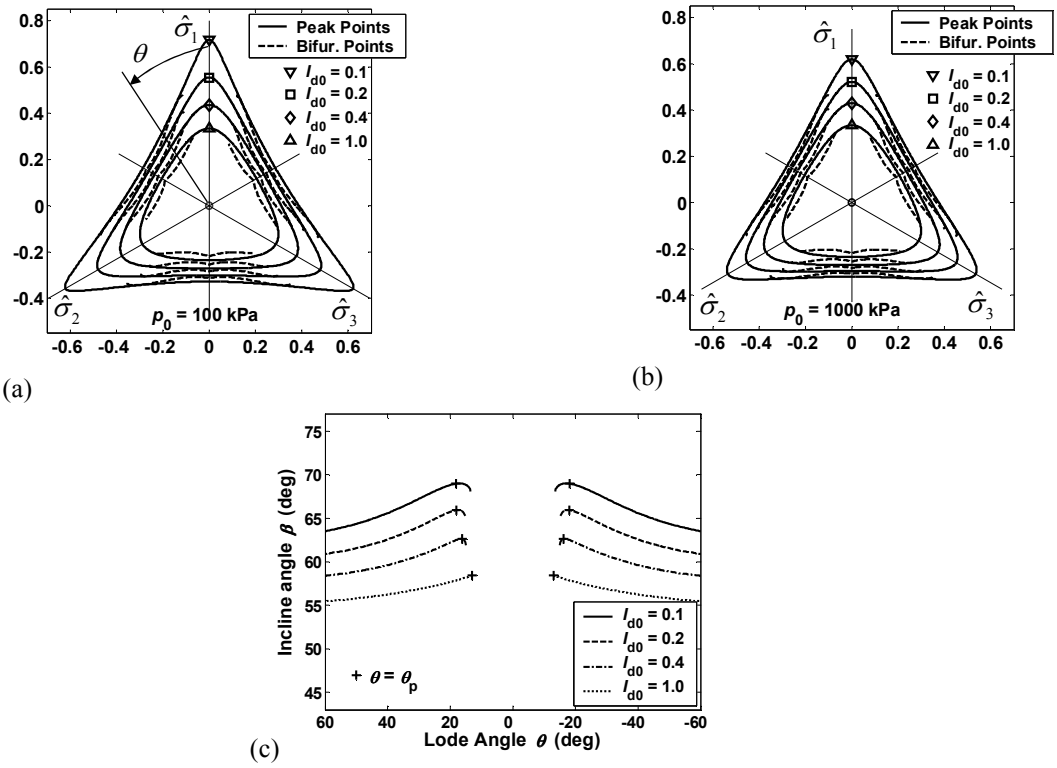

Figure 2. Bifurcation in true triaxial tests: Bifurcation points and peak points represented in deviatoric stress plane for tests with (a) $p_0 = 100\text{kPa}$ and (b) $p_0 = 1000\text{kPa}$. (c) Variation of inclination angle of shear plane with respect to different stress paths defined by the Lode angle.

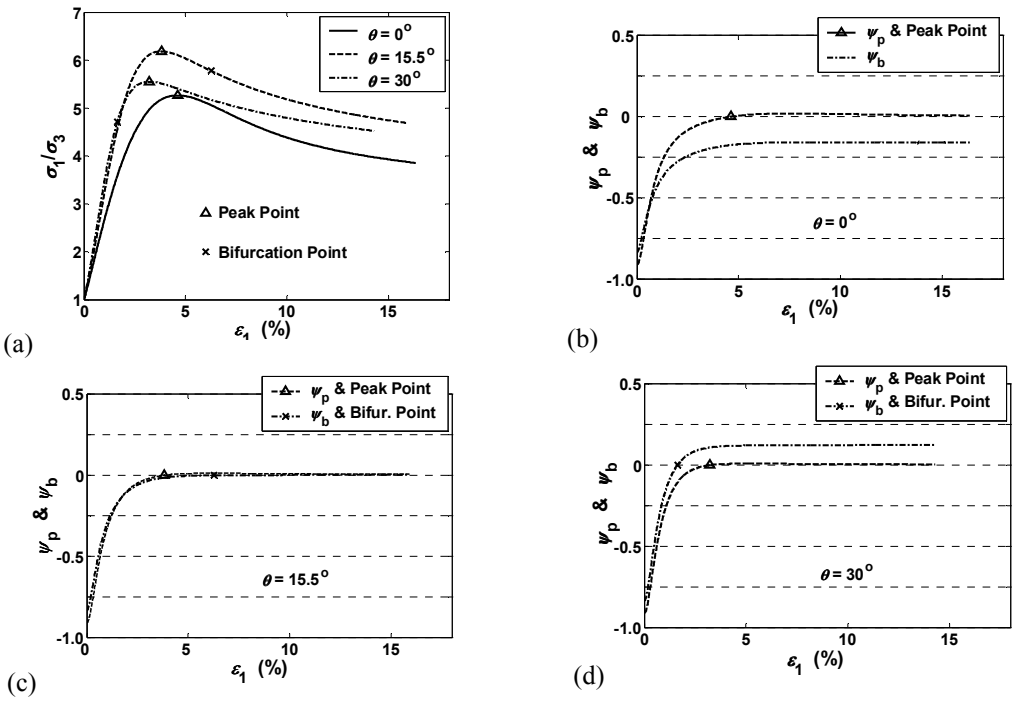

Figure 3. Results from tests with $p_0 = 100\text{kPa}$ and $I_{d0} = 0.3$: (a) Evolution of stress ratio along stress paths of $\theta = 0^\circ, 15.5^\circ$ and 30° with bifurcation points and peak points marked. (b) Bifurcation condition is not met along stress path of $\theta = 0^\circ$; (c) Bifurcation condition is met after the peak along stress path of $\theta = 15.5^\circ$; (d) Bifurcation is met before the peak.

A bifurcation point is found along stress paths with a Lode angle $|\theta| \approx 14^\circ$ (this value varies slightly with initial density) or greater, including the stress path for triaxial extension ($\theta = 30^\circ$). Uniform diffuse failure is predicted in tests along the stress paths with a Lode angle smaller than that, this includes the stress path for triaxial compression (which corresponds to $\theta = 0^\circ$). Stress path for biaxial compression test has a varying Lode angle, which is around 30 degrees at the bifurcation point.

The predicted shear planes are found to have their normal oriented in the direction of the intermediate principal stress σ_2. The orientation of shear plane can therefore be characterized with an inclination angle β defined in a similar way as in biaxial compression tests. The variations of the inclination angle with respect to the Lode angle obtained from tests of various initial densities are presented in Figure 2(c). It can be seen that the inclination angle for shear plane increases with the initial density. For a given initial density, there is a Lode angle $\theta = \theta_p$ which corresponds to a stress path along which the bifurcation point coincides with the peak point. This is marked in Figure 2(c). It also corresponds to the stress path with the greatest shear plane inclination angle.

It is interesting to note that for initially dense or medium dense samples, shear bifurcation may occur in hardening regime before the uniform peak or in softening regime after the uniform peak. For tests along stress paths with $|\theta| < \theta_p$, bifurcation point is encounter in hardening regime before the peak. There are some stress paths with $|\theta| < \theta_p$, shear bifurcation occurs in softening regime after the peak. To demonstrate this fact, the values of functions ψ_p and $\psi_b|_{\max}$ obtained from tests along stress paths of $\theta = 0^\circ, 15.5^\circ$ and 30° with an initial state of $p_0 = 100\text{kPa}$ and $I_{d0} = 0.3$ are presented in Figure 3. Peak points and bifurcation points are marked on stress ratio-strain curves (Figure 3(a)). In triaxial compression test ($\theta = 0^\circ$), shear bifurcation does not occur as indicated by the evolution of function $\psi_b|_{\max}$ shown in Figure 3(b). It can also be seen that the bifurcation condition is met in the softening regime along stress path of $\theta = 15.5^\circ$, and in the hardening regime along the stress path of $\theta = 30^\circ$ (Figure 3(c) and 3(d)). These results are qualitatively in agreement with the experimental observations by Wang and Lade [4].

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